INTEGRATED DESIGN OF STRUCTURE/CONTROLLER FOR PARALLEL INVERTED PENDULUM SYSTEMS

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Keywords: Simultaneous optimal design, Integrated design of structure/controller, Parallel inverted pendulum systems

Abstract: An integrated design of structure/controller for parallel inverted pendulum systems is presented. In practical parallel inverted pendulum systems, existence of a realizable stabilizing controller is inevitably determined depending on the position of center of gravity of two pendulums. In this paper these parameters are set as structural parameters in structural systems and a descriptor form representation is used to express the dynamics of parallel inverted pendulum systems. A state feedback gain and structural parameters are determined based on a design of linear quadratic regulator(LQR), in which a generalized Ricatti equation in LQR problem for the descriptor form representation is reduced to LMI conditions. Main contribution of this paper is to give a method to determine the position of center of gravity of two pendulums for parallel inverted pendulum systems in a sense of suitable for control. Based on the obtained structural parameters, some experimental works are executed. Experimental results show the effectiveness of the proposed design method.

1 INTRODUCTION

Recently, simultaneous optimal design for structural and control systems, in other words, integrated design of structural and control systems has attracted an attention of control engineers (Hale, et al., 1985) and (Pil, et al., 1996), in which structure and control system design parameters are simultaneously optimized to minimize a certain objective function. In most cases, control theory gives a controller design method for the given model of the plant. Control theory seldom suggests a design of the plant itself. In order to obtain a superior mechanism with high performance, it is necessary to prepare the structural systems with higher controllability in the design stage.

In (Seto, et al., 1989), a simultaneous optimal design method for structure and control systems for an optical servosystems is discussed, while in (Ando, et al., 2003) this method is modified and applied to a displacement expanders of magnetic recording test stands. (Kim, et al., 2003) proposed a method of simultaneous optimization for state feedback gains and structural parameters. The system formulation is given using descriptor form representation of the plant under LMI(Linear Matrix Inequality) constraint.

In this paper, we propose a simultaneous optimal design method of structure and control systems for a parallel (double) inverted pendulum systems. It is well known that stabilization problem of parallel inverted pendulum systems is very difficult compared with the stabilization problem of single inverted pendulum systems and series-type (double) inverted pendulum systems. Especially, in practical parallel inverted pendulum systems, essential controllability is highly depends on the length and weight of two pendulums. Furthermore, existence of a realizable stabilizing controller is inevitably determined depending on the position of center of gravity of two pendulums. If the length and the position of center of gravity of the pendulums are not proper, realizable stabilizing controller hardly exists in practical systems.

Analysis and stabilization control of parallel inverted pendulum systems is addressed in (Kawatani, et al., 1993) and (Sugie, et al., 1993). In (Kawatani, et al., 1993), state feedback controller with full state observer is designed via arbitrary pole assignment. While in (Sugie, et al., 1993), a two-degree-of-freedom controller is designed based on H_∞ loop shaping design procedure. In both cases, stabilization is succeeded. However, although analysis for the length of the pendulum is executed in both cases, analysis for the position of center of gravity of the

Ishii C., Yamamoto S. and Hashimoto H. (2004).
INTEGRATED DESIGN OF STRUCTURE/CONTROLLER FOR PARALLEL INVERTED PENDULUM SYSTEMS.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 108-113
DOI: 10.5220/0001134601080113

pendulums is not executed. In both literatures, suitable position of center of gravity of two pendulums for control is determined through experiments with adding additional weight in each pendulum by trial and error. Thus, in structural systems, shape of the pendulum in a sense of suitable for control in terms of position of the center of gravity is not considered.

Therefore, in this paper we focus on the position of center of gravity of the pendulums, and set these parameters as structural parameters in structural systems. Main contribution of this paper is to give a method to determine the position of center of gravity of two pendulums for parallel inverted pendulum systems in a sense of suitable for control. More clearly, we propose a simultaneous quasi-optimal design method for state feedback gains and structural parameters based on design of linear quadratic regulator(LQR).

In the design method, firstly the plant is described by a descriptor form representation instead of regular state space representation. This allows to show the structural parameters linearly in model of the plant. Secondly, generalized Ricatti equation in LQR problem for the descriptor form representation is rewritten to TMI. By adding a proper convexifying function to TMI, BMI is obtained from TMI. Finally, fixing structural parameters and controller gain matrices alternately, BMI reduces to LMI. Then, each feasibility problem with LMI can be solved. This procedure is repeated until the repetition index becomes specified value. Thus, state feedback gain and structural parameters are obtained.

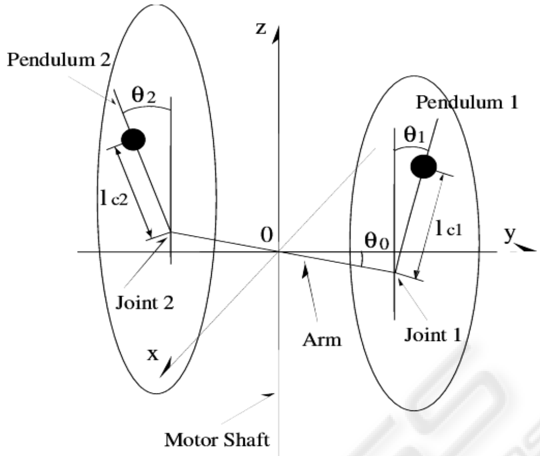
Based on the obtained structural parameters, in other words, the position of center of gravity of the pendulums, we added additional weights to the pendulums so that the real position of center of gravity becomes obtained value, and executed experimental work for stabilization. Besides, as for the obtained structural parameters, in terms of structural systems design, verification through various experiments with changing the position of center of gravity are carried out.

2 MODEL OF PARALLEL INVERTED PENDULUM

Consider the parallel inverted pendulum system shown in Fig.1.

The dynamics are given by

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + D\dot{\boldsymbol{\theta}} + G(\boldsymbol{\theta}) = \boldsymbol{\tau}, \quad (1)$$



θ_0 : rotation angle of arm
θ_1 : rotation angle of pendulum 1
θ_2 : rotation angle of pendulum 2
m_0: mass of arm
m_1: mass of pendulum 1
m_2: mass of pendulum 2
l_{01} : length of arm from origin to joint 1
l_{02} : length of arm from origin to joint 2
l_1 : length of pendulum 1
l_2 : length of pendulum 2
l_{c0} : center of gravity of arm
l_{c1} : center of gravity of pendulum 1
l_{c2} : center of gravity of pendulum 2
J_0 : moment of inertia of arm
J_1 : moment of inertia of pendulum 1
J_2 : moment of inertia of pendulum 2
d_0 : viscosity coefficient of joint 0
d_1 : viscosity coefficient of joint 1
d_2 : viscosity coefficient of joint 2
g : acceleration of gravity

Figure 1: Model of Parallel Inverted Pendulum

$$\begin{aligned}
\boldsymbol{\theta} &= \left[\begin{array}{ccc} \theta_0 & \theta_1 & \theta_2 \end{array}\right]^T, \ \boldsymbol{\tau} = \left[\begin{array}{ccc} \tau_0 & 0 & 0 \end{array}\right]^T, \\
M(\boldsymbol{\theta}) &= \begin{bmatrix} *1 & f_3\cos\theta_1 & f_7\cos\theta_2 \\ f_3\cos\theta_1 & f_4 & 0 \\ f_7\cos\theta_2 & 0 & f_8 \end{bmatrix}, \\
*1 &= f_1 + f_2\sin^2\theta_1 + f_6\sin^2\theta_2, \\
C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= \begin{bmatrix} *2 & -f_3\dot{\theta}_1\sin\theta_1 & -f_7\dot{\theta}_2\sin\theta_2 \\ *3 & 0 & 0 \\ *4 & 0 & 0 \end{bmatrix}, \\
*2 &= 2f_2\dot{\theta}_1\sin\theta_1\cos\theta_1 + 2f_6\dot{\theta}_2\sin\theta_2\cos\theta_2, \\
*3 &= -f_2\dot{\theta}_0\sin\theta_1\cos\theta_1, \ *4 = -f_6\dot{\theta}_0\sin\theta_2\cos\theta_2, \\
D &= \begin{bmatrix} d_0 & 0 & 0 \\ 0 & d_1 & 0 \\ 0 & 0 & d_2 \end{bmatrix}, \ G(\boldsymbol{\theta}) = \begin{bmatrix} 0 \\ -f_5\sin\theta_1 \\ -f_9\sin\theta_2 \end{bmatrix}.
\end{aligned}$$

where $\boldsymbol{\theta}$ is angle vector of arm and pendulum, $M(\boldsymbol{\theta})$ is moment of inertia matrix, $C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}$ is nonlinear vector containing Coriolis and Centrifugal forces, D

is viscosity matrix, $G(\boldsymbol{\theta})$ is gravitational vector and τ_0 is input torque for arm. Moreover, the following notations are used for simplicity.

$$\begin{aligned}
f_1 &= J_0 + m_0 l_{c0}^2 + m_1 l_{01}^2 + m_2 l_{02}^2, \\
f_2 &= m_1 l_{c1}^2, \\
f_3 &= m_1 l_{01} l_{c1}, \\
f_4 &= J_1 + m_1 l_{c1}^2, \\
f_5 &= m_1 g l_{c1}, \\
f_6 &= m_2 l_{c2}^2, \\
f_7 &= m_2 l_{02} l_{c2}, \\
f_8 &= J_2 + m_2 l_{c2}^2, \\
f_9 &= m_2 g l_{c2}.
\end{aligned}$$

Consider a linear approximation for the system (1) around the equilibrium $\boldsymbol{\theta}_0 = [\ 0\ \ 0\ \ 0\ ]^T$. Then, linearized system of (1) is described by

$$M_l \ddot{\boldsymbol{\theta}} + D\dot{\boldsymbol{\theta}} + G_l \boldsymbol{\theta} = \boldsymbol{\tau}, \tag{2}$$

where

$$M_l = \begin{bmatrix} f_1 & f_3 & f_7 \\ f_3 & f_4 & 0 \\ f_7 & 0 & f_8 \end{bmatrix},\ G_l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -f_5 & 0 \\ 0 & 0 & -f_9 \end{bmatrix}.$$

Define state variable $\boldsymbol{x}$ by $\boldsymbol{x} = [\ \boldsymbol{\theta}\ \ \dot{\boldsymbol{\theta}}\ ]^T$. Then, descriptor form representation of the parallel inverted pendulum system is given by

$$E\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{\tau}, \tag{3}$$

where

$$E = \begin{bmatrix} I & 0 \\ 0 & M_l \end{bmatrix}, A = \begin{bmatrix} 0 & I \\ -G_l & -D \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

Note that E is a positive symmetric matrix. Hence, equation (3) can be reformed to a regular state space representation by pre-multiplying E^{-1}. However, since E involves structural parameters, the descriptor form representation is used thorough this paper in order to make it easy to deal with structural parameters in structural systems.

In this paper, we focus on the position of center of gravity for two pendulums, so other parameters in parallel inverted pendulum systems are set as constant. Using a formula to calculate moment of inertia, f_4 and f_8 are expressed as follows.

$$\begin{aligned}
f_4 &= J_1 + m_1 l_{c1}^2 = \frac{1}{3} m_1 l_1^2 - m_1 l_1 l_{c1} + 2 m_1 l_{c1}^2 \\
f_8 &= J_2 + m_2 l_{c2}^2 = \frac{1}{3} m_2 l_2^2 - m_2 l_2 l_{c2} + 2 m_2 l_{c2}^2
\end{aligned}$$

In order to parameterize l_{c1} and l_{c2} linearly, the following approximation is adopted.

$$m_1 l_{c1}^2 \approx m_1 l_1 l_{c1},\ \ m_2 l_{c2}^2 \approx m_2 l_2 l_{c2}$$

Thus, f_4 and f_8 are described as

$$f_4 = \frac{1}{3} m_1 l_1^2 + m_1 l_1 l_{c1},\ \ f_8 = \frac{1}{3} m_2 l_2^2 + m_2 l_2 l_{c2}.$$

Let l_{c1} and l_{c2} be structural parameters in structural system. Then, M_l and G_l are described as follows.

$$M_l = M_0 + B_M \Sigma C_M,\ \ G_l = G_0 + B_G \Sigma C_G,$$

where

$$\begin{aligned}
M_0 &= \begin{bmatrix} f_1 & 0 & 0 \\ 0 & \frac{1}{3} m_1 l_1^2 & 0 \\ 0 & 0 & \frac{1}{3} m_2 l_2^2 \end{bmatrix}, \\
B_M &= \begin{bmatrix} 0 & m_1 l_{01} & 0 & m_2 l_{02} \\ m_1 l_{01} & m_1 l_1 & 0 & 0 \\ 0 & 0 & m_2 l_{02} & m_2 l_2 \end{bmatrix}, \\
C_M &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ G_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
B_G &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ -m_1 g & 0 & 0 & 0 \\ 0 & 0 & -m_2 g & 0 \end{bmatrix}, \\
C_G &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \\
\Sigma &= \begin{bmatrix} l_{c1} & 0 & 0 & 0 \\ 0 & l_{c1} & 0 & 0 \\ 0 & 0 & l_{c2} & 0 \\ 0 & 0 & 0 & l_{c_2} \end{bmatrix}.
\end{aligned}$$

Note that Σ is diagonal matrix whose elements are composed by structural parameters. Thus, descriptor form representation (3) is rewritten as

$$(E_0 + B_E \Sigma C_E)\dot{\boldsymbol{x}} = (A_0 + B_A \Sigma C_A)\boldsymbol{x} + B_0 \tau_0, \tag{4}$$

where

$$\begin{aligned}
E_0 &= \begin{bmatrix} I & 0 \\ 0 & M_0 \end{bmatrix},\ B_E = \begin{bmatrix} 0 \\ B_M \end{bmatrix}, \\
A_0 &= \begin{bmatrix} 0 & I \\ -G_0 & -D \end{bmatrix},\ B_A = \begin{bmatrix} 0 \\ -B_G \end{bmatrix}, \\
C_E &= [\,0\ \ C_M\,],\ C_A = [\,C_G\ \ 0\,], \\
B_0 &= [\,0\ 0\ 0\ 1\ 0\ 0\,]^T.
\end{aligned}$$

3 INTEGRATED DESIGN OF STRUCTURE/CONTROLLER

For the system (4), consider the linear quadratic regulator(LQR) problem, which is stated as follows.

LQR Problem:

Find a state feedback gain which minimizes the following quadratic cost function

$$J = \int_0^\infty \left(\boldsymbol{x}^T Q \boldsymbol{x} + \tau_0^T R \tau_0\right) dt, \tag{5}$$

where $Q \geq 0$ and $R > 0$ are arbitrary matrices with suitable dimension. ■

It is well known that the solution of the problem is given as follows.

Theorem 1:

Assume that (E, A, B) in the system (3) is impulse controllable and finite dynamics stabilizable. Then, the state feedback gain which minimizes the quadratic cost function (5) is given by $K = -R^{-1}B^T X_g$, where X_g is a solution of the following generalized Ricatti equation.

$$A^T X_g + Y_g A + Q - Y_g B R^{-1} B^T X_g = 0, \quad (6)$$

$$E^T X_g = Y_g E. \quad (7)$$

Then, minimum value of the cost function is given by $J_{\min} = \boldsymbol{x}_0 E^T X_g \boldsymbol{x}_0$. ■

Noting that there exists E^{-1}, Ricatti equation (6) is rewritten as follows.

$$\begin{aligned} &PE^{-1}A + A^T E^{-T} P \\ &\quad -PE^{-1}BR^{-1}B^T E^{-T} P + Q = 0, \end{aligned} \quad (8)$$

where $P = E^T X_g$.

Hereafter for theoretical development, consider the Ricatti inequality instead of Ricatti equation (8).

Using a change of variable and Schur Complement and noting equation (4), finally the following matrix inequality is obtained.

$$\begin{bmatrix} *1 & *2 & *3 \\ *2^T & -I & 0 \\ *3^T & 0 & -R^{-1} \end{bmatrix} < 0, \quad (9)$$

where

$$\begin{aligned} *1 &= (A_0 + B_A \Sigma C_A) X (E_0 + B_E \Sigma C_E)^T \\ &\quad + (E_0 + B_E \Sigma C_E) X (A_0 + B_A \Sigma C_A)^T \\ &\quad - B_0 M (E_0 + B_E \Sigma C_E)^T - (E_0 + B_E \Sigma C_E) M^T B_0^T, \\ *2 &= (E_0 + B_E \Sigma C_E) X H^T, \\ *3 &= (E_0 + B_E \Sigma C_E) M^T, \\ X &= P^{-1}, \ Q = H^T H, \ M = FX, \ F = -K. \end{aligned}$$

(9) is a trilinear matrix inequality(TMI) of the product of two variables X and Σ. $(1,1)$-block of (9) is described as

$$\begin{aligned} &A(\Sigma) X E^T(\Sigma) + E(\Sigma) X A^T(\Sigma) \\ &-B_0 M E^T(\Sigma) - E(\Sigma) M^T B_0^T \\ = \ &(A(\Sigma)X - BM)(A(\Sigma)X - BM)^T + E(\Sigma)E^T(\Sigma) \\ &-(A(\Sigma)X - BM - E(\Sigma))(A(\Sigma)X - BM - E(\Sigma))^T, \end{aligned}$$

where $A(\Sigma) = A_0 + B_A \Sigma C_A$ and $E(\Sigma) = E_0 + B_E \Sigma C_E$. Define $G(\Sigma, X, M)$ as

$$G(\Sigma, X, M) = A(\Sigma)X - BM - E(\Sigma). \quad (10)$$

By adding a positive semi-definite function, so-called convexifying function proposed in (Simomura, et al., 1993) given by

$$\begin{aligned} &(G(\Sigma, X, M) - G(\Sigma_f, X_f, M_f)) \\ &\quad \times (G(\Sigma, X, M) - G(\Sigma_f, X_f, M_f))^T \geq 0 \end{aligned} \quad (11)$$

to $(1,1)$-block in (9), finally the following bilinear matrix inequality(BMI) is obtained as a sufficient condition of (9).

$$\begin{bmatrix} *4 & *5 & *6 & *7 & *8 \\ *5^T & -I & 0 & 0 & 0 \\ *6^T & 0 & -R^{-1} & 0 & 0 \\ *7^T & 0 & 0 & -I & 0 \\ *8^T & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (12)$$

where

$$\begin{aligned} *4 &= G(\Sigma_f, X_f, M_f) G^T(\Sigma_f, X_f, M_f) \\ &\quad - G(\Sigma, X, M) G^T(\Sigma_f, X_f, M_f) \\ &\quad - G(\Sigma_f, X_f, M_f) G^T(\Sigma, X, M) \\ *5 &= E(\Sigma) X H^T, \\ *6 &= E(\Sigma) M^T, \\ *7 &= A(\Sigma) X - BM, \\ *8 &= E(\Sigma), \end{aligned}$$

Σ_f, X_f and M_f are proper matrices with suitable dimention.

BMI problem can be solved recursively by fixing each variable alternately. Thus, fixing (X, M) and Σ alternately, each feasibility problem with LMI can be solved. However, there is no guarantee that the parameters are adjusted desirably if only feasibility problem is considered. Therefore, suitable index function should be introduced to obtain desirable parameters.

Generally, it is said that pendulum having its position of center of gravity in high position can be stabilized easier than that having it in low position, and when the difference of natural frequency between two pendulums is large, stabilization becomes easy.

In terms of these reasons, we try to raise the position of center of gravity for long pendulum and lower the position of center of gravity for short pendulum. To this end, we consider to make the following index function small.

$$J_l = -\frac{l_{c1}}{l_{c1}^0} + \frac{l_{c2}}{l_{c2}^0}, \quad (13)$$

where l_{c1}^0 and l_{c2}^0 are original position of center of gravity for two pendulums. Note that the index function (13) is linear function related to the variable $\{l_{c1} \ l_{c2}\}$. Hence, it can be solved by minimization problem in the framework of LMI.

Thus, the algorithm to obtain a solution is stated as follows.

- Set an initial value $\Sigma_0 = \mathrm{diag}\{\, l_{c1}^0 \; l_{c_1}^0 \; l_{c2}^0 \; l_{c2}^0 \,\}$, and find (X_0, M_0) subject to (9), where l_{c1}^0 and l_{c2}^0 are original position of center of gravity for two pendulums.
- Set a repetition index values ε_1 and ε_2, and set $k = 0$.

Repeat

- Minimization problem:
  By fixing $(X, M) = (X_f, M_f) = (X_k, M_k)$, find Σ_{k+1} such that index function J_l given by (13) is minimized subject to (12).
- Feasibility problem:
  By fixing $\Sigma = \Sigma_f = \Sigma_{k+1}$, find (X_{k+1}, M_{k+1}) subject to (12).
- Set $k = k + 1$.

Until $|l_{c1}^0 - l_{c1}^k| > \varepsilon_1$ or $|l_{c2}^0 - l_{c2}^k| > \varepsilon_2$.

Note that parameters l_{c1}^k and l_{c2}^k may not converge to certain values, because mixed minimization and feasibility problem is solved in this approach, however, it has practical significance since control performance of the plant may be improved compared with the case in which structural parameters are not adjusted.

4 EXPERIMENTS

Original parameters of pendulum are shown in Table 1.

Table 1: Parameters of pendulum

l_1	l_2	l_{c1}	l_{c2}	m_1	m_2
0.4	0.23	0.15	0.078	0.11	0.075

Applying the proposed technique to a practical parallel inverted pendulum, finally one of the quasi-optimal parameter values $l_{c1} = 0.175$ and $l_{c2} = 0.07$ are obtained. Additional weight is added on proper position in each pendulum so that the position of center of gravity becomes obtained quasi-optimal value. Then, the mass of the pendulums are also changed as $m_1 = 0.14$ and $m_2 = 0.09$.

In general, minimum value of the cost function (5) obtained by solving LMI is larger than the one of obtained by solving Ricatti equation. Hence, state feedback gain is redesigned by solving corresponding Ricatti equation using the obtained optimal structural parameters. For $Q = \mathrm{diag}\{1.0 \times 10^3 \;\; 1.0 \times 10^6 \;\; 500 \;\; 0.1 \;\; 1 \;\; 1\}$, the following state feedback gain is obtained.

$$K = [\; -14.142 \;\; -1508.2 \;\; 1015.8 \;\; -15.503 \;\; -260.76 \;\; 131.48 \,]. \quad (14)$$

Besides, minimal order observer is used to estimate $\{\dot{\theta}_0,\ \dot{\theta}_1,\ \dot{\theta}_2\}$. Poles of the observer were assigned as $\{-60,\ -60,\ -60\}$.

Firstly, experimental works were carried out for the parallel inverted pendulum system with original parameters using the control system designed for its original parameters. However, none of experiments succeeded. Then, experimental works using the control system designed for obtained optimal parameters were executed. Experimental results are shown in Fig.2.

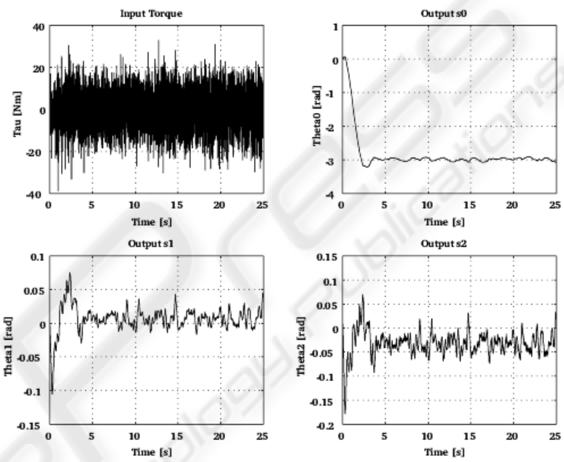


Figure 2: Experimental Results

As shown in Fig.2, stabilization of both pendulums are succeeded.

5 VERIFICATION

In order to evaluate the effectiveness of the proposed control system in terms of structural systems design, the following experiments were executed. First, fixing the position of center of gravity of one of the pendulums, several controllers are designed changing parameters for the position of center of gravity of another pendulum little by little in controller design. Then, for each controller, experimental works for stabilization control were carried out by shifting the real position of center of gravity of the pendulum in actual system little by little.

Experimental results for fixing $l_{c2} = 0.0725$ and changing conditions of l_{c1} are shown in Table 2. On the other hands, experimental results for fixing $l_{c1} = 0.172$ and changing conditions of l_{c2} are shown in Table 3.

From Table 2 and Table 3, it can be said that $l_{c1} = 0.172$ and $l_{c2} = 0.0725$ are the robustest value of the center of gravity for pendulums in both aspects

Table 2: Changing l_{c1} with $l_{c2} = 0.0725$ (fixed)

	0.180	0.178	0.176	0.174	0.172	0.170
0.180	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
0.178	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
0.176	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\times$	$\times$
0.174	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\odot$	$\times$
0.172	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\odot$	$\odot$	$\bigcirc$
0.170	$\bigcirc$	$\bigcirc$	$\odot$	$\odot$	$\odot$	$\odot$

Row : Real position of center of gravity
Column : Parameter used to design controller
$\times$: Failure, $\bigcirc$: Success (Good), $\odot$: Success (Very good)

Table 3: Changing l_{c2} with $l_{c1} = 0.172$ (fixed)

	0.0825	0.080	0.0775	0.075	0.0725	0.070
0.0825	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
0.0800	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
0.0775	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
0.0750	$\times$	$\times$	$\bigcirc$	$\times$	$\times$	$\times$
0.0725	$\bigcirc$	$\odot$	$\odot$	$\odot$	$\bigcirc$	$\bigcirc$
0.0700	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

Row : Real position of center of gravity
Column : Parameter used to design controller
$\times$: Failure, $\bigcirc$: Success (Good), $\odot$: Success (Very good)

of structural and control systems. In terms of above observations, we conclude that the effectiveness of the proposed integrated design of structure/controller for parallel inverted pendulum systems was verified.

6 CONCLUSIONS

In this paper, we proposed an integrated design of structure/controller for parallel inverted pendulum systems, in which the position of center of gravity of the pendulums are set as structural parameters in structural systems, and state feedback gain and structural parameters are determined based on a design of linear quadratic regulator(LQR). There are two features in the proposed method. The first one is that a descriptor form representation is used to express the dynamics of parallel inverted pendulum systems in order to show the structural parameters linearly. The second one is that a generalized Ricatti equation in LQR problem for the descriptor form representation is finally reduced to LMI conditions. To the best of our knowledge, this is the first paper which gives the position of center of gravity of pendulums for parallel inverted pendulum systems analytically in a sense of suitable for control. Based on the obtained structural parameters, we executed experimental works for stabilization. Experimental works show the effectiveness of the proposed design method.

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