AN ADAPTIVE SLIDING-MODE FUZZY CONTROL (ASMFC) APPROACH FOR A CLASS OF NONLINEAR SYSTEMS

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Keywords: Sliding mode control, Fuzzy controller, TSK fuzzy model, Feedback linearizable systems, Adaptive fuzzy identification

Abstract: This paper uses the concept of sliding-mode control (SMC), as a special approach in nonlinear control theory, in aiding the design of a fuzzy controller. The mathematical specifics of the presented approach are given along with its performance analysis. It was concluded that the new approach with distinctive characteristics holds potential for coping with difficult control problems for a class of complex (generally nonlinear) systems.

1 INTRODUCTION

In the previous literature, fuzzy control, especially adaptive and self-learning fuzzy control, has been successfully applied for complex nonlinear control problems (Driankov, Hellendoorn and Reinfrank, 1993; Passino and Yurkovich, 1998; Wang, 1993; Hwang and Lin, 1992; Takagi and Sugeno, 1983, 1985; Jang, 1992a, 1992b). One of the most attractive features of adaptive fuzzy control is that linguistic knowledge elicited from domain expert or available input-output data set can be conveniently incorporated into the design process of fuzzy controller.

In a sliding-mode controller (SMC), the sliding region is generally a hyper-plane. In the simple case of 2-D, the sliding region is simply a line. Separated by this sliding line, control force is switched to its maximum at one side and minimum at another. In the theory of SMC, it is usually presumed that the SMC controller can switch from one extreme to another extreme arbitrarily fast. Based on this assumption, the trajectory can remain along this line once it reaches it. In practice, nevertheless, it is well known in SMC theory that the trajectory of the system always chatter around this sliding line, rather than sliding strictly along it (Hung, Gao and Hung, 1993; Slotine and Li, 1991). Thus the output of the SMC controller alternates its sign along the switch line.

The synergism of fuzzy control and SMC has also been a hot research topic (Palm, 1992; Palm, Driankov and Hellendoorn, 1996; Palm and Stutz, 2003). One reason, from the perspective of the basic property of a control system—stability property, may be that the mathematically strict stability analysis for a fuzzy controller is hard to establish and guarantee in general cases, contrarily that for a sliding-mode controller can be well resolved. Another advantage offered by the SMC method includes its capability for decoupling high-dimensional systems into a body of lower-dimensional sub-systems to achieve the dimensionality reduction for a complex multi-input multi-output (MIMO) control system (Hung, Gao and Hung, 1993). This advantage may be beneficial for avoiding the curse of dimensionality inherent in a fuzzy inference system (FIS) even with moderately number of input variables (Jang, 1993; Chen and Tsao, 1989).

In this paper, to improve the transient performance of fuzzy controller, the state-space of control system is partitioned into a number of local cells, across individual cell state-space the sliding hyper-plane of SMC controller within its cell is designed separately in an adaptive fashion. The paper is organized in the following way. Firstly some basics of SMC are briefly introduced. In section III, the detailed approach of adaptive sliding-mode fuzzy control (ASMFC) is developed. Finally its performance and unique features are discussed.

Zhang J. and Böhme J. (2004).
AN ADAPTIVE SLIDING-MODE FUZZY CONTROL (ASMFC) APPROACH FOR A CLASS OF NONLINEAR SYSTEMS.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 193-197
DOI: 10.5220/0001124801930197

2 BASICS OF SMC METHOD

Let us consider a class of continuous-time nonlinear dynamical system which is feedback linearizable and of the canonical form:

$$\begin{cases} x^{(n)}(t) = f\left[x^{(n-1)}(t), x^{(n-2)}(t), \cdots, \dot{x}(t), x(t)\right] + bu(t) \\ y(t) = x(t) \end{cases} \tag{1}$$

where $f[\cdot]$ is an unknown continuous function (generally nonlinear), b>0 is the controller gain, $x(t) \in \Re$ is the system's state variable, and $u(t) \in \Re, y(t) \in \Re$ are the input variable and output variable of the system, respectively. Our goal is to force the state vector of the system (1) (where the superscript τ denotes the vector transpose)

$$\mathbf{x}(\mathbf{t}) = \left[x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)\right]^{\tau}$$

to follow a predefined reference trajectory

$$\mathbf{x}_r(\mathbf{t}) = \left[x_r(t), \dot{x}_r(t), \cdots, x_r^{(n-1)}(t)\right]^{\tau}.$$

Define the tracking error vector as the difference between the actual states and desired states, i.e.,

$$\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{x_r}(t) \tag{2}$$

then the control problem can be formalized as: find a control law $u(t)$ such that $\lim_{t \to \infty} \mathbf{e}(t) = 0$.

A candidate of such a control law is

$$u(t) = -F[\mathbf{x}(\mathbf{t}), \Theta] + x_r^{(n)} + \mathbf{m}(t)^{\tau} \mathbf{e}(t) + w(t) \tag{3}$$

where $w(t)$ is an auxiliary control input to be determined, $F[\cdot]$ is a proper function with sufficiently rich parameter set Θ used to well approximate unknown function $f[\cdot]$ in eqn. (1), i.e., $F = \hat{f}$ may be implemented by an adaptive fuzzy model (Jang, 1992a; Jang, 1992b; Jang, 1993), and $\mathbf{m}^{\tau} = [m_n, m_{n-1}, \cdots, 1]$ is an properly chosen vector that controls the performance of the closed-loop system with the control law (3). With this control law, the resulting closed-loop system is a linear one as

$$e^{(n)}(t) + m_1 e^{(n-1)} + \cdots + m_n e(t) = [f(\mathbf{x}(\mathbf{t})) - F(\mathbf{x}(\mathbf{t}), \Theta)] + w(t) \tag{4}$$

Our suggested control approach is formulated as the following procedures:

1. Use a parameterized adaptive fuzzy model to approximate $f[\cdot]$, i.e., adaptively update the parameter vector of fuzzy model such that for $\forall \mathbf{x}(\mathbf{t}) \in \Re^n$ and an upper bound of error $\varepsilon > 0$,
$$|F[\mathbf{x}(\mathbf{t}), \Theta] - f[\mathbf{x}(\mathbf{t})]| \le \varepsilon. \tag{5}$$
2. Apply the SMC approach to design $w(t)$ to guarantee the global stability property of the close-loop system.

Using the standard SMC design approach, define an error measure below:

$$s(t) = \lambda e^{(n-1)}(t) + (n-1)\lambda e^{(n-2)}(t) + \cdots + e \tag{6}$$

where constant $\lambda > 0$. Then the equation $s(t) = 0$ is called a switching surface in space $\Re^n$ on which $\mathbf{e}(\mathbf{t})$ approaches to zero exponentially, i.e., asymptotical tracking performance is achieved.

For simplicity, introduce a kind of differential operator to express the above differential polynomial as

$$s(t) = \left(\lambda + \frac{d}{dt}\right)^{n-1} e(t) = \left[\lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, 1\right] e(t). \tag{7}$$

The control law in eqn. (3) guarantees the system state trajectory, whatever the initial condition may be, will approach and subsequently maintain on the sliding surface $s(t) = 0$, if the condition

$$s(t) \cdot \dot{s}(t) \le -\eta |s(t)| \tag{8}$$

holds. Here η is a positive constant, which restricts that the state trajectory hits the sliding surface in a finite time (Hung, Gao and Hung, 1993; Slotine and Li, 1991). Thus $\mathbf{e}(\mathbf{t}) \to 0$ exponentially with a time constant $(n-1)/\lambda$.

Taking

$$\mathbf{m}^{\tau} = \left[0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, \lambda\right] \tag{9}$$

differentiating eqn. (6), and inserting eqn. (5) into it yield

$$\dot{s}(t) = (f(\mathbf{x}(\mathbf{t})) - F(\mathbf{x}(\mathbf{t}), \Theta)) + w(t). \tag{10}$$

and

$$\frac{d|s(t)|}{dt} = [(f(\mathbf{x}(\mathbf{t})) - F(\mathbf{x}(\mathbf{t}), \Theta)) + w(t)]\operatorname{sgn}(s). \tag{11}$$

Then condition (8) always maintains if we choose

$$w(t) = -(\varepsilon + \eta)\operatorname{sgn}(s). \tag{12}$$

By substituting eqns. (9) and (11) into eqn. (3), eventually we have the control law

$$u(t) = -F(\mathbf{x}, \Theta) + x_r^{(n)} + \left[0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, \lambda\right] e(t) - (\varepsilon + \eta)\operatorname{sgn}(s). \tag{13}$$

then the closed-loop system (4) can asymptotically track the reference state trajectory specified beforehand with guarantee of global stability property (Jang, 1992b).

3 CASE STUDY: A 2-D FEEDBACK LINEARIZYBLE NONLINEAR SYSTEM

Consider a 2nd-order system described by state-space equation

$$\ddot{x}(t) = f[\dot{x}(t), x(t)] + bu(t) \quad (14)$$

One feasible control law of SMC for 2^{nd}-order system eqn. (14) may be chosen as:

$$u = -k \cdot g_{\phi}(s) + v \quad (15)$$

where v is an equivalent control used when the system state lies in the sliding mode, constant $k > 0$ represents the maximum output of SMC controller. According to eqn. (6), the switching hyper-plane is

$$s = \dot{e} + \lambda e = 0. \quad (16)$$

There are many ways to define $g_{\phi}(s)$ in eqn. (15) for different purposes. Three candidate functions for define $g_{\phi}(s)$ are given here:

1. Sign function, i.e.,

$$g_{\phi}(s) = \text{sgn}(s) = \begin{cases} -1, & \text{for } s < 0 \\ 1, & \text{for } s > 0 \end{cases} \quad (17)$$

Introduction of the sign function $\text{sgn}(s)$ often causes chattering problem for a SMC controller. One way to alleviate the problem is to use another nonlinear function below.

2. Saturation function, i.e.,

$$g_{\phi}(s) = sat(\frac{s}{\phi}) = \begin{cases} \frac{s}{\phi}, & \text{for } \left|\frac{s}{\phi}\right| \le 1 \\ \text{sgn}(\frac{s}{\phi}), & \text{for } \left|\frac{s}{\phi}\right| > 1 \end{cases} \quad (18)$$

where ϕ is a constant that determines the width of the boundary layer around the switching surface.

In actuality, the control law resulting from this selection of $g_{\phi}(s)$ is a continuous approximation of the ideal relay control (Hung, Gao and Hung, 1993; Slotine and Li, 1991). Another possible variant is as follows.

3. Hyperbolic tangent function, i.e.,

$$g_{\phi}(s) = \tanh(\frac{s}{\phi}) \quad (19)$$

In all the above three cases, provided sufficiently large k, SMC controller of form (15) has been shown to be asymptotically stable (Hung, Gao and Hung, 1993; Slotine and Li, 1991).

For a 2-D system, the controller structure and the corresponding control surface are illustrated in Figure 1.

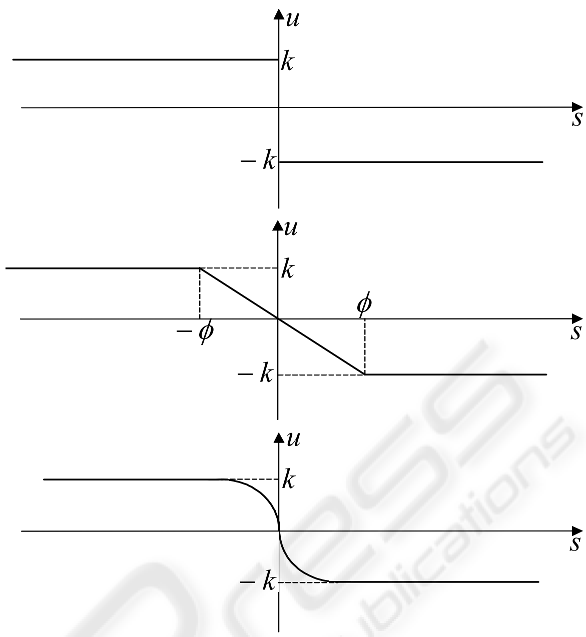

Figure 1: Three examples of SMC controllers in 2-D case.

4 ADAPTIVE SLIDING-MODE FUZZY CONTROL (ASMFC)

From the perspective of optimal control theory, SMC falls into the category of time sub-optimal control. As is well known in optimal control theory, the result of a time optimal control problem for a regulator with set-point input is a type of *Bang-Bang control* with respect to a nonlinear switching curve shown in Figure 2. Figure 2 also illustrates the control surface resulting from the nonlinear switching function.

Since a fuzzy inference system (FIS) can integrate and coordinate different control algorithms in a seamless way by using fuzzy decision-making logic according to the available fuzzy knowledge and data base, we can directly incorporate the design conception of SMC into the development of a fuzzy controller without causing any undesirable effects. In Takagi-Sugeno-Kang (TSK) fuzzy model, the output of each fuzzy *if-then* rule is explicitly and generally expressed as a linear combination of controller inputs plus a constant term (Takagi and Sugeno, 1983; Takagi and Sugeno, 1985; Hoffmann and Nelles, 2001).

In fact, the rule output can also be a more generally nonlinear function of the rule input variables. In this section, we express the output of each fuzzy rule, i.e., the control output when the states enter into a local cell space, a switching function of state vector. In this way, we carefully design a new adaptive fuzzy controller by borrowing the notion of SMC, which actually leads to an adaptive sliding-mode fuzzy control approach presented in this short paper. In our approach, the parameters in the output of each fuzzy rule that covers different cell of state space are determined by different SMCs that operate over the corresponding cell state-space, whose concept was proposed by Chen and Tsao (1989).

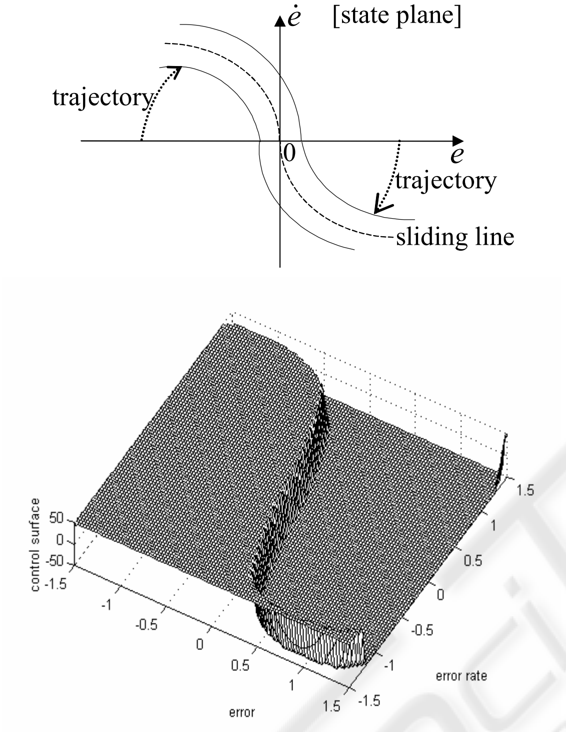

Figure 2: Nonlinear switching curve and control surface.

For the ASMFC controller, the error and the rate of error are taken as the its inputs. Its *l*-th fuzzy *if-then* rule in the rule base takes the format of

R^l : if e is F_1^l and $\dot{e}$ is F_2^l,

$$\text{then } u^l = ksat\left(\frac{\dot{e}+\lambda_l e + c_l}{\phi_l}\right) \qquad (20)$$

where F_1^l and F_2^l represent the linguistic label, i.e., input fuzzy set, which can be characterized by proper parameterized membership function defined over the corresponding universe of discourse.

With only a small number of *if-then* rules, ASMFC can generate a complex nonlinear switching function, which is difficult to achieve by standard SMC method. Also note that the rule output in expression (20) need not to be a saturation function, it could be either a sign function or hyperbolic tangential function described before.

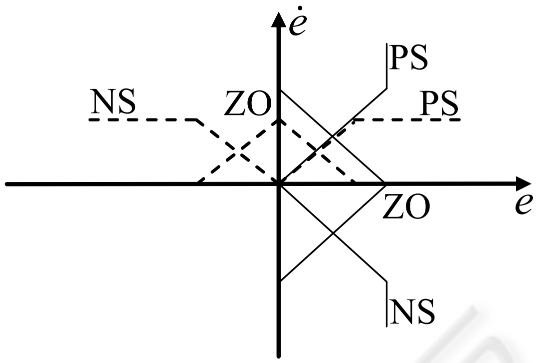

Figure 3: Fuzzification of error e and its rate $\dot{e}$.

In the case of a 2-D system, the switching line can be either a function of e , or a function of $\dot{e}$. In this case only a very small number of fuzzy rule patches are required to cover the switching function of single variable. Therefore in an ASMFC controller the number of *if-then* fuzzy rules is reduced to a reasonable and manageable amount and thus the curse of dimensionality arising from multi-variable fuzzy controller can be avoided.

To approximate the switching curve shown in Figure 2, we assign 3 linguistic labels (described by their own properly-parameterized membership functions) to input variable e and $\dot{e}$, respectively. The fuzzification of e and $\dot{e}$ is illustrated in Figure 3, where symbols 'ZO', 'NS', 'PS' represent the corresponding linguistic terms 'zero', 'negative small', and 'positive small', respectively. In this case, we partition the universe of discourse of both input variables into 3 overlapping fuzzy subsets, and hence we have 9 fuzzy rules in the rule-base of fuzzy controller and the state space is partitioned into 9 localized cells. Extensive simulation experiments have demonstrated that the 9-rule base suffices to well approximate the desired switching curve. The control surface and sliding surface are shown in Figure 4.

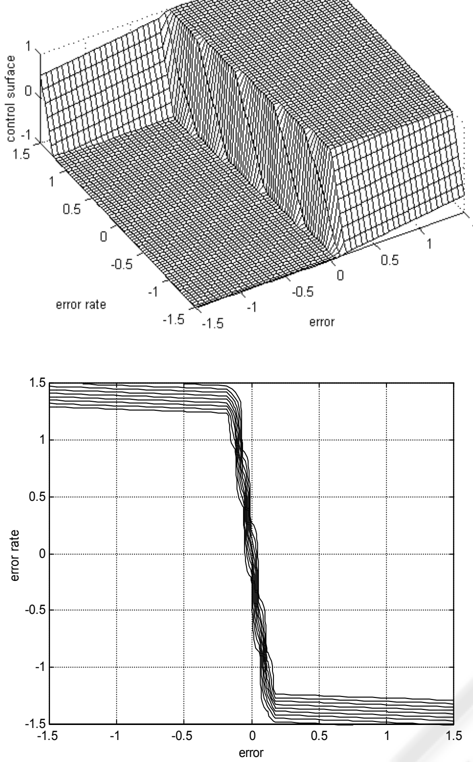

Figure 4: Control surface and sliding surface for an ASMFC controller with only 9 fuzzy if-then rules.

5 CONCLUSIONS

In this short article an adaptive sliding mode fuzzy control approach is proposed with some analysis of its property for addressing nonlinear control problems. This approach combines the concept of a branch of nonlinear control theory, namely SMC, and that of a fuzzy inference system that can uniformly approximate any nonlinear function with arbitrary degree of accuracy. In this sense, global stability of the control system designed by this approach can be mathematically established (Jang, 1992b; Hung, Gao and Hung, 1993; Slotine and Li, 1991). Nevertheless, be aware that the presented approach seems only applicable to the class of nonlinear systems over which the feedback linearization technique can be performed.

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