POSITION CONTROL OF AN ELECTRO-HYDRAULIC SERVOSYSTEM

A non-linear backstepping approach

Claude Kaddissi, Jean-Pierre Kenné, Maarouf Saad

École de Technologie Supérieure, 1100 Notre-Dame West street, Quebec H3C 1K3, Canada

Keywords: Electro-hydraulic systems modeling, Non-linear control, Backstepping.

Abstract: This paper studies the control of an electro-hydraulic servosystem using a non-linear backstepping based approach. These systems are known to be highly non-linear due to many factors as leakage, friction and especially the fluid flow expression through the servo-valve. Another fact, often neglected or avoided, is that such systems have a non-differentiable mathematical model for bi-directional applications. All these facets are pointed out in the proposed model of this paper. Therefore, the control of this class of systems should be based on non-linear strategies. Many experiments showed the failure of classic control with electro-hydraulic systems, unless operating in the neighborhood of a desired value or reference signal. The backstepping is used here to overcome all the non-linear effects. The model discontinuity is solved in this paper, by approximating the non-differentiable function by a sigmoid, so that the backstepping could be used without restrictions. In fact, simulation results show the effectiveness of the proposed approach in terms of guaranteed stability and zero tracking error.

1 INTRODUCTION

An electro-hydraulic system is composed of multiple components: A pump, which feeds the system with fluid oil from an oil container; An accumulator, which acts as an auxiliary of energy integrated in the hydraulic circuit; A relief valve on the other hand to compensate the increase of pressure if any; A hydraulic actuator to drive a given load at a desired position, its displacement direction, speed and acceleration are determined by a servo-valve. Note that oil exiting the hydraulic actuator is driven through the servo-valve back to the tank.

Electro-hydraulic systems became increasingly current in many kinds of industrial equipments and processes. Such applications include rolling and paper mills, aircraft's and all kinds of automation including cars industry where linear movements, fast response, and accurate positioning with heavy loads are needed. This is principally because of the great power capacity with good dynamic response and system solution that they can offer, as compared to DC and AC motors. However, as a result of the ever-demanding complexity with these applications, considering non-linearity and mathematical model singularity, the traditional constant-gain controllers have become inadequate. *Lim* (1997) applied simple poles placement for a linearized model of an electro-hydraulic system. *Plahuta et al.* (1997) tried a retroaction strategy for variable displacement hydraulic actuator and this by using two cascaded control loops for position and speed control. *Zeng and Hu* (1999) used a PDF algorithm (Pseudo-Derivative Feedback) where the integrator part of a PID controller was placed in the direct path. Experiments and simulations showed that factors resulting in dynamic variations are beyond the capacity of these controllers. And there are also many of these factors to take into account, such as load variations, changes of transducers characteristics and properties of the hydraulic fluid, changes of servo components dynamics and other system components, etc. As a result, many robust and adaptive control methods have been used. *Yu and Kuo* (1996) employed an indirect adaptive controller for speed feedback, based on pole placement. Sliding mode controller has been used by *Fink and Singh* (1998), in order to regulate the pressure drop due to the load across the actuator. On the other hand fuzzy logic control has been

Kaddissi C., Kenné J. and Saad M. (2004).
POSITION CONTROL OF AN ELECTRO-HYDRAULIC SERVOSYSTEM - A non-linear backstepping approach.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 270-276
DOI: 10.5220/0001134402700276

employed by *Yongqian et al*. (1998), the controller was based on a decision rules matrix of the error. Results were very closed to neural network based controller as in *Kandil et al. 1999*, where learning was accomplished through the output error retro-propagation and thus system knowledge was not necessary. Simulation results in these advanced strategies were very successful in most cases, aside minor transient state problems or small steady state error. However, stability is not guaranteed in these approaches. This is because in such cases stability is studied on discrete domains and one cannot expect system behaviour at limits. Another point to consider is that in spite of ensuring system robustness, most control strategies might generate a control law with high amplitude, which causes system saturation. Such is the case of feedback linéarizations. Therefore one should think to benefit of system non-linearities that offer more manoeuvrability to avoid saturating control signals.
This paper proposes a backstepping approach, tacking into account all system non-linearities. A major advantage of the backstepping approach is its flexibility to build a control law by avoiding the cancellation of useful non-linearities. In addition modification is brought so the system model can be used for bi-directional applications. Under such modification the use of this approach ensures global stability of the system, and generates low amplitude control signal.

The paper is organised as follow. Section 2 presents the dynamic model of the electro-hydraulic system with emphasis on non-linearity and non-differentiability. In section 3, we shall present the design of a backstepping control strategy according to the system properties. In section 4 Simulation results and comparisons will be done. Some conclusions will be carried out in the last section.

2 SYSTEM MODELING

Consider the hydraulic system shown in figure 1. In this hydraulic circuit the DC electric motor drives the pump at constant speed. The pump in turn delivers oil flow from the tank to the rest of the components. Normally, the pressure P_s at the pump discharge depends on the load, however it is made constant due to the presence of an accumulator and a relief valve. In fact, the accumulator acts as an additional pressure source in case needed. On the other side the relief valve compensates the pressure increase due to big loads, by returning the additional

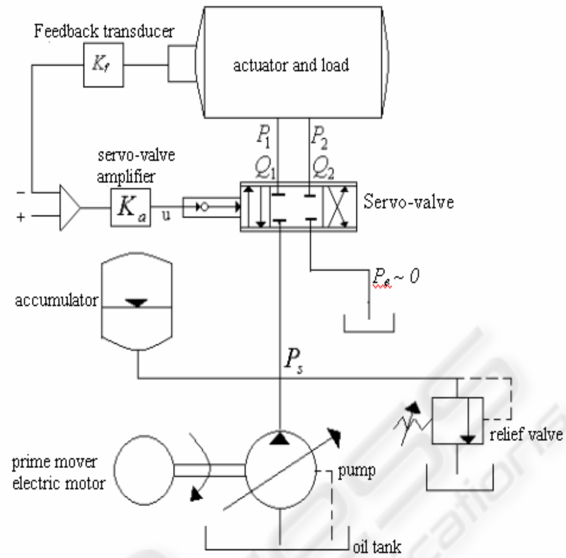

Figure 1: Schematic of hydraulic servo-system

amount of flow to the tank. A hydraulic rotary actuator drives the load. The actuator rotation is due to the oil flow coming from the servo-valve, the latter determines its direction, speed and acceleration through convenient position of its spool. The control signal being generated by the controller designed in this paper actuates the spool to the right position. We should note that the oil returns to the tank from the servo-valve at atmospheric pressure and we assume that the latter is a single stage servo-valve, critically centred and the orifices are matched and symmetrical.

The dynamic equation for a servo-valve spool movement can be given by (*LeQuoc et al*., 1990):

$$\tau_v \dot{A}_v + A_v = Ku \tag{1}$$

Where u is the control input, K is the servo–valve constant, τ_v is its time constant and A_v is the valve opening area. The flow rate from and to the servo-valve, through the valve orifices, assuming small leakage, are given as:

$$Q_1 = Q_2 = C_d A_v \sqrt{\frac{P_s - P_L}{\rho}} \tag{2}$$

Where P_L is the differential pressure due to load.

$$\begin{cases} P_L = P_1 - P_2 & \text{in the positive direction} \\ P_L = P_2 - P_1 & \text{in the negative direction} \end{cases}$$

$P_s = P_1 + P_2$ is the source pressure, C_d is the flow discharge coefficient and ρ is the fluid oil mass density.
Since oil viscosity might vary with temperature, it should be considered in the actuator dynamics along with oil leakage. Thus we give the compressibility equation as:

$$\frac{V}{2\beta}\dot{P}_L = C_d A_v \sqrt{\frac{P_s - sign(A_v)P_L}{\rho}} - D_m\dot{\theta} - C_L P_L \tag{3}$$

We define C_L as the load leakage coefficient, β is the fluid bulk modulus, θ is the output angular position, V is the oil volume under compression in one chamber of the actuator and D_m is the actuator volumetric displacement.

Now we consider the hydraulic actuator equation of motion given by Newton's first law. Neglecting the frictional torque we have:

$$J\ddot{\theta} = D_m(P_1 - P_2) - B\dot{\theta} - T_L \tag{4}$$

T_L is the load torque, B viscous damping coefficient and J the actuator inertia.
Note that in equations (3) the non-linear term due to the flow expression and the non-differentiable *sign* function that stands for changing in motion direction, are at the origin of such systems complexity.

Finally choosing,
$x_1 = \theta$, $x_2 = \omega = \dot{\theta}$, $x_3 = P_L$, $x_4 = A_v$ as state variables; the system can be easily described with a 4th order non-linear state space model.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = w_a x_3 - w_b x_2 - w_c \\ \dot{x}_3 = p_a x_4 \sqrt{P_s - x_3.sign(x_4)} - p_b x_3 - p_c x_2 \\ \dot{x}_4 = -r_a x_4 + r_b u \\ y = x_1 \end{cases} \tag{5}$$

Where r_a, r_b, p_a, p_b, p_c, w_a, w_b, w_c, are appropriate constants given by,

$$r_a = \frac{1}{\tau_v},\ r_b = \frac{K}{\tau_v},\ p_a = \frac{2\beta C_d}{V\sqrt{\rho}},\ p_b = \frac{2\beta C_L}{V},$$

$$p_c = \frac{2\beta D_m}{V},\ w_a = \frac{D_m}{J},\ w_b = \frac{B}{J},\ w_c = \frac{T_L}{J}$$

3 BACKSTEPPING BASED NON-LINEAR CONTROL

In this section, the non-linear backstepping, as presented by *Hassan (2002)*, will be used to control the electro-hydraulic system presented in figure 1. The control law is derived based on a lyapunov function, to ensure an input-output stability of the system. This method has been employed before by *Ursu and Popescu (2002)* with a reduced order model of an electro-hydraulic system, for position control; and with a 4th order model for load pressure control, all with positive desired trajectories or $sign(x_4) = 1$. Later and for better tracking characteristics the backstepping was applied to a 5th order electro-hydraulic system model *(Ursu, F. et al. 2003).* However the same assumption was held (i.e. a positive reference for position). Conversely, it is very vital to consider the case of a trajectory that takes positive and negative values. Else, the system application will loose generality, and will bypass a large number of applications where electro-hydraulic systems are used. One of those is electro-hydraulic active suspension, where a hydraulic actuator has to ensure a minimum vertical displacement of the car body. Thus, it is evident that under road stochastic fluctuations, the servo-valve will direct the flow to the actuator in either ways, depending if the car is crossing a bump or a pothole. Here, this method will be used with the system model (5) for position control and appropriate modification will be brought to account for a positive-negative varying reference.

We denote by $e_i = x_i - x_{id}$ for $i = 1,...,4$ the error between each state variable and its desired trajectory. Let us choose a candidate lyapunov function defined by,

$$V_1 = \frac{\rho_1 e_1^2}{2} \tag{6}$$

Then its derivative is given by,

$$\dot{V}_1 = \rho_1 e_1(\dot{x}_1 - \dot{x}_{1d}) = \rho_1 e_1(e_2 + x_{2d} - \dot{x}_{1d})$$

Thus, taking

$$x_{2d} = \dot{x}_{1d} - k_1 e_1 \tag{7}$$

Renders,

$$\dot{V}_1 = -k_1\rho_1 e_1^2 + \rho_1 e_1 e_2 \tag{8}$$

In a second step we shall take,

$$V_2 = V_1 + \frac{\rho_2 e_2^2}{2} \tag{9}$$

And its derivative,

$$\dot{V}_2 = \dot{V}_1 + \rho_2 e_2 \dot{e}_2$$
$$= -k_1 \rho_1 e_1^2 + e_2[\rho_1 e_1 - \rho_2 w_b x_2 + \rho_2 w_a e_3 + \rho_2 w_a x_{3d} - \rho_2 w_c - \rho_2 \dot{x}_{2d}]$$

Here, taking

$$x_{3d} = \frac{w_b}{w_a} x_2 - \frac{\rho_1}{\rho_2 w_a} e_1 + \frac{1}{w_a} \dot{x}_{2d} - \frac{k_2}{\rho_2 w_a} e_2 + \frac{w_c}{w_a} \tag{10}$$

Will give,

$$\dot{V}_2 = -k_1 \rho_1 e_1^2 - k_2 e_2^2 + \rho_2 w_a e_2 e_3 \tag{11}$$

Next, consider

$$V_3 = V_2 + \frac{\rho_3 e_3^2}{2} \tag{12}$$

Its derivative is given by,

$$\dot{V}_3 = \dot{V}_2 + \rho_3 e_3 \dot{e}_3$$
$$= -k_1 \rho_1 e_1^2 - k_2 e_2^2 + e_3[\rho_2 w_a e_2 + \rho_3(-p_c x_2 - p_b x_3 + p_a \sqrt{P_s - sign(x_4) x_3}(e_4 + x_{4d}) - \dot{x}_{3d})]$$

By choosing,

$$x_{4d} = \frac{(p_c x_2 + p_b x_3 + \dot{x}_{3d} - \frac{\rho_2 w_a}{\rho_3} e_2 - k_3 e_3)}{p_a \sqrt{P_s - sign(x_4) x_3}} \tag{13}$$

We have,

$$\dot{V}_3 = -k_1 \rho_1 e_1^2 - k_2 e_2^2 - k_3 \rho_3 e_3^2 + \rho_3 p_a e_3 e_4 \sqrt{P_a - sign(x_4) x_3} \tag{14}$$

Finally we consider,

$$V_4 = V_3 + \frac{\rho_4 e_4^2}{2} \tag{15}$$

Next we derive equation (15) and obtain,

$$\dot{V}_4 = \dot{V}_3 + \rho_4 e_4 \dot{e}_4$$
$$= -k_1 \rho_1 e_1^2 - k_2 e_2^2 - k_3 \rho_3 e_3^2 + e_4[\rho_4(-r_a x_4 + r_b u) - \rho_4 \dot{x}_{4d} + \rho_3 p_a e_3 \sqrt{P_s - sign(x_4) x_3}]$$

By setting the control u as stated by equation (16)

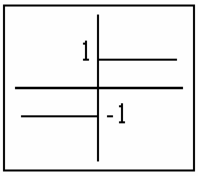

Figure 2:
sign function

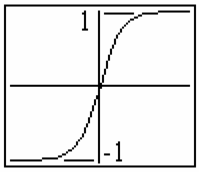

Figure 3:
sigmoid function

$$u = \frac{1}{r_b}\left(r_a x_4 + \dot{x}_{4d} - \frac{\rho_3 p_a}{\rho_4} e_3 \sqrt{P_s - sign(x_4) x_3} - \frac{k_4}{\rho_4} e_4 \right) \tag{16}$$

We should get the following result,

$$\dot{V}_4 = -k_1 \rho_1 e_1^2 - k_2 e_2^2 - k_3 \rho_3 e_3^2 - k_4 e_4^2 < 0 \tag{17}$$

Unfortunately, this is not the case because the control signal u is a discontinuous signal, which involves the derivative of x_{4d} that contains a *sign*(x_4) function. Thus it is impossible to generate such control signal. For that reason and to remedy this problem, we introduce the sigmoid function defined by (see figure 2 and 3),

$$sigm(x) = \frac{1 - e^{-ax}}{1 + e^{-ax}} \tag{18}$$

This is a continuously differentiable function with the following properties,

$$a > 0$$

$$sigm(x) = \begin{cases} 1 & \text{if } ax \to \infty \\ 0 & \text{if } x = 0 \\ -1 & \text{if } ax \to -\infty \end{cases} \tag{19}$$

and,

$$\frac{d sigm(x)}{dx} = \frac{2a e^{-ax}}{(1 + e^{-ax})^2} \tag{20}$$

Note, that the rate at which sigm(x) converges to 1 or –1 depends on the slope ‘a/2’.
Now we can rewrite the equation (13) as,

$$x_{4d} = \frac{(p_c x_2 + p_b x_3 + \dot{x}_{3d} - \frac{\rho_2 w_a}{\rho_3} e_2 - k_3 e_3)}{p_a \sqrt{P_s - sigm(x_4) x_3}} \tag{21}$$

Therefore we can solve for control signal u in (16) by

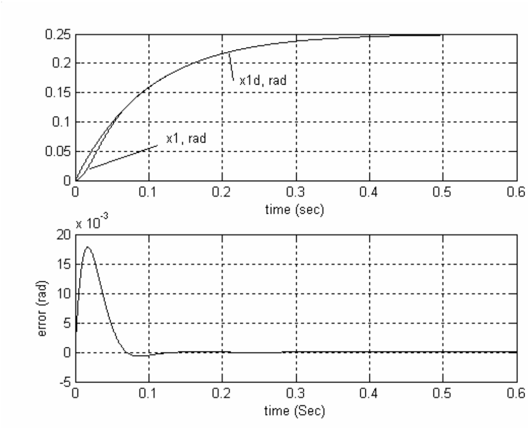

Figure 4: Position vs. desired position and tracking error

$$u = \frac{1}{r_b}\left(r_a x_4 + \dot{x}_{4d} - \frac{\rho_3 p_a}{\rho_4} e_3 \sqrt{P_s - sigm(x_4) x_3} - \frac{k_4}{\rho_4} e_4 \right) \tag{22}$$

That gives,

$$\dot{V}_4 = -k_1 \rho_1 e_1^2 - k_2 e_2^2 - k_3 \rho_3 e_3^2 - k_4 e_4^2 < 0$$

We can state now, that $\dot{V}_4 < 0$ for every $e_i \neq 0$, thus we conclude that the control law found in (22) renders the system globally asymptotically stable.

In the next section we present the simulation of the controlled system.

4 SIMULATIONS AND RESULTS

We shall present in this section, simulation results to reveal the backstepping efficiency and robustness in all cases (see appendix for control parameters values).
First, let us choose a desired trajectory for position that tends to a constant,

$$x_{1d} = x_{1f}\left(1 - e^{-\frac{t}{t_r}} \right) > 0 \tag{23}$$

Thus, $x_{1d} \to x_{1f}$ when $t \to \infty$, with a time constant t_r. Where we set x_{1f} = 0.25rad and t_r = 0.1 sec.

Graphics are given in figures 4 and 5. Figure 4, shows how the system output x_1 follows the desired trajectory with an excellent transient state and a zero tracking error in steady state. On the other hand figure 5 shows a smooth control signal generated by

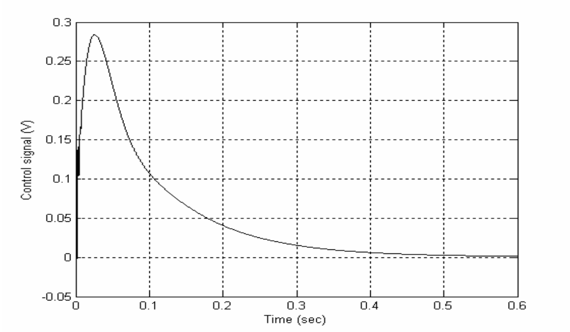

Figure 5: Control signal

equation (22), which drives the servo-valve spool x_4 to the convenient positions, far from saturating amplitudes. Once desired position reached, x_4 holds at zero hence no oil will flow to the actuator.

Next we will consider the case of a 'sine function' for the desired position with 1 amplitude and 2.5 rad/s frequency. Hence at zero crossing the valve spool will have to move in the negative direction as well as the actuator. Figure 6 and 7 shows the comparison of the control signal when the model in (5) is simulated with the *sign* function and the *sigmoid* function respectively. It is obvious in the first case, that when the actuator changes its direction the discontinuous *sign* term gives infinite control amplitude. While, a smooth control is obtained with the proposed *sigmoid* function.
Furthermore, as seen from figure 8 the *sigmoid* converges exponentially towards a *sign* with constant a = 2500. Finally as seen from figure 9; the backstepping used with the new system model, achieves a perfect tracking of the desired position. This approach is compared with the results of a classical PID controller based on pole placement. It is obvious; the PID fails to achieve good tracking, which results in large steady state error.
We conclude that, the backstepping effectiveness is not affected by the approximation we made. The latter allows a smooth continuous control signal as in figure 7, which justify at the end of our work the choice for this approach and the proposition we brought to the system model.

5 CONCLUSION

In this paper we studied the position control of an electro-hydraulic servo-system. The control law we established is based on the non-linear backstepping. Our goal was to account for the system non-linearity and non-differentiability, and to show how system stability is globally guaranteed. We saw how mathematical model non-differentiability prohibits

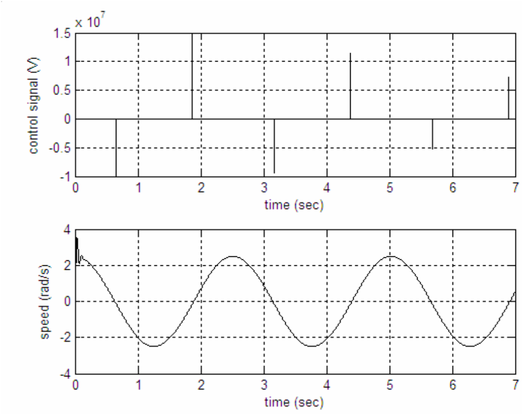

Figure 6: Control signal and actuator speed when sign function is used

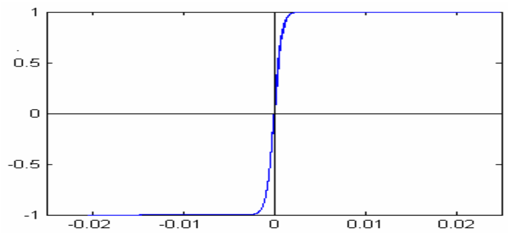

Figure 8: sigm(x_4) compared to sign(x_4)

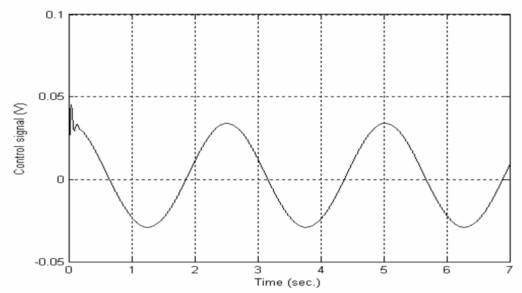

Figure 7: Control signal when sigmoid is used

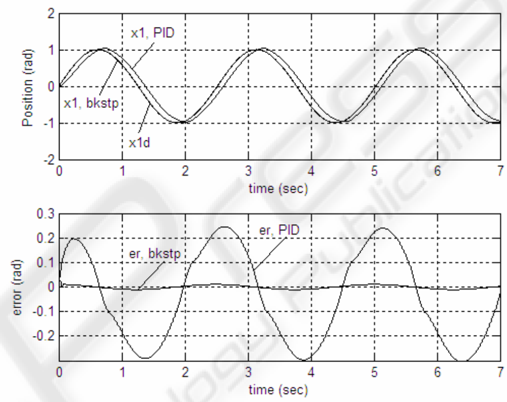

Figure 9: Position vs. desired position and tracking error for backstepping and PID

successful control. Introducing the sigmoid function brought solution to the non-differentiable aspect and gave the model a smoother expression allowing successful control via the same strategy. Comparison with classical PID showed the effectiveness of this control strategy, as it ensures perfect tracking with small transient and steady state error. Our future work consists of real time implementation of this control law to reveal its effectiveness and bring improvements if necessary. We are looking as well to industrial applications of electro-hydraulic systems, especially in cars industry. Our fields of interests are basically electro-hydraulic active suspension control and power transmission control systems, for better ride quality. Those are the most competent nowadays applications for hydraulic servo-systems; especially that road means of transports are facing a great competition on behalf of the air and maritime transportations means.

REFERENCES

Fink, A., & Singh, T. (1998). *Discrete sliding mode controller for pressure control with an electrohydraulic servovalve.* Paper presented at the control Applications, 1998. Proceedings of the 1998 IEEE International Conference on.

Hassan, K. (2002). *Non-Linear Systems* (Third ed.): Prentice Hall, Upper Saddle River NJ07458.

Kandil, N., LeQuoc, S., & Saad, M. (1999). On-Line Trained Neural Controllers for Nonlinear Hydraulic System. *14th World Congress of IFAC*, 323-328.

LeQuoc, S., Cheng, R. M. H., & Leung, K. H. (1990). Tuning an electrohydraulic servovalve to obtain a high amplitude ratio and a low resonance peak. *Journal of Fluid Control, 20*(3Mar), 30-49.

Lim, T. J. (1997). *Pole placement control of an electrohydraulic servo motor.* Paper presented at the Power Electronics and Drive Systems, 1997. Proceedings., 1997 International Conference on.

Plahuta, M. J., Franchek, M. A., & Stern, H. (1997). Robust controller design for a variable displacement hydraulic motor. *American Society of Mechanical Engineers, The Fluid Power and Systems Technology Division (Publication) FPST. Proceedings of the 1997 ASME International Mechanical Engineering Congress and Exposition, Nov 16-21 1997, 4*, 169-176.

Ursu, I., F. Popescu, & Ursu, F. (2003, May 29-31,). *Control synthesis for electrohydraulic servo mathematical model.* Paper presented at the Proceedings of the CAIM 2003, the 11th International Conference on Applied and Industrial Mathematics, Oradea, Romania.

Ursu, I., & Popescu, F. (2002, October 11-13,). *Backstepping control synthesis for position and force nonlinear hydraulic servoactuators.* Paper presented at the Book of Abstracts of the CAIM 2002, 10th International Conference on Applied and Industrial Mathematics,. Pitesti, Romania, October 11-13, p. 16.

Yongqian, Z., LeQuoc, S., & Saad, M. (1998). *Nonlinear fuzzy control on a hydraulic servo system.* Paper presented at the American Control Conference, 1998. Proceedings of the 1998.

Yu, W.-S., & Kuo, T.-S. (1996). Robust indirect adaptive control of the electrohydraulic velocity control systems. *IEE Proceedings: Control Theory and Applications, 143*(5), 448-454.

Zeng, W., & Hu, J. (1999). Application of intelligent PDF control algorithm to an electrohydraulic position servo system. *IEEE/ASME International Conference on Advanced Intelligent Mechatronics, AIM Proceedings of the 1999 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM '99), Sep 19-Sep 23 1999*, 233-238.

APPENDIX

Table 1: Hydraulic servo-system parameters

Servo-valve time constant	τ_v	$3.18\text{x}10^{-3}$ sec
Servo-valve amplifier gain	K	$0.0397 cm^2/V$
Flow discharge coefficient	C_d	0.63
Fluid bulk modulus	β	$7.995\text{x}10^3$ daN/cm^2
Actuator chamber volume	V	135.4 cm^3
Supply pressure	P_S	68.94 daN/cm^2
Fluid mass density	ρ	$\frac{85}{981x10^5}$ g/cm^3
Leakage coefficient	C_L	0.09047 cm^5/daN.s
Actuator displacement	D_m	2.802 cm^3/rad
Viscous damping coeff.	B	0.766 daN.s.cm
Actuator Inertial load	J	0.0481 daN.cm.s^2
Actuator load torque	T_L	11.2 daN.cm

Table 2: Control parameters

k_1	10
k_2	2.5
k_3	900
k_4	800
ρ_1	400
ρ_2	0.05
ρ_3	0.003
ρ_4	1700