ON THE EFFICIENCY OF A CERTAIN CLASS OF NOISE REMOVAL ALGORITHMS IN SOLVING IMAGE PROCESSING TASKS

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Keywords: Noise removal, image processing, regression, filtering, multiresolution analysis, wavelet transform, statistical image restoration techniques, least mean squares techniques

Abstract: The investigated noise removal algorithms are HRBA, HSBA, HBA, AMVR, PNRA, MMSE, MNR, MNR2 and NFPCA. The multiresolution support provides a suitable framework for noise filtering and for restoration purposes by noise suppression. The techniques used in the paper are mainly based on the statistically significant wavelet coefficients specifying the support. The performed tests reveal that the use of the multiresolution support proves powerful and offers a versatile way to handle noise of different classes of distributions.

1 INTRODUCTION

The restoration techniques are usually oriented toward modeling the type of degradation in order to infer the inverse process for recovering the original image. Some of the techniques (HRBA, HSBA, HBA, PNRA) presented in the sequel aim to improve the quality of the filtered images using a certain amount of information globally extracted from the whole set of samples consisting of filtered and non-filtered ones. The AMVR algorithm allows the removal of the normal/uniform noise whatever the mean of the noise is.

The multiresolution support provides a suitable framework for noise filtering and for restoration by suppressing the noise. The MNR technique is essentially based on the statistical significance of the wavelet coefficients specifying the support.

An important feature of neural networks is the ability they have to learn from their environment, and, through learning to improve performance in some sense. In the following we restrict the development to the problem of feature extracting unsupervised neural networks derived on the base of the biologically motivated Hebbian self-organizing principle which is conjectured to govern the natural neural assemblies and the classical principal component analysis (PCA) method used by statisticians for almost a century for multivariate data analysis and feature extraction.

Cocianu C., State L., Panayiotis V. and Stefanescu V. (2004).
ON THE EFFICIENCY OF A CERTAIN CLASS OF NOISE REMOVAL ALGORITHMS IN SOLVING IMAGE PROCESSING TASKS.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 320-323
DOI: 10.5220/0001127603200323

2 ALGORITHMS FOR IMPROVING THE QUALITY OF FILTERED IMAGES

The research aimed the comparison of the performances of our restoration algorithms HRBA, HSBA, HBA, PNRA (Cocianu, 2002) and NFPCA against well known algorithms that are currently used for solving this type of problem.

Let X be the given image, $\left\{X_1^{(2)}, X_2^{(2)},..., X_N^{(2)}\right\}$ a sample of the $X^{(\eta)} = X + \eta$, $\eta \sim N(\mu, \Sigma)$ and $\left\{X_1^{(1)}, X_2^{(1)},..., X_N^{(1)}\right\}$ the sample of the filtered random vector $F\left(X^{(\eta)}\right)$. Let $\mu^{(1)} = E\left(F\left(X^{(\eta)}\right)\right)$, $\mu^{(2)} = E\left(X^{(\eta)}\right)$ be the mean vectors and Σ_{11}, Σ_{22} the covariance matrices. The HRBA is (State, 2001).

Step 1. Compute $\left\{X_1^{(1)}, X_2^{(1)},..., X_N^{(1)}\right\}$ by applying the binomial filter to $\left\{X_1^{(2)}, X_2^{(2)},..., X_N^{(2)}\right\}$.

Step 2. For each row $1 \le i \le r$, compute

$$\hat{\mu}^{(p)}(i) = \frac{1}{N}\sum_{k=1}^{N} X_k^{(p)}(i), \quad p = 1,2$$

$$\hat{\Sigma}_{ts}(i) = \frac{1}{N-1}\sum_{k=1}^{N}\left(X_k^{(t)}(i) - \hat{\mu}^{(t)}(i)\right)\left(X_k^{(s)}(i) - \hat{\mu}^{s(s)}(i)\right),$$

$t, s = 1,2$

Step 3. For each $1 \le i \le r$, compute $T\left(\hat{\mu}^{(1)}(i)\right)$ by applying a threshold filter to $\hat{\mu}^{(1)}(i)$

Step 4. Compute

$$\overline{X}(i) = T\left(\hat{\mu}^{(1)}(i)\right) + \rho\hat{\Sigma}_{12}(i)\left(\hat{\Sigma}_{22}(i)\right)^{+}\left(\left(\hat{\mu}^{(2)}(i)\right) - \hat{\mu}^{(1)}(i)\right)$$

where ρ is a noise-preventing suitable constant.

The HSBA is (State, 2001),

Step 1. Compute the sample $\left\{X_1^{(1)}, X_2^{(1)},..., X_N^{(1)}\right\}$ as described in Step 1 of the HRBA

Step 2. For each $1 \le i \le r$, do Step 3 until Step 7

Step 3. Compute $\hat{\mu}^{(p)}(i) = \frac{1}{N}\sum_{k=1}^{N} X_k^{(p)}(i)$, $p = 1,2$

$$\hat{\Sigma}_p(i) = \frac{1}{N-1}\sum_{k=1}^{N}\left(X_k^{(p)}(i) - \hat{\mu}^{(p)}(i)\right)\left(X_k^{(p)}(i) - \hat{\mu}^{(p)}(i)\right)^T$$

Step 4. Compute $S_w(i) = \hat{\Sigma}_1(i) + \hat{\Sigma}_2(i)$ and

$$S_m(i) = S_w(i) + \left(\hat{\mu}^{(1)}(i) - \hat{\mu}^{(2)}(i)\right)\left(\hat{\mu}^{(1)}(i) - \hat{\mu}^{(2)}(i)\right)^T$$

Step 5. Compute the eigenvalues $\left(\lambda_1(i),...,\lambda_n(i)\right)$ and the eigenvectors $\left(\Phi_1(i),...,\Phi_n(i)\right)$ of $S_m(i)$. Select the largest t eigenvalues and let $\Lambda_t(i) = diag\left(\lambda_1(i),...,\lambda_t(i)\right)$, $\Phi^{(t)}(i) = \left(\Phi_1(i),...,\Phi_t(i)\right)$

$$K(i) = \left(\Phi^{(t)}(i)\Lambda_t^{-\frac{1}{2}}(i)\right)^T S_w(i)\left(\Phi^{(t)}(i)\Lambda_t^{-\frac{1}{2}}(i)\right)$$

Step 6. Compute $\Psi(i)$ the matrix having the columns the unit eigenvectors of $K(i)$. The most informative features responsable for the class separability are given by $A(i) = \Phi^t(i)\Lambda^{-\frac{1}{2}}(i)\Psi(i)$.

Step 7. Compute the row $\overline{X}(i)$ of the restored image $\overline{X}$, $\overline{X}(i) = T\left(\hat{\mu}^{(1)}(i)\right) + \sigma A(i)A^{+}(i)T\left(\hat{\mu}^{(1)}(i)\right)$, where σ is a noise-preventing constant, $0 < \sigma < 1$.

The HBA image restoration algorithm is based on the Bhattacharyya distance, (State, 2001)

Step 1. Compute the sample $\left\{X_1^{(1)}, X_2^{(1)},..., X_N^{(1)}\right\}$ as described in Step 1 of HRBA

Step 2. For each $1 \le i \le r$, do Step 3 until Step 7

Step 3. Compute $\hat{\mu}^{(p)}(i) = \frac{1}{N}\sum_{k=1}^{N} X_k^{(p)}(i)$, $p = 1,2$

$$\hat{\Sigma}_p(i) = \frac{1}{N-1}\sum_{k=1}^{N}\left(X_k^{(p)}(i) - \hat{\mu}^{(p)}(i)\right)\left(X_k^{(p)}(i) - \hat{\mu}^{(p)}(i)\right)^T$$

Step 4. Compute the Bhattacharyya distance $\mu\left(\frac{1}{2}, i\right)$.

Step 5 Compute the eigenvalues $\left(\lambda_1,...,\lambda_c\right)$ and the eigenvectors Φ_i, $i = 1, c$ of $\hat{\Sigma}_2^{-1}(i)\hat{\Sigma}_1(i)$.

Step 6 Arrange the eigenvalues in the decreasing order of $\frac{\left[\Phi_s^T\left(\hat{\mu}^{(2)}(i) - \hat{\mu}^{(1)}(i)\right)\right]^2}{1 + \lambda_s} + \ln\left(2 + \lambda_s + \frac{1}{\lambda_s}\right)$ and select the feature matrix $M(i) = \left(\Phi_1,...,\Phi_k\right)^T$.

Step 7. Compute

$$\overline{X}(i) = T\left(\hat{\mu}^{(1)}(i)\right) + \sigma M(i)M^T(i)T\left(\hat{\mu}^{(1)}(i)\right),$$

where σ is a noise-preventing constant, $0 < \sigma < 1$.

The PNRA is based on the innovations algorithm of the best linear predictors . Let X_0 be a $R \times C$ image, $R \ge 1$, $C \ge 1$, whose pixels are colored on a N level gray scale. We assume that the input is represented by a sample $\{X_l^{(1)}, l=1,...,n\}$ on $X^{(1)}=X_0+\eta$ (State, 2000). Using a binomial mask B and the contrast enhancement operator P resulted by Lagrange interpolation (Cocianu, 1997), we get the variants $X^{(2)}=P(B(X^{(1)}))$, $X^{(3)}=P(X^{(2)})$. For each $r=1,...,R$ and $c=1,...,C$ we define (State, 2000),

$$z^{(i)} = \left(X^{(i)} - E\left(X^{(i)}\right)\right), \; i = 1,2,3.$$

Let $\{X_t\}_{t \in Z}$ be a zero mean stochastic process, $K(i,j)$ its autocorrelation function and $\hat{X}_{n+1} = \begin{cases} 0, & n = 0 \\ P_{H_n} X_{n+1}, & n \ge 1 \end{cases}$, $v_n = \left\| X_{n+1} - \hat{X}_{n+1} \right\|^2$. If, for any $n \ge 1$, $[K(i,j)]_{i,j=1,...,n}$ is a non degenerated matrix, then we get (Brockwell, 1985)

$$\begin{cases} v_0 = K(1,1) \\ \theta_{n,n-k} = v_k^{-1}\left(K(n+1,k+1) - \sum_{j=0}^{k} \theta_{k,k-j}\theta_{n,n-j} v_j \right), \\ \quad k = \overline{0,n-1} \\ v_n = K(n+1,n+1) - \sum_{j=0}^{k} \theta^2{}_{n,n-j} v_j \end{cases}$$

Aiming the removal of the residual noise, we apply to each sample the transform,

$$z_i^{(4)} = z_i^3 + (-1)^i \frac{M}{p}, \; p > 1, \; i = \overline{1,n}$$

In order to develop an approximation scheme for $z_l^{(4)}$, l=1,...,n, we note that (State, 2000),

$$K(4,j) = K(3,j) + \frac{1}{n-1}\alpha \sum_{i=1}^{n} (-1)^i z_i^{(j)} ,$$

where $\alpha = \dfrac{M}{p}$. The description of the AMVR algorithm is (Cocianu, 2002),

Input The $L \times C$ image Y representing a normal/uniform disturbed version of the initial image X, $Y(l,c) = X(l,c) + \eta_{l,c}^0$, $1 \le l \le L, 1 \le c \le C$, where $\eta_{l,c}^0$ is a sample of the random variable $\eta_{l,c}$ distributed either $N(\mu_{l,c}, \sigma_{l,c}^2)$ or $U(\mu_{l,c}, \sigma_{l,c}^2)$.

Step 1. Generate the sample of images $\{X_1, X_2, ..., X_n\}$, where

$$X_i(l,c) = Y(l,c) - \eta_{l,c}^i, \; 1 \le l \le L, 1 \le c \le C$$

and $\eta_{l,c}^i$ is a sample of the random variable $\eta_{l,c}$.

Step 2. Compute

$$\overline{X}(l,c) = \frac{1}{n}\sum_{i=1}^{n} X_i(l,c), \; 1 \le l \le L, 1 \le c \le C .$$

Step 3. Compute the estimation $\hat{X}$ of X using the adaptive filter MMSE, $\hat{X} = MMSE(\overline{X})$.

The multiresolution support provides a suitable framework for noise filtering and for restoration with noise suppression. The procedure used is to determine statistically significant wavelet coefficients and from this to specify the multiresolution support, therefore a statistical image model is used as an integral part of the image processing. The support is used subsequently to hand-craft the filtering processing. The MNR algorithm is (Stark, 1995),

Input: The image X_0, the number of the resolution levels p and the heuristic thresold k.

Step 1. Compute the image variants $\{X_j\}_{j=1,p}$ and the wavelet coefficients using the "À Trous" algorithm (Stark, 1995)

$$X_j(r,c) = \sum_l \sum_k h(l,k) X_{j-1}\left(r + 2^{j-1}l, c + 2^{j-1}k\right)$$

$$\omega_j(r,c) = X_{j-1}(r,c) - X_j(r,c).$$

Step 2. Apply the test: $\omega_j(r,c)$ is significant if and only if $|\omega_j(r,c)| \ge k\sigma_j$, for $j = 1,..., p$

Step 3. Compute the restored image,

$$\tilde{X}(r,c) = X_p(r,c) + \sum_{j=1}^{p} g(\sigma_j, \omega_j(r,c))\omega_j(r,c).$$

In the following, we present a generalization of the MNR algorithm based on the multiresolution support set for noise removal in case of arbitrary mean (Cocianu, 2003). Let g be the original "clean" image, $\eta \sim N(m,\sigma^2)$ and the analyzed image $f = g + \eta$. The sampled variants of f, g and η obtained using the two-dimensional filter φ are

$c_0(x,y) = \langle f(l,c), \varphi(x-l, y-c) \rangle$,

$I_0(x,y) = \langle g(l,c), \varphi(x-l, y-c) \rangle$,

$E_0(x,y) = \langle \eta(l,c), \varphi(x-l, y-c) \rangle$, $c_0 = I_0 + E_0$.

The wavelet coefficients computed by the algorithm "À Trous" are $\omega_j^{c_0}(x,y) = \omega_j^{I_0}(x,y) + \omega_j^{E_0}(x,y)$, where $\frac{1}{2}\psi\left(\frac{x}{2}\right) = \phi(x) - \frac{1}{2}\phi\left(\frac{x}{2}\right)$. For any pixel (x,y), we get $c_p(x,y) = I_p(x,y) + E_p(x,y)$.The mean of the noise can be decreased using the following algorithm.

Step1. Determine the images $E^{(i)}$, $1 \le i \le n$, by superimposing noise sampled from $N(m,\sigma^2)$ on the "white wall" image.

Step2. For all j, $1 \le j \le p$, compute c_j, $E_j^{(i)}$, $1 \le i \le n$ and the coefficients $\omega_j^{c_0}$, $\omega_j^{E^{(i)}}$ using the "À Trous" algorithm.

Step 3. Compute the image $\tilde{I}$ by,

$$\tilde{I}(x,y) = \frac{1}{n}\sum_{i=1}^{n} \Big[c_p(x,y) - E_p^{(i)}(x,y) + \\ + \sum_{j=1}^{p} \left(\omega_j^{c_0}(x,y) - \omega_j^{E^{(i)}}(x,y) \right) \Big] .$$

Step 4. Compute a variant of the original image I_0 using the multiresolution filtering based on the statistically significant wavelet coefficients.

An alternative approach in solving image restoration task can be performed by PCA neural network. The idea is to use features extracted from the noise in order to compensate the lost information and improve the quality of images. The NFPCA algorithm is presented in the following. Let I^0 be a

RxC matrix ($C = nC_1$, $2 \le n < C$) representing the initial image of *L* gray levels and *I* the distorted variant resulted from I^0 by superimposing noise $N(0,\Sigma)$, $\forall i = 1,...,R$, $k = n(j-1),...,nj$, $j = 1,...,C_1$, $I_{i,j}(k) = I^0_{i,j}(k) + \eta(k)$. The restoration process of the image *I* is described as follows.

Step 1. Compute the image *I'* by decorrelating the noise component, $I'_{i,j} = \Phi^T I_{i,j} = \Phi^T I^0_{i,j} + \eta'$, $\eta' = \Phi^T \eta \sim N(0,\Sigma')$, $\Sigma' = \Phi^T \Sigma \Phi = \Lambda$, where

$\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_n\}$.

Step 2. The noise component η' is removed for each pixel *P* of the image *I'* using the multirezolution support of *I'* by the labeling method of each wavelet coefficient of P, resulting *I"*.

$I''_{i,j} = MST(I'_{i,j}) \cong \Phi^T I^0_{i,j}$, $\forall i = 1,...,R$, $j = 1,...,C_1$.

Step 3. An approximation $\tilde{I} \cong I^0$ of the initial image I^0 is produced by applying the inverse transform of T_{Φ^T} to *I"*.

3 COMPARATIVE ANALYSIS ON THE PERFORMANCE OF THE NOISE REMOVAL ALGORITHMS

A series of experiments were performed, different 256 gray level images being preprocessed aiming the contrast enhancement, increasing enlightens and noise removing by filtering them. Our experiments use the averaging and respectively binomial filtering techniques. The parameters involved in the mentioned algorithms were tuned taking into account the following factors: the distortion degree of the inputs, the particular smoothing filter, the volume of the resulting accepted data (Cocianu, 2002).

A synthesis of the comparative analysis on the quality and efficiency corresponding to the restoration algorithms presented in the paper is supplied in Table 1, Table 2, Table 3 and Table 4.

Table 1

Restoration algorithm	Mean error/pixel N(30,150)	Mean error/pixel N(50,200)
Mean	9.422317	12.346784
HRBA	9.333114	11.747860
HSBA	9.022712	11.500245
HBA	9.370968	11.484837

Table 2

Restoration algorithm	Mean error/pixel N(50,200)	Mean error/pixel N(90,250)
Mean	12.346784	102.528893
The innovation algorithm	11.647346	94.912895

Table 3

Restoration algorithm	Type of noise	Mean error/pixel
MMSE	U(40,70)	50.58
AMVR		8,07
MMSE	N(50,100)	46.58
AMVR		9.39
MNR2		12.23
NFPCA		10.67

Table 4

Restoration algorithm	Type of noise	Mean error/pixel
MNR(h_1)	N(0,100)	11.6
MNR(h_2)		9.53
MNR(h_1)	N(0,200)	14.16
MNR(h_2)		11.74

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