MOBILE ROBOT LOCALIZATION USING LINEAR SYSTEM MODEL

Xu Zezhong[a,b], Liu Jilin[a]
a,Department of Information Science and Electronic Engineering,Zhejiang University, 310027, Hangzhou, China
b, Department of Computer Science and Engineering,Hangzhou Institute of Electronic Engineering, Hangzhou, China

Keywords: Mobile robot localization, Nonlinear state estimation, Extended Kalman filter, Linearization error.

Abstract: Localization is a fundamental problem for mobile robot autonomous navigation. EKF is an efficient tool for position estimation, but it suffers from linearization errors due to linear approximation of nonlinear system equations. In this paper we describe a position estimation method for mobile robot. Process and measurement equations are linear by appropriately constructing the state vector and system models. The position of mobile robot is estimated recursively based on optimal KF. It avoids linear approximation of nonlinear system equations and is free of linearization error. All these techniques have been implemented on our mobile robot ATRVII equipped with 2D laser rangefinder SICK.

1 INTRODUCTION

The mobile robot localization problem, especially for pose tracking, is one fundamental problem in mobile robotics. Dead reckoning technique is a solution to estimate the position of mobile robot. However, due to the uncertainties on odometer modeling errors and occasional slipping errors, it generally suffers from cumulative errors that increase without bounds (Borenstein et al. 1995). In order to overcome this disadvantage, probabilistic localization method (Olson 2000; Thrun 2000) is proposed by taking into account various uncertainties on system state and sensor measurement. In this way, the mobile robot localization problem can be described as state filtering or state estimation problem.

The optimal minimum mean square error (MMSE) estimation is conditional expectation conditioned on all prior observations. If system equations are linear, the optimal state estimation solution is the Kalman filter (KF) algorithm (Kalman 1960). Unfortunately, system equations are generally nonlinear in mobile robotics. System state estimation requires a complete description of the conditional probability density with unbounded number of parameters. To estimate position of robot, various suboptimal solutions for nonlinear estimation problem have been proposed based on different approximation techniques.

There are three types of approximation techniques. The first is to approximate probability distribution over state space. Grid-based Markov Localization (Fox et al. 1998) is a discrete approximation of probability density distribution over all possible position and Monte Carlo Localization (Fox et al. 1999) approximates the posterior probability distribution over the state space with a set of particles. The second is to approximate nonlinear system equation with linear function. Extended Kalman filter (EKF) (Kalman and Bucy 1961) and various variants apply the optimal Kalman filter to deal with nonlinear estimation problems by replacing nonlinear equations with linear approximation. The third is to compute conditional expectation with efficient numerical approximation method. Gaussian filter (Ito et al. 2000) and Gaussian sum filter (Alspach 1972) resort to numerical integration techniques to compute conditional mean and conditional covariance.

The Kalman filter has brought revolutionary improvement in stochastic estimation problem since its introduction in 1960. The linear KF was developed to provide optimal state estimation of linear system with noisy measurement. The EKF works by approximating the nonlinear process and measurement equations about the current state estimation. Classical EKF is a first order approximation filter. It approximates nonlinear function with first order Taylor series expansion evaluated at current state estimation. However, EKF

Zezhong X. and Jilin L. (2004).
MOBILE ROBOT LOCALIZATION USING LINEAR SYSTEM MODEL.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 243-248
DOI: 10.5220/0001136202430248
Copyright © SciTePress

has two drawbacks. One is that this approximation is not accurate if area around current estimation is very nonlinear. Another is Jacobian matrix computation. Some nonlinear functions may not be differentiable, even if it is differential, computing the derivatives may still be hard.

In order to improve the performance of state estimation, various modified EKF methods are proposed. The second-order filter (Nam et al. 1999) is known to achieve better precision than the first-order filter at the price of complex computation. Higher order filter (Kelly 1994) is proposed by taking into account higher order terms in the Taylor series expansion, but the more complex computations are involved. Iterated EKF (Bellaire et al. 1995) is designed to improve the precision of approximation by approximating measurement equation about the corrected posterior estimation instead of the predicted prior estimation. A range-direction-cosine EKF (Mehra 1971) transforms nonlinear measurement equation into linear function by choosing a suitable coordinates system. A modified EKF (Wall et al. 1997) algorithm offers better performance by approximating nonlinear equations about state vector computed from deterministic equation rather than state vector estimated from stochastic equation. Interlaced EKF (Glielmo et al. 1999) partitions the state vector into several parts. Each filter works independently and considers the other parts of state vector as known parameters. A modified EKF (Chui et al. 1990) improves the filtering performance by modifying the centre of Taylor series approximation. The linear regression Kalman filter (Lefebvre et al. 2002) approximates the process and measurement functions by statistical linear regression of the function with some sampling points. A typical LRKF is unscented Kalman filter (UKF) (Julier et al. 1995; Wan et al. 2000). It approximates nonlinear process and measurement function with sigma points. A NMSKF (Lefebvre et al. 2002) linearize process and measurement equations in a higher-dimensional state space. It is applicable to state estimation for static system and for a limited class of dynamic systems.

In this paper, we apply KF to estimate recursively the position of robot. The process and measurement equation are linear by constructing appropriately the system state and system models. With linear system equations, the position estimation of robot is optimal.

An outline of this paper is as follows. In Section 2 we describe the coordinate system. In Section 3 we describe the system state space and system models. Process equation and measurement equation are defined. In Section 4 an efficient filter algorithm with linear process equation and linear measurement equation is described. Experimental results are presented in Section 5. Finally we conclude in Section 6.

2 COORDINATE SYSTEM

There are four coordinate systems. The first is world coordinate system denoted with $X_W O_W Y_W$. The second is odometer coordinate system denoted with $X_O O_O Y_O$. The third is scanner coordinate system denoted with $X_S O_S Y_S$. The fourth is the local coordinate system of mobile robot, denoted with $X_L O_L Y_L$. Robot local coordinate system has the same orientation and origin as the odometer coordinate system. Scanner coordinate system has the same orientation as robot local coordinate system. They are shown in Fig.1.

Robot local coordinate system, sensor coordinate

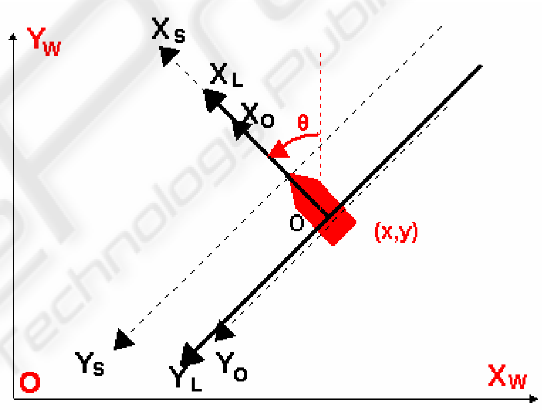

Figure 1: coordinate system.

system and odometer coordinate system are defined according to DIN70000. All angle are in the range of $-180^O \ldots 180^O$. They are shown in Fig. 2.

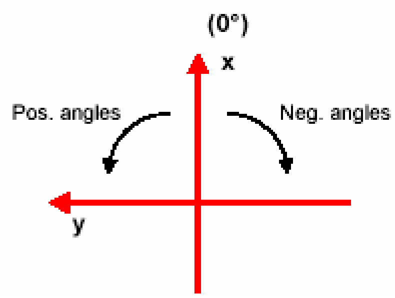

Figure 2: DIN70000

In this paper, only world coordinate system and robot local coordinate system are considered. All values represented with scanner and odometer coordinate system are transformed into robot local coordinate system.

3 STATE SPACE AND SYSTEM MODEL

Mobile robot moves in an indoor environment. Robot is equipped with an odometer and a laser rangefinder. The data from the odometer is used to predict the position of robot. Laser rangefinder scans environment information and extracts natural landmark, corner, to correct the predicted position of robot. Data from sensor odometer and rangefinder are all with uncertainty. The Kalman filter has proven to be a valuable tool for mobile robot position estimation given that the initial state and covariance are known. The position of robot, $[x, y, \theta]^T$, is considered as system state. Process equation is based on odometer model and the world coordinate of landmark is considered as observation information. The position of robot is recursively estimated as it evolves through time.

3.1 State space

We assume that environment is a 2D plane and represent this 2D plane with a world coordinates system $X_WO_WY_W$. Another is robot's local coordinates system $X_LO_LY_L$. The configuration of robot is represented with a three-dimensional state vector $[x, y, \theta]^T$. (x, y) represents the position of robot. θ represents the orientation or heading of the robot. It is defined in the counterclockwise. The value of θ is from –π to π. It is shown in Fig.1.

The position prediction of robot is according to the motion model. The observation prediction is according to the observation model and the predicted position of robot is corrected according to updated rules.

3.2 Motion model

It is assumed that the robot moves along a circular arc at each step. The position transition is based on odometer information. Date from odometer is $U_k = (D_k, \gamma_k)$ at step k. D_k is the distance traveled along the arc and γ_k is the change in motion direction. $R_k = D_k / \gamma_k$ is the radius of arc. According to motion model shown in Fig.3, the deterministic process equation is:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \frac{\mathbf{D}_k}{\gamma_k}\left(\cos(\theta_k + \gamma_k) - \cos(\theta_k)\right)$$

$$\mathbf{y}_{k+1} = \mathbf{y}_k + \frac{\mathbf{D}_k}{\gamma_k}\left(\sin(\theta_k + \gamma_k) - \sin(\theta_k)\right)$$

$$\theta_{k+1} = \theta_k + \gamma_k$$

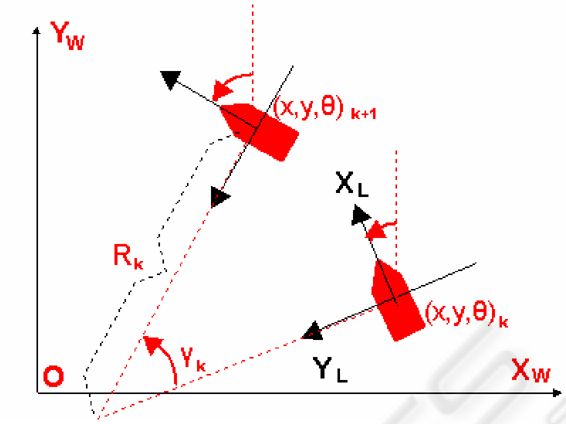

Figure 3: Motion model.

3.3 Observation model

Laser rangefinder scans environmental information and extracts landmark. A landmark is a typical feature of environment. In this paper the corner is considered as natural landmark. A range scan is segmented and merged. The corner is extracted from current laser scan and is represented with (x_L, y_L) in local coordinates system. The corner is represented with (x_W, y_W) in world coordinates system. We use the position (x_W, y_W) of a corner as the observation value. The position (x_L, y_L) of corner is computed in local coordinates system. It is translated into world coordinates system as observation prediction based on current estimated position of robot. It is a two-dimensional observation vector. According to observation model shown in Fig.4, the deterministic measurement equation is:

$$\mathbf{x}_W = \mathbf{x}_k - \mathbf{x}_L \sin(\theta_k) - \mathbf{y}_L \cos(\theta_k)$$

$$\mathbf{y}_W = \mathbf{y}_k + \mathbf{x}_L \cos(\theta_k) - \mathbf{y}_L \sin(\theta_k)$$

4 LOCALIZATION WITH KF

EKF approximates nonlinear system equations with first-order terms of Taylor series and induces linearization errors. If the initial position is assumed as Gaussian distribution, the result position is not Gaussian distribution after a nonlinear transformation. So, it is incomplete that only position expectation and position covariance are recursively estimated. It is not an optimal estimation solution for mobile robot localization with nonlinear system equations.

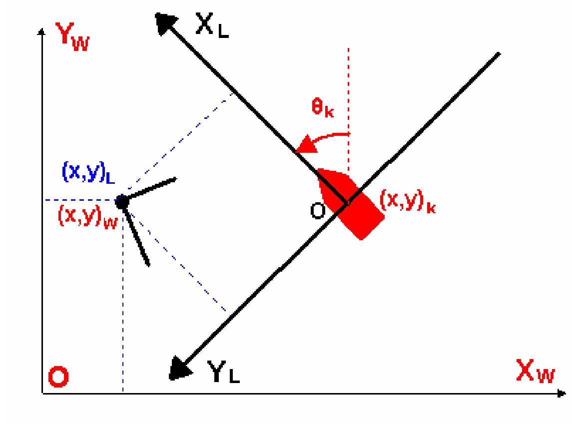

Figure 4: Observation model.

We apply optimal linear Kalman filter to deal with mobile robot localization by constructing appropriately system state vector and system models. In this way, the process equation and measurement equation are linear. So, we estimate the position of robot directly with a linear Kalman filter. This estimation solution is optimal in MMSE sense. State vector X_k, observation vector Z_k and input vector U_k are defined as:

$$\mathbf{X}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{y}_k \\ \mathbf{s}_k \\ \mathbf{c}_k \end{bmatrix} \quad \mathbf{Z}_k = \begin{bmatrix} \mathbf{x}_W \\ \mathbf{y}_W \end{bmatrix} \quad \mathbf{U}_k = \begin{bmatrix} \mathbf{D}_k \\ \gamma_k \end{bmatrix}$$

We extend system state to four-dimensional vector $[x_k, y_k, s_k, c_k]^T$. (x_k, y_k) is the position of robot. (s_k, c_k) is the sin and cosine of orientation θ of robot. System state is estimated recursively predicted and updated. The position (x_k, y_k, θ_k) of robot and its variance are computed from mean and variance of system state $[x_k, y_k, s_k, c_k]^T$. Observation vector $Z=[x_W, y_W]^T$ is the position of corner in world coordinates system. By replacing $\sin(\theta_k)$ and $\cos(\theta_k)$ in system models with s_k and c_k, we get linear process equation and linear measurement equation. The process equation is:

$$\begin{aligned} \mathbf{X}_{k+1|k} &= \mathbf{f}(\mathbf{X}_{k|k}, \mathbf{U}_k) + \omega_k \\ &= \begin{bmatrix} \mathbf{x}_{k|k} + \frac{\mathbf{D}_k}{\gamma_k}\left(\mathbf{c}_{k|k}\cos(\gamma_k) - \mathbf{s}_{k|k}\sin(\gamma_k) - \mathbf{c}_{k|k}\right) \\ \mathbf{y}_{k|k} + \frac{\mathbf{D}_k}{\gamma_k}\left(\mathbf{s}_{k|k}\cos(\gamma_k) + \mathbf{c}_{k|k}\sin(\gamma_k) - \mathbf{s}_{k|k}\right) \\ \mathbf{s}_{k|k}\cos(\gamma_k) + \mathbf{c}_{k|k}\sin(\gamma_k) \\ \mathbf{c}_{k|k}\cos(\gamma_k) - \mathbf{s}_{k|k}\sin(\gamma_k) \end{bmatrix} + \omega_k \\ &= \begin{bmatrix} 1 & 0 & -\frac{\mathbf{D}_k}{\gamma_k}\sin(\gamma_k) & \frac{\mathbf{D}_k}{\gamma_k}(\cos(\gamma_k) - 1) \\ 0 & 1 & \frac{\mathbf{D}_k}{\gamma_k}(\cos(\gamma_k) - 1) & \frac{\mathbf{D}_k}{\gamma_k}\sin(\gamma_k) \\ 0 & 0 & \cos(\gamma_k) & \sin(\gamma_k) \\ 0 & 0 & -\sin(\gamma_k) & \cos(\gamma_k) \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k|k} \\ \mathbf{y}_{k|k} \\ \mathbf{s}_{k|k} \\ \mathbf{c}_{k|k} \end{bmatrix} + \omega_k \\ &= \mathbf{F}_k * \mathbf{X}_{k|k} + \omega_k \end{aligned}$$

Where ω_k is noise vector representing uncertainty on odometer modeling and slipping. Its mean is zero and variance is Q.

The measurement equation is:

$$\begin{aligned} \mathbf{Z}_k &= \mathbf{h}(\mathbf{X}_{k+1|k}) + \upsilon_k \\ &= \begin{bmatrix} \mathbf{x}_{k+1|k} - \mathbf{x}_L\mathbf{s}_{k+1|k} - \mathbf{y}_L\mathbf{c}_{k+1|k} \\ \mathbf{y}_{k+1|k} + \mathbf{x}_L\mathbf{c}_{k+1|k} - \mathbf{y}_L\mathbf{s}_{k+1|k} \end{bmatrix} + \upsilon_k \\ &= \begin{bmatrix} 1 & 0 & -\mathbf{x}_L & -\mathbf{y}_L \\ 0 & 1 & -\mathbf{y}_L & \mathbf{x}_L \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k+1|k} \\ \mathbf{y}_{k+1|k} \\ \mathbf{s}_{k+1|k} \\ \mathbf{c}_{k+1|k} \end{bmatrix} + \upsilon_k \\ &= \mathbf{H}_k * \mathbf{X}_{k+1|k} + \upsilon_k \end{aligned}$$

Where υ_k is noise vector representing uncertainty on sensor measurement. Its mean is zero and variance is R.

Matrix F_k only depends on the input vector and is uncorrelated with current position estimation. Matrix H_k only depends on the position of landmark and is uncorrelated with current position prediction. So, process and measurement equations are linear. The position of mobile robot is predicted and corrected according to following update rules:

$$\hat{\mathbf{X}}_{k+1|k} = \mathbf{F}_k * \hat{\mathbf{X}}_{k|k}$$

$$\mathbf{P}_{k+1|k} = \mathbf{F}_k * \mathbf{P}_{k|k} * \mathbf{F}_k^{\mathbf{T}} + \mathbf{Q}$$

$$\hat{\mathbf{X}}_{k+1|k+1} = \hat{\mathbf{X}}_{k+1|k} + \mathbf{K}_k\left(\mathbf{Z}_k - \mathbf{H}_k * \hat{\mathbf{X}}_{k+1|k}\right)$$

$$\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}_k * \mathbf{S}_k * \mathbf{K}_k^{\mathbf{T}}$$

$$\mathbf{S}_k = \mathbf{H}_k * \mathbf{P}_{k+1|k} * \mathbf{H}_k^{\mathbf{T}} + \mathbf{R}$$

$$\mathbf{K}_k = \mathbf{P}_{k+1|k} * \mathbf{H}_k^{\mathbf{T}} * \mathbf{S}_k^{-1}$$

From step k to step k+1, system state mean and covariance are predicted and corrected recursively. The resulting position estimation of robot is an optimal estimation. If the initial position of robot is assumed as Gaussian distribution, the probability distribution of position after each step is still Gaussian. It is reasonable that only position mean and variance are recursively computed.

5 EXPERIMENTAL RESULT

In this section, we demonstrate the experimental result of proposed position estimation method. By extending the state space of robot to four-dimensional state vector, the system process and measurement equations are linear. Localization based on linear system model avoids linearization error due to linear approximation of nonlinear system equations.

In this section, we demonstrate the experimental results of proposed position estimation method with our mobile robot ATRVII. Fig. 5 is robot equipped with laser range rangefinder. Robot moves around our lab, scans landmark features and estimate its position and heading.

Figure 5: Mobile robot.

Fig. 6 shows the estimated position errors with one sigma confidence limit. Fig. 7 shows innovation sequence and variance. Experimental results show that the filter proposed here is consistent and convergent.

With the same parameter, the position of robot is estimated with classical EKF. Fig. 8 shows difference of estimation results between KF and EKF. Experimental results show that the Linear filter gives more conservative estimation result than nonlinear filter.

KF is an optimal state estimation for linear system. Position estimation based on EKF induces linearization error and brings additional uncertainty. linearization error due to linear approximation of

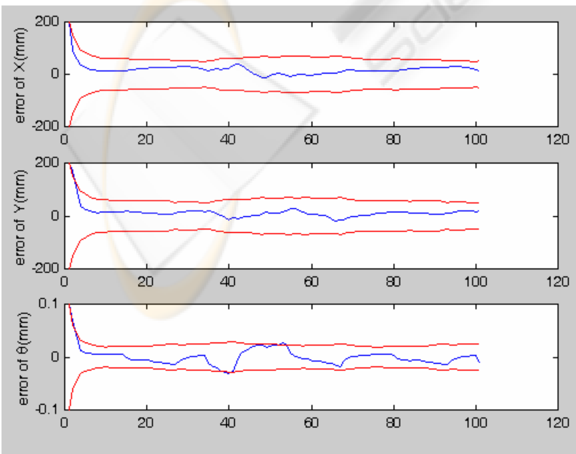

Figure 6: Estimated position error with KF.

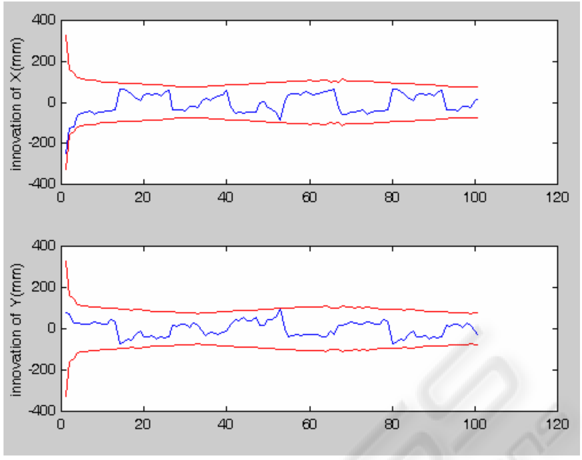

Figure 7: Innovation and innovation variance with KF.

nonlinear process equation affects mainly the long-term position estimation. linearization error due to linear approximation of nonlinear measurement equation affects mainly the short-term position estimation. By constructing appropriately state vector and system models, we use optimal Kalman filter for mobile robot position estimation. It is free of linearization errors.

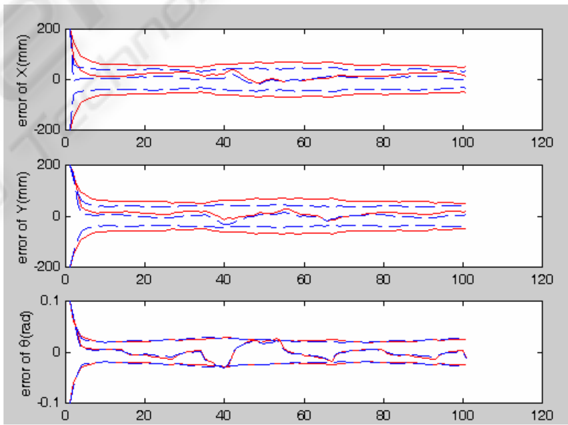

Figure 8: Position estimation with KF and EKF. solid line for KF and dashed line for EKF.

6 CONCLUSION

The extended Kalman filter has been widely used as a position estimation method for mobile robot localization and simultaneous localization and map building (SLAM) problem. However, The classical extended Kalman filter for this application suffers from a fundamental flaw. Linear approximation of nonlinear system equations with first-order Taylor series induces linearization error. In this paper, a position estimation method with linear process and measurement equations is developed. Process and measurement equations are linear by appropriately constructing state vector and system models. With linear process and measurement function, we apply linear Kalman filter to estimate optimally the position of robot. It avoids linear approximation of nonlinear system equations and is free of linearization error. The filter is consistent and convergent. Comparing with EKF, it gives more conservative estimation result.

REFERENCES

Alspach, D. and Sorenson, H. 1972. Nonlinear Bayesian estimation using Gaussian sum approximations. IEEE *Transactions on Automatic Control*, vol. AC-17, pp. 439-448.

Bellaire, R.L., Kamen E.W. and Zabin, S.M. 1995. A new nonlinear iterated filter with applications to target tracking. In SPIE Proceedings *on Signal and Data Processing of Small Targets*, vol. 2561, pp. 240-251.

Borenstein, J. and Feng, L. 1995. Correction of Systematic Odometer Errors in Mobile Robots. In Proceedings of the 1995 International Conference *on Intelligent Robots and Systems (lROS'95)*, Pittsburgh, Pennsylvania, pp. 569-574.

Chui, C.K., Chen, G. and Chui, H. 1990. Modified extended Kalman filtering and parallel system parameter identification, IEEE Transactions *on Automatic Control*, 35 (1): 100-104.

Fox, D., Burgard, W., Thrun, S. and Cremers, A.B. 1998. Position Estimation for Mobile Robots in Dynamic Environments. *AAAI/IAAI 1998*: 983-988.

Fox, D., Burgard, W., Dellaert F. and Thrun, S. 1999. Monte Carlo Localization: Efficient Position Estimation for Mobile Robots. *AAAI/IAAI 1999*: 343-349.

Glielmo, L., Setola R. and Vasca, F. 1999. An Interlaced Extended Kalman Filter, IEEE Transactions *on Automatic Control*, vol. 44, no. 8, pp. 1546-1549.

Ito, K. and Xiong, K. 2000. Gaussian Filters for Nonlinear Filtering Problems. IEEE Transactions *on Automatic Control*, 45(5): 910-927.

Julier, S., Uhlmann J. and Durrant-Whyte, H. 1995. A new approach for filtering nonlinear systems. In Proceedings of the *American Control Conference*, pp.1628-1632.

Kalman, R.E. 1960. A New Approach to Linear Filtering and Prediction Problems. Journal of *Basic Engineering Transactions of the ASME*, pp.35-45.

Kalman, R.E. and Bucy, R. 1961. New results in linear filtering and prediction theory. Journal of *Basic Engineering Transactions of the ASME*, vol. 83, pp. 95-108.

Kelly, A.J. 1994. A 3D State Space Formulation of a Navigation Kalman Filter for Autonomous Vehicles, *CMU Robotics Institute Technical Report* CMU-RI-TR-94-19.

Lefebvre, T., Bruyninckx, H. and De Schutter, J. 2002. Comment on “A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators”. IEEE Transactions On *Automatic Control*, Vol. 47, no. 8, pp.1406-1408.

Lefebvre, T., Bruyninckx, H. and De Schutter, J. 2003. Non-linear Autonomous Compliant Motion with a Non-minimal State Kalman Filter. In International Conference on *advanced Robotics*, pp.136-141.

Mehra, R.K. 1971. A Comparison of Several Nonlinear Filters for Reentry Vehicle Tracking. IEEE Transactions on *Automatic Control*, AC-16(4): 307-319.

Nam, K.H. and Tahk, M.J. 1999. A second-order stochastic filter involving coordinate transformation. IEEE Transactions on *Automatic Control*, Vol. 44, No.3, pp. 603-608.

Olson, C.F. 2000. Probabilistic self-localization for mobile robots. IEEE Transactions on *Robotics and Automation*, vol. 16(1), pp. 55-66.

Thrun, S. 2000. Probabilistic algorithms in robotics, *AI Magazine*, 21(4): 93-109.

Wall, D.S. and Gaston, F.M.F. 1997. Modified extended kalman filtering. In Proceedings of International Conference on *Digital Signal Processing,* pp.703-706.

Wan, E.A. and van der Merwe, R. 2000. The unscented Kalman filter for nonlinear estimation. In Proceedings of Conf. *Adaptive Systems for Signal Processing, Communication and Control*, Canada.