FURTHER RESULTS ON OUTPUT FEEDBACK CONTROL OF DISCRETE LINEAR REPETITIVE PROCESSES

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Abstract: Repetitive processes are a distinct class of 2D systems (i.e. information propagation in two independent directions) of both systems theoretic and applications interest. They cannot be controlled by direct extension of existing techniques from either standard (termed 1D here) or 2D systems theory. Here we give new results on the relatively open problem of the design of physically based control laws using an LMI setting. These results are for the sub-class of so-called discrete linear repetitive processes which arise in applications areas such as iterative learning control.

1 INTRODUCTION

Repetitive processes are a distinct class of 2D systems of both system theoretic and applications interest. The essential unique characteristic of such a process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(p),\ 0 \leq p \leq \alpha$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of next pass profile $y_{k+1}(p),\ 0 \leq p \leq \alpha,\ k \geq 0$.

Physical examples of repetitive processes include long-wall coal cutting and metal rolling operations (see, for example, (Benton, 2000)). Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications include classes of iterative learning control (ILC) schemes (Amann et al., 1998) and iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle (Roberts, 2000). In the case of ILC for the linear dynamics case, the stability theory for so-called differential and discrete linear repetitive processes is the essential basis for a rigorous stability/convergence theory for such algorithms.

One unique feature of repetitive processes in comparison to other classes of 2D linear systems is that it is possible to define physically meaningful control laws for their dynamics. For example, in the ILC application, one such family of control laws is composed of state feedback control action on the current pass combined with information 'feedforward' from the previous pass (or trial in the ILC context) which, of course, has already been generated and is therefore available for use.

In the general case of repetitive processes it is clearly highly desirable to have an analysis setting where control laws can be designed for stability and/or performance. In which context, previous work has shown that an LMI re-formulation of the stability conditions for discrete linear repetitive processes

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leads naturally to design algorithms to ensure closed loop stability along the pass under control laws of this form — see, for example, (Gałkowski et al., 2002).

To implement a control law which uses the current pass state vector will, in general, require an observer to estimate the elements in this vector which are not directly measurable. As an alternative, this paper shows how to use the LMI setting to design control laws which only require pass profile information (which has already been generated and hence is available control action) for implementation. Note here that LMI based methods have also been investigated as a means of stability analysis and controller design for 2D discrete linear systems described by the well known Roesser (Roesser, 1975) and Fornasini Marchesini (Fornasini and Marchesini, 1978) state space models, see, for example, (Hinamoto, 1997; Du and Xie, 2002). Discrete linear repetitive processes have strong structural links with such systems class of systems and some results can be exchanged between these classes of linear systems. The key novelty in this paper is the use of physically motivated control schemes which are actuated only by pass profile information (in previous work the current pass state feedback vector was used and hence the possible need for an observer to implement such a scheme) and also the development of numerically feasible design algorithms for them.

Throughout this paper, the null matrix and the identity matrix with the required dimensions are denoted by 0 and I, respectively. Moreover, $M > 0$ (< 0) denotes a real symmetric positive (negative) definite matrix. We use $(*)$ to denote the transpose of matrix blocks in some of the LMIs employed (which are required to be symmetric).

2 BACKGROUND

Following (Rogers and Owens, 1992), the state-space model of a discrete linear repetitive process has the following form over $0 \leq p \leq \alpha,\ k \geq 0$,

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{aligned} \tag{1}$$

Here on pass k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector and $u_k(p) \in \mathbb{R}^l$ is the vector of control inputs.

To complete the process description, it is necessary to specify the boundary conditions, i.e. the state initial vector on each pass and the initial pass profile. Here no loss of generality arises from assuming $x_{k+1}(0) = d_{k+1} \in \mathbb{R}^n$ $k \geq 0$, and $y_0(p) = f(p) \in \mathbb{R}^m$, where d_{k+1} is a vector with known constant entries and $f(p)$ is a vector whose entries are known functions of p over $[0, \alpha]$. (For ease of presentation, we will make no further explicit reference to the boundary conditions in this paper.)

The stability theory (Rogers and Owens, 1992) for linear repetitive processes consists of two distinct concepts but here it is the stronger of these which is required. This is termed stability along the pass and several equivalent sets of necessary and sufficient conditions for processes described by (1) to have this property are known, but here the essential starting point is based on the so-called 2D characteristic polynomial for these processes given next.

Define the delay operators $z_1,\ z_2$ in the along the pass (p) and pass-to-pass (k) directions respectively as

$$x_k(p) := z_1 x_k(p+1), \quad x_k(p) := z_2 x_{k+1}(p) \tag{2}$$

Then the 2D characteristic polynomial for processes described by (1) is defined as

$$\mathcal{C}(z_1, z_2) = det \begin{bmatrix} I - z_1 A & -z_1 B_0 \\ -z_2 C & I - z_2 D_0 \end{bmatrix} \tag{3}$$

and it can be shown (Rogers and Owens, 1992) that stability along the pass holds if, and only if,

$$\mathcal{C}(z_1, z_2) \neq 0, \ \forall\, |z_1| \leq 1,\ |z_2| \leq 1$$

Note that stability along the pass can also be expressed in the form

$$\begin{aligned} \mathcal{C}(z_1, z_2) = det(I - z_1 \widehat{A}_1 - z_2 \widehat{A}_2) \neq 0, \\ \forall\, |z_1| \leq 1,\ |z_2| \leq 1 \end{aligned} \tag{4}$$

where

$$\widehat{A}_1 = \begin{bmatrix} A & B_0 \\ 0 & 0 \end{bmatrix}, \quad \widehat{A}_2 = \begin{bmatrix} 0 & 0 \\ C & D_0 \end{bmatrix} \tag{5}$$

In this work, we use the following LMI based sufficient condition derived from (4) which, unlike all other existing stability tests, leads immediately (see below) to systematic methods for control law design. The proof of this result can be found in (Gałkowski et al., 2002).

Theorem 1 *A discrete linear repetitive process described by (1) is stable along the pass if there exist matrices $Y > 0$ and $Z > 0$ such that the following LMI holds*

$$\begin{bmatrix} Y - Z & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y & \widehat{A}_2 Y & -Y \end{bmatrix} < 0$$

The control law considered in previous work has the following form over $0 \leq p \leq \alpha,\ k \geq 0$

$$u_{k+1}(p) = K_1 x_{k+1}(p) + K_2 y_k(p) := K \begin{bmatrix} x_{k+1}(p) \\ y_k(p) \end{bmatrix} \tag{6}$$

where K_1 and K_2 are appropriately dimensioned matrices to be designed. In effect, this control law uses feedback of the current state vector (which is assumed to be available for use) and 'feedforward' of the previous pass profile vector. Note that in repetitive processes the term 'feedforward' is used to describe the case where state or pass profile information from the previous pass (or passes) is used as (part of) the input to a control law applied on the current pass, i.e. to information which is propagated in the pass-to-pass (k) direction. The basic result for the design of this control law for closed loop stability along the pass is as follows.

Theorem 2 *(Gałkowski et al., 2002) Consider a discrete linear repetitive process of the form described by (1) subject to a control law of the form (6). Then the resulting closed loop process is stable along the pass if there exist matrices $Y > 0$, $Z > 0$, and N such that the following LMI holds.*

$$\begin{bmatrix} Z-Y & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 N & \widehat{A}_2 Y + \widehat{B}_2 N & -Y \end{bmatrix} < 0 \quad (7)$$

where $\widehat{A}_1$, $\widehat{A}_2$ are given in (5) and

$$\widehat{B}_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ \widehat{B}_2 = \begin{bmatrix} 0 \\ D \end{bmatrix} \quad (8)$$

If (7) holds, then a stabilizing K in the control law (6) is given by

$$K = NY^{-1} \quad (9)$$

3 OUTPUT FEEDBACK BASED CONTROLLER DESIGN

In many cases the state vector $x_{k+1}(p)$ may not be available or, at best, only some of its entries are. Hence, we now consider the use of output based feedback based control laws to achieve closed loop stability along the pass. The first law considered has the following form over
$0 \leq p \leq \alpha,\ k \geq 0$.

$$u_{k+1}(p) = \widetilde{K}_1 y_{k+1}(p) + \widetilde{K}_2 y_k(p) \quad (10)$$

This control law is, in general, weaker than that of (6) and examples are easily given where stability along the pass can be achieved using (6) but not (10).

To consider the effect of a controller of the form (10) on the process dynamics, first substitute the pass profile (second) equation of (1) into (10) to obtain (assuming the required matrix inverse exists)

$$\begin{aligned} u_{k+1}(p) =& (I_l - \widetilde{K}_1 D)^{-1} \widetilde{K}_1 C x_{k+1}(p) \\ &+ (I_l - \widetilde{K}_1 D)^{-1} [\widetilde{K}_2 + \widetilde{K}_1 D_0] y_k(p) \end{aligned} \quad (11)$$

and hence (11) can be treated as a particular case of (6) with

$$\begin{aligned} K_1 =& (I_l - \widetilde{K}_1 D)^{-1} \widetilde{K}_1 C \\ K_2 =& (I_l - \widetilde{K}_1 D)^{-1} (\widetilde{K}_2 + \widetilde{K}_1 D_0) \end{aligned} \quad (12)$$

This route may, however, encounter serious numerical difficulties (arising from the fact that they are a set of matrix nonlinear algebraic equations) and hence we proceed by rewriting these last equations to obtain

$$\begin{aligned} (I_l - \widetilde{K}_1 D) K_1 =& \widetilde{K}_1 C \\ (I_l - \widetilde{K}_1 D) K_2 =& \widetilde{K}_2 + \widetilde{K}_1 D_0 \end{aligned} \quad (13)$$

and assume that

$$K_1 = L_1 C \quad (14)$$

Then it follows immediately that

$$\begin{aligned} &\widetilde{K}_1 = L_1 (I + DL_1)^{-1} \\ &\widetilde{K}_2 = [I - L_1 (I + DL_1)^{-1} D] K_2 - \\ &\quad - L_1 (I + DL_1)^{-1} D_0 \end{aligned} \quad (15)$$

for any L_1 such that $I + DL_1$ is nonsingular, and we have the following result.

Theorem 3 *Suppose that a discrete linear repetitive process of the form described by (1) is subject to a control law of the form (10) and that (14) holds. Then the resulting closed loop process is stable along the pass if there exist matrices $Y > 0$, $Z > 0, X > 0$ and N such that the following LMI holds*

$$\begin{aligned} &\begin{bmatrix} Z-Y & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 N \widetilde{C} & \widehat{A}_2 Y + \widehat{B}_2 N \widetilde{C} & -Y \end{bmatrix} < 0 \\ &X \widetilde{C} = \widetilde{C} Y \end{aligned} \quad (16)$$

where $\widehat{B}_1$, $\widehat{B}_2$, $\widehat{A}_1$, $\widehat{A}_2, N$ are defined as in Theorem 2, and

$$\widetilde{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \quad (17)$$

Also if this condition holds, the controller matrices $\widetilde{K}_1$ and $\widetilde{K}_2$ can be obtained from (15), where

$$[L_1 \ \ K_2] = NX^{-1} \quad (18)$$

and it is required that $I + DL_1$ is nonsingular.

Proof: From (18) we have that $N = LX$, $L := \begin{bmatrix} L_1 & K_2 \end{bmatrix}$, and substitution into the LMI of (16) now gives

$$\begin{aligned} &\begin{bmatrix} Z-Y & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 L X \widetilde{C} & \widehat{A}_2 Y + \widehat{B}_2 L X \widetilde{C} & -Y \end{bmatrix} < 0 \\ &X \widetilde{C} = \widetilde{C} Y \end{aligned}$$

or with the equality constraint applied

$$\begin{bmatrix} Z - Y & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 L\widetilde{C}Y & \widehat{A}_2 Y + \widehat{B}_2 L\widetilde{C}Y & -Y \end{bmatrix} < 0$$

Finally, set $L\widetilde{C} = K$ to obtain the LMI stabilisation condition (i.e Theorem 1 applied to the closed loop process)

$$\begin{bmatrix} Z - Y & (*) & (*) \\ 0 & -Z & (*) \\ (\widehat{A}_1 + \widehat{B}_1 K)Y & (\widehat{A}_2 + \widehat{B}_2 K)Y & -Y \end{bmatrix} < 0$$

and the proof is complete.

The design developed above is easily implemented using LMI toolboxes, such as Scilab or Matlab, but has one real disadvantage in that it is based on a sufficient but not necessary stability condition. This means that there could well be a high degree of conservativeness in the sense that in many cases it will fail to produce a stabilising controller when one actually exists. To avoid, or lower, the level of conservativeness present, we next develop an extension of the control law considered in this section based on the additional use of the delayed current pass profile and pass-to-pass profile information.

4 EXTENDED OUTPUT FEEDBACK BASED CONTROLLER DESIGN

The control law considered in this section has the following form and is, in effect, (10) augmented at point p by additive contributions from points $p-1$ on the current pass and p on the previous pass respectively

$$\begin{aligned} u_{k+1}(p) =& \widetilde{K}_1 y_{k+1}(p) + \widetilde{K}_2 y_k(p) \\ &+ \widetilde{K}_3 y_k(p-1) + \widetilde{K}_4 y_{k-1}(p) \end{aligned} \quad (19)$$

Substituting the second equation of (1) into the control law (19) now yields

$$\begin{aligned} u_{k+1}(p) =& (I - \widetilde{K}_1 D)^{-1} \Big(\widetilde{K}_1 C x_{k+1}(p) \\ &+ [\widetilde{K}_2 + \widetilde{K}_1 D_0] y_k(p) \\ &+ \widetilde{K}_3 y_k(p-1) + \widetilde{K}_4 y_{k-1}(p) \Big) \end{aligned} \quad (20)$$

which is the particular case of the so-called extended, mixed state, pass profile controller

$$\begin{aligned} u_{k+1}(p) =& K_1 x_{k+1}(p) + K_2 y_k(p) \\ &+ K_3 y_k(p-1) + K_4 y_{k-1}(p) \end{aligned} \quad (21)$$

This last control law is, in effect, an extension of that of the previous section but here it is used as an intermediate step in the computation of the matrices $\widetilde{K}_i,\ i = 1, \ldots, 4$, through use of the following result.

Theorem 4 *Suppose that a discrete linear repetitive process of the form described by (1) is subject to a control law of the form (21) and that (14) holds. Then the resulting closed loop process is stable along the pass if there exist matrices $Y > 0$, $X = diag(X_1, X_2, X_3, X_4) > 0$, $Z > 0$ and N such that*

$$\begin{bmatrix} Y - Z & (*) & (*) \\ 0 & -Z & (*) \\ \widehat{A}_1 Y + \widehat{B}_1 N\widehat{C} & \widehat{A}_2 Y + \widehat{B}_2 N\widehat{C} & -Y \end{bmatrix} < 0 \quad (22)$$

$$X\widehat{C} = \widehat{C}Y$$

where

$$\widehat{A}_1 = \begin{bmatrix} A & -I & 0 & B_0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \widehat{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C & 0 & -I & D_0 \end{bmatrix},$$

$$\widehat{B}_1 = \begin{bmatrix} B & 0 & 0 & B \\ 0 & B & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \widehat{B}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & D & 0 \\ D & 0 & 0 & D \end{bmatrix},$$

$$N = \begin{bmatrix} N_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -N_3 \\ 0 & 0 & 0 & -N_4 \\ 0 & 0 & 0 & N_2 \end{bmatrix}, \ \widehat{C} = \begin{bmatrix} C & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \quad (23)$$

with

$$\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_3 \\ 0 & 0 & 0 & K_4 \\ 0 & 0 & 0 & K_2 \end{bmatrix} = NX^{-1} \quad (24)$$

Also if (22) holds, the controller matrices $\widetilde{K}_1$ and $\widetilde{K}_2$ can be computed using (15) and then

$$\begin{aligned} \widetilde{K}_3 =& [I - L_1(I + DL_1)^{-1} D] K_3 \\ \widetilde{K}_4 =& [I - L_1(I + DL_1)^{-1} D] K_4 \end{aligned} \quad (25)$$

where it is assumed that $I + DL_1$ is nonsingular.

Proof. Substitute (21) into (1) and using (14) we obtain the closed loop state space model

$$\begin{aligned} x_{k+1}(p+1) =& (A + BL_1 C) x_{k+1}(p) \\ &+ (B_0 + BK_2) y_k(p) + BK_3 y_k(p-1) \\ &+ BK_4 y_{k-1}(p), \\ y_{k+1}(p) =& (C + DL_1 C) x_{k+1}(p) \\ &+ (D_0 + DK_2) y_k(p) + DK_3 y_k(p-1) \\ &+ DK_4 y_{k-1}(p) \end{aligned} \quad (26)$$

This last description is not in the form to which Theorem 1 can be applied but it is possible to obtain an equivalent state space model for which this is the case. Here the route is by using the delay operators of (2) and the 2D characteristic polynomial. To begin, rewrite (26) by introducing the substitutions

$$l := k+1 \qquad v_l(p) := y_{k+1}(p) \tag{27}$$

and then apply (2) to obtain

$$\begin{aligned} x_l(p) =& z_1(A+BL_1C)x_l(p) \\ &+ z_1(B_0+BK_2)v_l(p) + z_2^2 BK_3 v_l(p) \\ &+ z_1 z_2 BK_4 v_l(p) \\ v_l(p) =& z_2(C+DL_1C)x_l(p) \\ &+ z_2(D_0+DK_2)v_l(p) + z_1 z_2 DK_4 v_l(p) + \\ &+ z_2^2 DK_4 v_l(p) \end{aligned} \tag{28}$$

and introduce

$$\mathcal{C}_c(z_1,z_2) := \\ det \begin{bmatrix} I - z_1\widetilde{A} & -z_1\widetilde{B}_0 - z_1^2F_1 - z_1z_2F_3 \\ -z_2\widetilde{C} & I - z_2\widetilde{D}_0 - z_1z_2F_2 - z_2^2F_4 \end{bmatrix}$$

Application of appropriate elementary operations (which leave the determinant invariant) to the right-hand side of this last expression now yields

$$det \begin{bmatrix} I - z_1\widetilde{A} & z_1 I & 0 & -z_1\widetilde{B}_0 \\ 0 & I & 0 & z_1F_1 + z_2F_3 \\ 0 & 0 & I & z_1F_2 + z_2F_4 \\ -z_2\widetilde{C} & 0 & z_2 I & I - z_2\widetilde{D}_0 \end{bmatrix} \tag{29}$$

where

$$\begin{aligned} \widetilde{A} &= A + BL_1C, & \widetilde{B}_0 &= B_0 + BK_2 \\ \widetilde{C} &= C + DL_1C, & \widetilde{D}_0 &= D_0 + DK_2 \\ F_1 &= BK_3, & F_2 &= DK_3 \\ F_3 &= BK_4, & F_4 &= DK_4 \end{aligned}$$

At this stage, the closed loop state space model has a 2D characteristic polynomial which is of the form required for use in (4) (and therefore Theorem 1 can be directly applied) with

$$\widetilde{A}_1 = \begin{bmatrix} \widetilde{A} & -I & 0 & \widetilde{B}_0 \\ 0 & 0 & 0 & -F_1 \\ 0 & 0 & 0 & -F_2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \widetilde{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -F_3 \\ 0 & 0 & 0 & -F_4 \\ \widetilde{C} & 0 & -I & \widetilde{D}_0 \end{bmatrix} \tag{30}$$

Application of Theorem 1 together with some obvious algebraic operations now yield directly the LMI of (22) as a sufficient condition for closed loop stability along the pass. Finally, by comparing (21) and (20) we have that $\widetilde{K}_1$ and $\widetilde{K}_2$ can be computed using (15) and $\widetilde{K}_3$ and $\widetilde{K}_4$ using (25), provided $I + DL_1$ nonsingular, and the proof is complete.

As a numerical example, consider the following process which can be shown to be unstable along the pass

$$A = \begin{bmatrix} -1.36 & -1.29 & -0.8 \\ 0.15 & 0.34 & 0 \\ -0.19 & 0 & -1.36 \end{bmatrix}, \ B_0 = \begin{bmatrix} 0.44 & 0.51 \\ 0.93 & 0.14 \\ 0.65 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.18 & -2.35 & 0.8 \\ 1.07 & -2.5 & 0.5 \\ -0.43 & 0.8 & 2.82 \end{bmatrix}, \ C = \begin{bmatrix} -0.38 & 0 & -0.37 \\ 0 & 0 & -0.98 \end{bmatrix}$$

$$D = \begin{bmatrix} -2.85 & -0.65 & -2.5 \\ -0.28 & -2.98 & 1.96 \end{bmatrix}, \ D_0 = \begin{bmatrix} -1.15 & 0 \\ -0.42 & 1.13 \end{bmatrix}$$

Then here Theorem 2 (explicit contribution from the current pass state vector) is successful but Theorem 3 is not. Theorem 4 is, however, successful with the following solution matrices $Y > 0$, $Z > 0$, $X = diag(X_1, X_2, X_3, X_4)$

$$X_1 = \begin{bmatrix} 101.1824 & 208.8786 \\ 208.8786 & 492.3833 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 2.1312 & 0 & 0 \\ 0 & 1.6720 & -2.7704 \\ 0 & -2.7704 & 17.2027 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 133.4038 & 173.4733 \\ 173.4733 & 238.0715 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 2619.8248 & 3243.2668 \\ 3243.2668 & 4033.1097 \end{bmatrix}$$

and N is of the structure defined in (23) where

$$N_1 = \begin{bmatrix} 280.3089 & 516.6972 \\ 84.8 & 155.6090 \\ -159.7937 & -310.9259 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} -281.4519 & -339.9495 \\ 898.5330 & 1115.4205 \\ -678.3573 & -847.8786 \end{bmatrix}$$

$$N_3 = 10^{-11} \times \begin{bmatrix} -0.2198 & -0.2865 \\ 0.0085 & 0.0110 \\ 0.1886 & 0.2458 \end{bmatrix}$$

$$N_4 = 10^{-11} \times \begin{bmatrix} 0.0478 & 0.0625 \\ 0.0172 & 0.0225 \\ -0.0156 & -0.0204 \end{bmatrix}$$

Hence

$$L_1 = \begin{bmatrix} 4.8613 & -1.0129 \\ 1.4944 & -0.3179 \\ -2.2186 & 0.3097 \end{bmatrix}$$

and K is of the structure (24) where

$$K_1 = \begin{bmatrix} -1.8473 & 0 & -0.8061 \\ -0.5679 & 0 & -0.2414 \\ 0.8431 & 0 & 0.5174 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.6894 & 0.4701 \\ 0.1328 & 0.1698 \\ 0.2964 & -0.4486 \end{bmatrix}$$

$$K_3 = 10^{-14} \times \begin{bmatrix} -0.9040 & 0.7980 \\ 0.0351 & -0.0309 \\ 0.7767 & -0.6855 \end{bmatrix}$$

$$K_4 = 10^{-14} \times \begin{bmatrix} 0.2108 & -0.1850 \\ 0.0757 & -0.0665 \\ -0.0686 & 0.0602 \end{bmatrix}$$

Finally, the output controller matrices for closed loop stability along the pass under the application of (19) computed using (15) and (25) are

$$\widetilde{K}_1 = \begin{bmatrix} 49.4899 & -40.7956 \\ 14.2722 & -11.7740 \\ -44.4902 & 36.4625 \end{bmatrix}$$

$$\widetilde{K}_2 = \begin{bmatrix} -1.7708 & 0.9557 \\ -0.1795 & 0.3063 \\ 1.2597 & -0.9670 \end{bmatrix}$$

$$\widetilde{K}_3 = 10^{-12} \times \begin{bmatrix} 0.3698 & -0.3263 \\ 0.1098 & -0.0968 \\ -0.3293 & 0.2905 \end{bmatrix}$$

$$\widetilde{K}_4 = 10^{-13} \times \begin{bmatrix} 0.6785 & -0.5950 \\ 0.1968 & -0.1726 \\ -0.6067 & 0.5321 \end{bmatrix}$$

Remark It is of interest to note that in the example here the elements of $\widetilde{K}_3$ and $\widetilde{K}_4$ are significantly smaller in magnitude than those in the other controller matrices. Also if these matrices are deleted from the control then it can be verified that the closed loop process is still stable along the pass but the design method of Theorem 3 fails. This feature also appears in numerous other numerical examples computed to date. Hence it can be conjured that this last design method can be exploited to reduce the degree of conservativeness due to the fact that the underlying LMI control law design is based on a sufficient but not necessary condition for stability along the pass.

5 CONCLUSIONS

One unique feature of repetitive processes in comparison to other classes of 2D systems is that it is possible define physically meaningful control laws for them. It is hence essential to have an analysis setting where such control laws can be designed for stability and/or performance.

Previous work has shown that, of the currently available tools, it is only an LMI based setting can meet this last specification. In this paper we have continued the development of control laws based on this analysis setting which critically remove the need to use current pass state feedback information.

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