STATE OBSERVER FOR NONLINEAR SYSTEMS: APPLICATION TO GRINDING PROCESS CONTROL

Seraphin C. Abou & Thien-My Dao
Mechanical / Manufacturing Engineering Department
Ecole de Technologie Supérieure
1100, rue Notre-Dame West
Montreal, (Quebec), H3C 1K3, Canada

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Abstract: Due to the measurement problems encountered in mineral processes, observers are appropriate ingredients of advanced model based control algorithm. The measurement problem can be solved by designing nonlinear observer. This paper discusses the way in which a state observer may be designed to control a special class of nonlinear systems. Focus is put on the pertinent applicability of the scope of these techniques, to control the dynamics of mills in mineral processes. The approach uses a small number of parameters to control the mill power draw affected by sudden changes within the system. It provides with principles and ability of the system to adapt to changing circumstances due to intermittent disturbances (like for instance changes in hardness of the raw material). Performance and stability analysis was developed. Using a generalised similarity transformation for the error dynamics, it is shown that under boundedness condition the proposed observer guarantees the global exponential convergence of the estimation error. This way, the nominal performance of the process is improved but the robust stability is not guaranteed to fully avoid the mill plugging.

1 INTRODUCTION

Grinding plants never operate at steady state but rather at perpetual transient states due to a variety of disturbances. The mathematical model was addressed in the way that combines disturbance parameters with material physical properties. It satisfies sufficient conditions which lead to determine the system at any instant in time.

In mineral processes, the application of modern model based control algorithms is hampered by the lack of accurate and cheap on-line sensors. The design of state observers, which reconstruct states out of a limited set of measurements, is a possible approach for dealing with the measurement problem. Due to the (time varying) nonlinear behaviour of grinding systems, the measurement problem can only be solved by designing nonlinear observers.

In general, observers design methodologies are based on (i) exact linearisation, (ii) local linearisation in original coordinates, (iii) local linearisation in observer coordinates, and (iv) high gain methods are considered (Misawa, 1989). Due to the process uncertainty, inherent in mineral processing, applicability and robustness analysis of the nonlinear observers have been performed. The stability properties analysed are with respect to zero, which is equilibrium for the proposed system. In this sense, our main restriction on the nominal system is that the subsystem be globally stable with variable viewed as a virtual control input. As a case study, wet grinding in continuous and fed-batch operation mode considered is described in Section 2. In Section 3 observer design is discussed in general while simulation results are presented in Section 4. The observer performance analysis is discussed in general in Section 5.

2 SYSTEM DESCRIPTION

A wet grinding shown in Fig.1 or dry grinding (cement processing) has been developed with the objective of studying the effects of many variables on particle size reduction in continuous grinding processes. Detailed phenomenological model that describes the charge behaviour has been developed and validated against real data (Abou, 1998).

Indeed, mineral processes present non-linear/chaotic dynamic behaviour. Considerable efforts have been developed in controlling such system, (Abou, 1997), (Weller, 1980). In (Abou, 1998), a comprehensive

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model integrates the physical mechanisms governing mineral processes and a fundamental understanding of the charge behaviour was expressed. It was pointed out that grinding media collisions and impacts on lifters induced non-linearity in materials breakage process. Due to inappropriate control of the motor charge, important engineering conclusions derived from the charge motion studies (Abou, 1997), recommend a focused study of the influence of the wear of both the grinding media and the lifters on the material size reduction quality. Further Investigation reveals that, an important factor of the poor quality of fine grinding is due to lacks of an appropriate control of the power draw of the mill. This causes increase of energy consumption, and production cost, (Austin, 1990).
To address practical results which could be transferred to industrial level, the key is the development of a practically an accurate grinding circuit control. That is to maximise the manoeuvrability at the low and the high speed rotating stability of ball mills when the material hardness and size or the slurry concentration change.

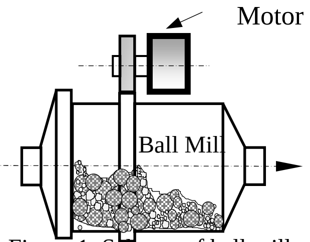

Figure.1: Schema of ball mill powering system

3 GRINDING PROCESS MODELLING

Grinding systems are power-intensive, and even the simplest ones; exhibit complex bifurcation behaviour in going from periodic motion to chaos. Such a complex behaviour has been noticed in the analyses of the dynamics of the charge of ball mill (Abou, 1998). It appears that, simple but nonlinear models are necessary to describe such a system. The main goal is to minimise the consumption energy, avoid strong impact which causes wear of lifters, and rotate the charge with optimal speed for required fine particle quality. Using the cross section of the ball mill shown in Fig.1, the mill action could be shown graphically by considering the change in position of the centre of gravity of ball and particle charge with increasing speed of rotation, Fig.2.

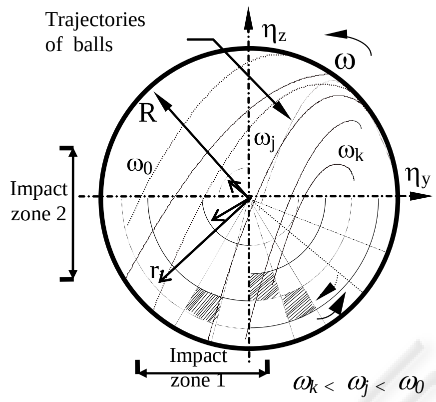

Figure 2: Ball movement with various rotation speed

Notice that the motor load is influenced by the filling percentage, the speed, the mill geometry and other relevant material properties such as stiffness and the coefficient of friction, etc… As shown in Fig.2, theoretical position of the charge at different rotation speed was first derived by (Davis, 1919) based on force balance.
Most research (Austin, 1990), have developed first order model to describe the system. However, their use in practical solutions context has a lack of their dependence on the physical parameters of the system. Since the problem is to develop the grinding process model for control purpose, the main objective in an advanced mathematical model formulation could base on the following basic control flowchart structure, Fig.3 to develop the process behaviour.

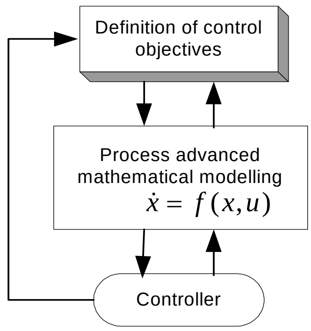

Figure 3: Control system design procedure

Notice that, besides in *batch mode*, grinding circuit can operate in c*ontinuous or fed-batch mode*. Based on the interpretation of the Fig.3, we are interested in the constitutive characteristics of the charge motion defined by the function $f(x, u)$, focusing on the specific parameters that better describe continuous grinding phenomena relationship. From a macroscopic standpoint, the internal breakage model can be formulated taking in account the specifics phenomena of particle transport and size reductions:

$$\frac{\partial}{\partial t}\left[x(.)\right]=\frac{\partial}{\partial z}\left[\Psi_n(.)\frac{\partial m_i}{\partial z}-\Psi_n(.)m_i\right] \quad (1)$$

where, m_i [kg], is particle mass of size ***i***

The left side term of equation (Abou, 1998) expresses the rate of mineral production, while the term at the right side indicates fine particle transport phenomena. In such a process with distributed parameters, function $\Psi_n(.)$ that characterises the particle size reduction, depends on many variables which are absolutely linked to system performance reliability. Therefore, without lacking for the physical sense for the process, we can write:

$$\Psi_n(.)=\Psi_n(x,u) \quad (2)$$

Thus, we note the variation of the volume **V** of the charge is important to the breakage mechanism as much as it is to the transport phenomena, but from a volumetric point of view both phenomena could be treated in a different way. Therefore, the fraction of the total mass broken within a tiny volume of the charge is assumed to be $\sigma(t)$:

$$\sigma(t)=\iiint_V a\rho_c dV \quad (3)$$

where ρ_c is the charge bulk density, a is defined as a mass volume of material of classes ***i,*** so that the flow rate of particle is:

$$\frac{d\sigma}{dt}=\iiint_V \frac{\partial(a\rho_c)}{\partial t}dV+\iiint_V a\rho_c\frac{d(dV)}{dt} \quad (4)$$

In worse case, where we associate to the breakage process, the flux due to the absolute motion of the particle, we could define the flux associate to the fluid. However, as the mass could not be transferred by conduction phenomena, the mass flux therefore, vanishes, so that we could write:

$$\frac{d\sigma}{dt}=-\iint_F \vec{J}_i\cdot d\vec{F}+\iiint_V \vartheta\sigma_p dV \quad (5)$$

where, $\vec{J}_i$:longitudinal diffusion flux of the mass in class ***i.*** ; ϑ :piecewise parameter; σ_p : local fine particle.

Based on equation (5) for the observer design, we assume that the mixing mechanism of powders in ball mills can be well described by a diffusion model and many factors such as the screen plate gape, balls quantity and energy consumed. We deduced that, the process could be defined as a multi-input multi-output nonlinear system of the form:

$$\dot{x}=F(x,u(t)) \qquad x(0)=x_o \quad (6)$$
$$y=g(x)$$

where $x\in\Omega$ is the state, $u\in R^r$ is the control input: $y\in R^m$ is the output; $x(0)$ is the initial state.

It is assumed that, $\forall$ ***t*** the state trajectory $x(t)$ is defined. In addition, the function $F(.)$ is continuously differentiable nonlinear function which represents the dynamics of the process and the disturbances.

We consider four state variables:

- the material grinding rate, $x_1(t)$
- the charge grindability, $x_2(t)$
- the material fineness , $x_3(t)$
- the raw material hardness, $x_4(t)$

The output is set as follows:

$$y(t)=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix}x(t) \quad (7)$$

Interactions of these parameters are not easily identified.

Assumption 1. As the proposed function $F(.)$ in (6) is assumed to be C^1, there exists a C^1 function $\zeta(x)$ such that

$$\dot{x}=F(x,\zeta(x)) \quad (8a)$$

Is globally asymptotically stable.

As result the system (6) could be designed in parameterised nonlinear mapping form as follows:

$$\dot{x}=f(x,y,u)+h(x,y,u) \quad (8b)$$

4 NONLINEAR OBSERVER DESIGN

For a linear dynamical system in equation (9), a well-known Luenburger basic linear observer theory is given as follows:

$$\dot{x}(t)=Ax(t)+Bu(t) \qquad x(t_o)=x_o \quad (9)$$
$$y(t)=Cx(t)$$

where $\dim x=n$; $\dim y=m$; $with\ n>m$; $\dim u=r$

$$\dot{\hat{x}}(t)=A\hat{x}(t)+Bu(t)+K(y(t)-C\hat{x}(t)) \quad (10)$$

The error dynamic equation is: $\dot{e}(t)=\tilde{A}e(t)$

As results, if the following conditions are satisfied:

<u>Conditions</u>

1. matrix **C** has $rank\ m<n$
2. the pair $\{C,A\}$ is completely observable
3. a $(n-m)$ x n transformation matrix **T** exists so that $KC=TA-\tilde{A}T$
4. eigenvalues of the state matrix $\tilde{A}$ have negative real parts.

such that $(A-KC)$ is stable, then the state error $e = x - \hat{x}$ converges to zero. In addition we have:

$$\mathrm{Re}\{\alpha_j(\tilde{A})\} < \mathrm{Re}\{\varphi_i(A)\} \tag{11}$$

In nonlinear system as described in (6), stability would not suffice as for Luenberger observers to guaranty its applicability. Complete controllability is required.

Even though the use of the nonlinear functions can make the observer more efficient, treating the system in the form as in (6) is a challenge. The Jacobian matrices of $F(x,t)$ and $g(x,t)$ in (6) with respect to $\boldsymbol{x}$ taken at $\boldsymbol{\alpha}$ is used to design a matrix $\Gamma(\alpha,t)$ which is a full rank, ($\alpha \in R^n$) and satisfies assumptions below. The system described in (1)-(5) is highly nonlinear, clearly, it is difficult to verify in practice assumptions that $\partial F(x,t)/\partial x$, $\partial g(x,t)/\partial x$ and their respective time derivatives are bounded. Therefore, there is a real incentive for finding possible ways to lessen the complexity of the computation of $\Gamma^{-1}(\hat{x},t)$.

Therefore, by eliminating some redundant terms, we are seeking an improvement of the proposed observer design for a special class of the system described in (6) using (8a) and (8b), for which assumptions 1 hold. Proceeding by analogy to the classical observer design approach in linear case for SISO, it is possible to extend the high gain observer design to MIMO cases, fig.4.

Keeping to the fact that the model described by equations (5) and (6) are exactly the same as another and have theoretical importance, the system could be treated as a special class of nonlinear system when unknown inputs are considered. In this sense, to avoid our investigations becoming extremely restricted circumstances where deficiencies become apparent, we introduced the following representation class to fairly well match the mill behaviour.

$$\begin{aligned} \dot{x} &= Ax + h(x,y,u) \\ y &= Cx \end{aligned} \tag{12}$$

Equation (12) is valid for each state of the system. The sufficient and necessary conditions that characterise the function $h(x,y,u)$ may be found in (Misawa, 1989). Therefore the following conditions are assumed.

Assumption 2: The observer state converges asymptotically to the state of the system, so that the state error is in the neighbourhood of zero. Therefore, the unmodeled dynamics subsystems have relative degree zero.

Assumption 3: The partial derivatives of $h(x,y,u)$ with respect to x and their respective time derivatives are bounded for all x and u, so that:

$$\mathrm{N}_{ij}(x,y,u) = \frac{\partial h_i}{\partial x_j} \tag{13}$$

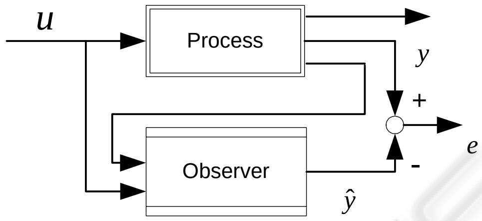

Figure 4: Nonlinear observer structure

We assumed x_o is an equilibrium point corresponding to zero input and output, i.e., $f(x_o)=0$; $h(x_o)=0$ Functions $f(.)$ *and* $h(.)$ are smooth.

We denote by $\delta_{\theta i}$ a diagonal matrix and **A** the constant matrix in Brunowsky form:

$$\delta_{\theta i} = diag(\frac{1}{\theta_i}, \frac{1}{\theta_i^2}, ..., \frac{1}{\theta_i^k}) \tag{14}$$

$$A_i = \begin{bmatrix} 0 & 1 & ... & 0 \\ 0 & 0 & 1 & 0 \\ 0 & . & . & ... \\ 0 & 0 & 0 & .... \end{bmatrix} \tag{15}$$

The design of parameters $\delta_{\theta i}$ must be large enough to compensate the system nonlinearity. Thus we shall assume:

<u>Assumptions 4</u>:

a. Matrix $\Gamma_i(x,y,u)$ is full rank and is defined as follow :

$$\Gamma_i(x,y,u) = \begin{bmatrix} C_i \\ C_i\Psi_i(x,y,u) \\ ..... \\ C_i\Psi_i^{n-1}(x,y,u) \end{bmatrix} \tag{16}$$

where $\Psi_i(x,y,u) = A_i + \mathrm{N}_{ij}(x,y,u)$

b. There exists a positive constant γ which is independent of θ and satisfies condition 4. such that:

$$\sup\left\{\left\|\delta_{\theta i}\dot{\Gamma}_i(x,y,u)\delta_{\theta i}^{-1}\right\|\right\} \le \gamma \tag{17}$$

c. $$\Lambda(x,y,u)=\begin{bmatrix}\varphi_n & 0 & . & 0\\ \varphi_{n-1} & 0 & 0 & 0\\ . & . & . & .\\ \varphi_1 & 0 & 0 & 0\end{bmatrix} \quad (18)$$

A high gain observer design for the class of nonlinear systems in equation (12) can be stated as follows:

$$\dot{\hat{x}}(t)=A_i\hat{x}(t)+h(\hat{x},y,u)+\delta_{\theta i}^{-1}K_i(y-C_i\hat{x}) \quad (19)$$

We define the estimation error as:

$$e(t)=x(t)-\hat{x}(t) \quad (20)$$

One major problem in updating the gain of observer in equation (19) lies in the computation of the symbolic inverse of the matrix $\Gamma_i(x,y,u)$. In many cases, this may become very complicated depending on the nonlinearities involved in the system. More precisely, at times, the matrix $\Gamma_i(\hat{x},y,u)$ may contain excessive number of terms and consequently, the real time implementation of the observer may become tedious (Iwasaki, 1999). This in turn will bring considerable simplification to the expression of the observer's gain.

To this end, $\Gamma_i(x,y,u)$ is consider lower triangular and non singular for all x and u.

$$x=\begin{bmatrix}x_1, & x_2, & ....., & x_n\end{bmatrix}^T \quad (21)$$

Based on equation (5) to express the system described in equation (12), the improvement of the observer in equation (19) is related to the simplification of the gain of the observer by elimination of the redundant terms.

For the grinding system, it is known that the motor load depends on the load within the mill that is tightly related to the input feed (raw material physical properties, tailings flow rates, energy...) and the output (flow rate, particle distribution ...). The evolution of the charge within the mill, (the hold-up) reproduces some unstable behaviour and is formulated as follows:

$$\dot{x}=\begin{cases}\dot{x}_1=A_1x+h_1(x_1,u)\\ \dot{x}_2=A_2x+h_2(x_1,x_2,u)\\ \quad ....\\ \dot{x}_n=A_nx+h_n(x_1,x_2,...,x_n,u)\end{cases} \quad (22)$$

$$y=\begin{cases}y_1=C_1x\\ y_2=C_2x\\ \quad ....\\ y_n=C_nx\end{cases} \quad (23)$$

Based on equations (8), the function $h(x,y,u)$ is as follows:

$$\begin{cases}h_1(x_1,u)=\varepsilon x_1u\\ h_2(x_1,x_2,u)=x_1^{\,2}x_2+x_2u\\ h_3(x_1,x_2,x_3,u)=\varepsilon x_2x_3\exp(u)\\ h_4(x_1,x_2,x_3,x_4,u)=x_3x_4+u\end{cases} \quad (24)$$

Based on equation (15) the matrix $\Lambda_i(x,y,u)$ is chosen such as $\Lambda_i(x,y,u)=L_i(x,y,u)C_i$

Similar to $\Gamma_i(x,y,u)$ we choose the matrix $Q_i(x,y,u)$ as follows:

$$Q_i(x,y,u)=\begin{pmatrix}C_i\\ C_i\tilde{A}_i(x,y,u)\\ ....\\ C_i\tilde{A}_i^{n-1}(x,y,u)\end{pmatrix} \quad (25)$$

where $\tilde{A}_i(x,y,u)=A_i+\Lambda_i(x,y,u)$

Further the similarity matrix transformation for the error dynamics is:

$$M_i(x,y,u)=Q_i^{-1}(x,y,u)\Lambda_i(x,y,u) \quad (26)$$

Based on the following theorem, [4.] equation (26) is valid.

Theorem:

Assume that system (19) satisfies assumptions ***a, b, c***.

There exist $\theta_o>0$ such that $\forall\theta\ge\theta_o$ we have, for all $\hat{x}(0)\in R^n$; $\|\hat{x}(0)-x(0)\|\le\lambda$ where λ is positive [4.]. Therefore equation (19) becomes as follows:

$$\dot{\hat{x}}(t)=A_i\hat{x}(t)+h(\hat{x},y,u)+M_i^{-1}\delta_{\theta i}^{-1}K_i(y-C_i\hat{x}) \quad (27)$$

Then the error dynamics is:

$$\dot{e}(t)=A_ie+h(x,y,u)-h(\hat{x},y,u) -M_i^{-1}(x,y,u)\left[L(\hat{x},y,u)+\delta_{\theta i}^{-1}K_i\right]C_ie \quad (28)$$

Note that the gain matrix K_i is chosen such that matrices $A_i-K_iC_i$ is Hurwitz, i.e. (all the eigenvalues of have negative real parts).

5 SIMULATION RESULTS

From the above equations the material grinding rate within the mill, $x_1(t)$; the charge grindability (i.e., total of material grinded per unity of energy), $x_2(t)$; the material fineness, $x_3(t)$ and the raw material hardness, $x_4(t)$ are presented as below.

By an easy manipulation of non linear equations we could choose conveniently the steady-state values. Others values are imposed from the model (Nijmeijer, 1990). Practical problem observed on industrial milling circuit is that, large changes of the material feed hardness causes instability in the system controlling. The values of the various coefficients in the model have been tuned in such a way that the model step responses fit with experimental step responses. Thirteen tonnes of material was initially loaded in the mill.

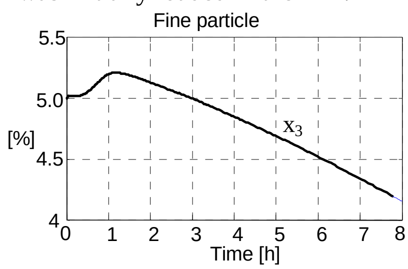

Figure 5: Percentage of fine particle

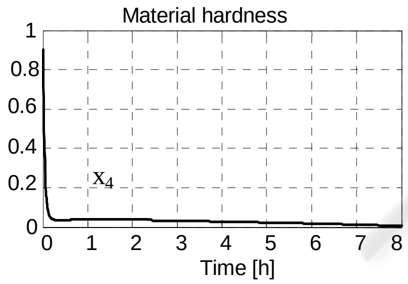

Figure 6: Material hardness variation

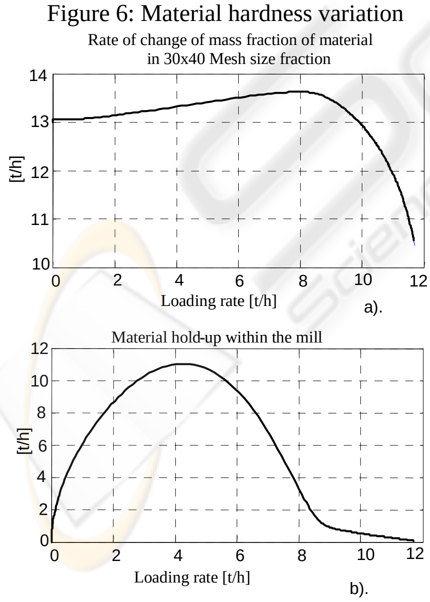

Figure 7: Illustration of loading rate dependence of the grinding performance

a) Effect of solid flow rate on grinding rate
b) Effect of solid flow rate on hold-up

Instead of trying to find a mathematical expression of disturbances, a state observer in equation (27) can be used to estimate it and compensate for it in real time.

As a result, for the system in equations (22)-(24), using equation (27) the estimation of the material grinding rate, $\hat{x}_1(t)$; the charge portion that is under going grinding per unity of input energy, $\hat{x}_2(t)$; the material fineness, $\hat{x}_3(t)$ and the raw material hardness, $\hat{x}_4(t)$ is as follows:

$$A\hat{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} \quad (29)$$

$$h(\hat{x},y,u) = \begin{bmatrix} \varepsilon u & 0 & 0 & 0 \\ 0 & -a & 0 & 0 \\ 0 & 0 & \varepsilon b \exp(u) & 0 \\ 0 & 0 & -c & 0 \end{bmatrix} \times \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} + \begin{bmatrix} o \\ d \\ 0 \\ 1 \end{bmatrix} u$$

(30)

$$\tilde{y} = y - C_i\hat{x} \quad (31)$$

$$\tilde{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} \quad (32)$$

$$\Upsilon = M_i^{-1}\delta_{\theta i}^{-1}K_i \quad (33)$$

$$\begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \\ \Upsilon_3 \\ \Upsilon_4 \end{bmatrix} = \begin{bmatrix} -\hat{x}_1^2 + 1 + \theta_1 k_{11} + \varepsilon u \\ -2(0.5\hat{x}_1^4 + \hat{x}_1^2 + 0.5\theta_1 k_{11}\hat{x}_1) - x_1x_2 + \theta_1 k_{11} + \theta^2{}_1 k_{12} \\ -\hat{x}_3^2 + \varepsilon\hat{x}_2 \exp(u) + 1 + \theta_2 k_{21} \\ \hat{x}_3^4 - (2 + \theta_2 k_{21})\hat{x}_3^2 - 2x_3x_4 + (\theta_2 k_{22} + k_{21})\theta_2 \end{bmatrix}$$

(34)

In figures 8, 9 are shown the simulation results for the observer and the system, while the tracking error is shown in fig.10, 11.

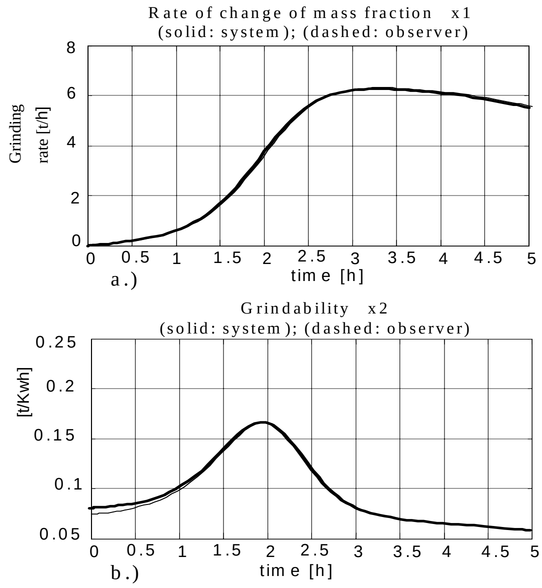

Figure 8: Output response of the system and the observer
a.) Rate of change of mass fraction
b.) Grindability

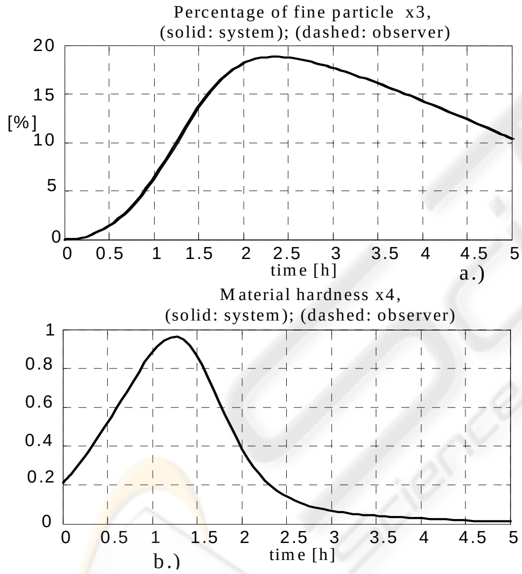

Figure 9: Output response of the system and the observer
a.) Percentage of fine particle
b.) Material hardness

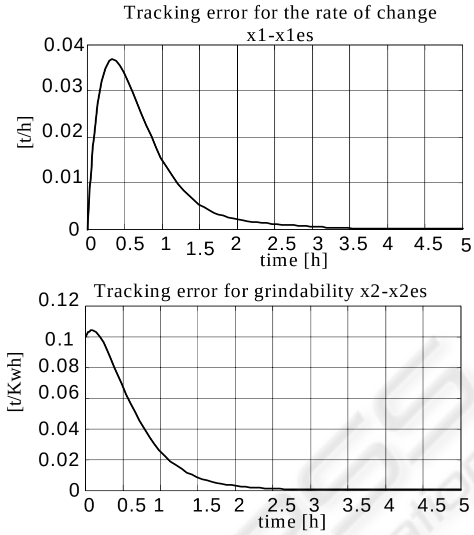

Figure 10: Grindability and rate of change tracking error

6 ROBUSTNESS AGAINST STRUCTURAL UNCERTAINTY

In practice, from the test results, correlation for samples could not be obtained because of different geological origins. However, experimental relationships between dynamic elastic parameters and Bond grindability, (Van Heerden, 1987), were used to valid the observer simulation results. In the dynamical analysis the dynamics of the error system, obtained by combining the experimental process results with the observer designed is analysed. Fig.12 shows that the observer is stable for unknown input.

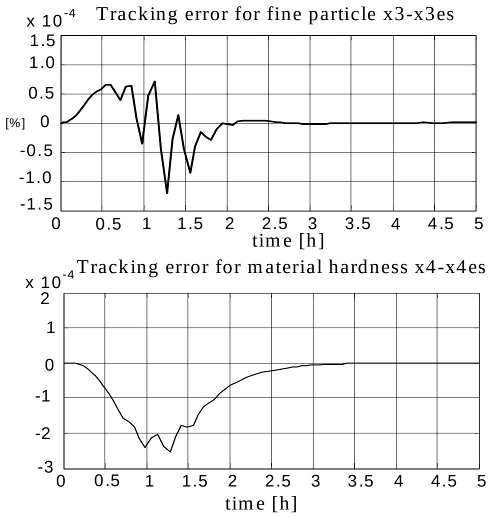

Figure 11: Fine particle percentage and material hardness tracking error

Figure 12: Observer convergence for unknown input

7 DISCUSSION

As a case study of mineral processing, the wet grinding ball mill operated in continuous or fed-batched mode has been studied. Simulation results showed that significant part of the steady state error is due to the model part and thus independent of the observer design methodology. The robustness against parametric and structural uncertainty can be increased, although this will increase the noise sensitivity. Since we herein want to track only the truly time-varying features of the process dynamics, the state observer designed strategy is satisfactory.

The load within the mill should be controlled at a well chosen level because too high levels of the load in the mill create process disturbances. The output product fineness depends on the solids rate flow. In view of the approximations involved in this treatment, the agreement between the observer and the model is remarkable. The estimation of the observer converges to zero exponentially.

8 CONCLUSION

We have described symbolic computations for reducing a nonlinear system to observable forms. These tools can be applied to systems that are linearly observable, locally observable with zero input or merely locally observable.

The key impact of this development lies in the system ability, to reduce material residence time, to flow information and material in a much-improved manner with the appropriate control strategy. Additional elements to be considered in the evaluation of the performance of the observer are distributed parameters effects due to the large sampling intervals often encountered in mineral applications.

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