SIMULTANEOUS LOCALIZATION AND MAPPING BASED ON MULTI-RATE FUSION OF LASER AND ENCODERS MEASUREMENTS

Leopoldo Armesto, Josep Tornero
Dept. of Systems Engineering and Control, Technical University of Valencia
Camino de Vera s/n, 46020, Spain

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Abstract: The SLAM problem in static environments with EKF is adapted for multi-rate sensor fusion of encoders and laser rangers. In addition, the formulation is general and can be adapted for any multi-rate sensor fusion application. The proposed algorithm, based on well-known techniques for feature extraction, data association and map building, is validated with some experimental results. This algorithm should been seen as a part of a complete autonomous robot navigation algorithm, also described in the paper.

1 INTRODUCTION

SLAM problem addresses the simultaneously locate and build a map using a mobile robot with no previous knowledge of robot initial localization and the map (environment). A number of approaches have already been proposed to solve the SLAM problem since the seminal paper (Smith et al., 1988) was presented. The most relevant of these are based on *grid-based methods* (Thrun et al., 1998) and *parametric methods* (Dissanayake et al., 2001), (Jensfelt and Christensen, 2001), (Castellanos et al., 2001).

Kalman filter approach of SLAM consists on joining the robot state and the set of landmark parameters of the environment. It is well known that landmark covariance decreases monotonically. In fact, in the limit, the determinant of the covariance matrix of a map containing more than one landmark converges to zero and is fully correlated (Dissanayake et al., 2001). The main advantage of this approach is that KF gives a robust, optimal recursive state estimation to fuse redundant information from different sensors, assuming Gaussian noise on the system and measurements.

Multi-rate fusion is used when sensors have different sampling rates. In any complex application, it is unrealistic to assume the same sampling period for all system variables. In mobile robots, sensors such as laser rangers, sonars, radars, encoders, GPS, vision systems, etc., have different sampling rates.

In this paper, we present a realistic approach to the SLAM problem, where sensor measurements are treated as system outputs at their sampling rates. In this approach data-missing problems are easily considered. In particular, we fuse encoder measurements at fast sampling and laser ranger measurements at slow sampling. The proposed multi-rate SLAM is more appropriate for real-time control applications, because it produces vehicle and map estimations at the fast sampling rate of the control.

2 SLAM WITH MULTI-RATE FUSION

2.1 Vehicle and Landmark Models

Let the robot pose be described by the following discrete dynamic equation,

$$\mathbf{x}_r(k+1) = \mathbf{f}_r(\mathbf{x}_r(k)) + \gamma_r(\mathbf{x}_r(k), \mathbf{w}(k))$$

with $\mathbf{x}_r(k) = [x(k)\ y(k)\ \theta(k)\ v(k)\ \omega(k)]^T$ the robot state vector with Cartesian positions $x(k)$ and $y(k)$, orientation $\theta(k)$, linear velocity $v(k)$ and angular velocity $\omega(k)$. The robot constant velocity model is,

$$\mathbf{f}_r(\mathbf{x}_r(k)) = \begin{bmatrix} x(k) + Tv(k)\cos(\theta(k)) \\ y(k) + Tv(k)\sin(\theta(k)) \\ \theta(k) + T\omega(k) \\ v(k) \\ \omega(k) \end{bmatrix}$$

where T is the sampling period.

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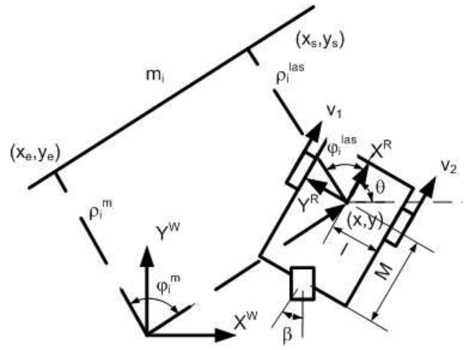


Figure 1: Vehicle and landmark variables definition

Inputs to the system are linear acceleration $a(k)$ and angular acceleration $\alpha(k)$ of the robot, which are unknown and therefore treated as system noises $\mathbf{w}(k) = [a(k)\ \alpha(k)]^T$ with Gaussian distribution $\mathbf{w}(k) \sim N(\mathbf{0}, \mathbf{Q}(k))$. The noise mapping to robot states is given by,

$$\gamma_r(\mathbf{x}_r(k), \mathbf{w}(k)) = \begin{bmatrix} \frac{T^2}{2} a(k)\cos(\theta(k)) \\ \frac{T^2}{2} a(k)\sin(\theta(k)) \\ \frac{T^2}{2}\alpha(k) \\ Ta(k) \\ T\alpha(k) \end{bmatrix}$$

Let $\mathcal{M}$ denote the map of the environment, which is a set of segment lines m_i. Each segment, m_i is described by a set of parameters: distance of a line to the origin $\rho_i^m(k)$, orientation $\varphi_i^m(k)$, start point $(x_{s,i}^m, y_{s,i}^m)$ and end point $(x_{e,i}^m, y_{e,i}^m)$, as shown in figure 1. The use of redundant information is due to the fact that $\rho_i^m(k)$ and $\varphi_i^m(k)$ are used in the map state vector $\mathbf{x}_{m,i}(k) = [\rho_i^m(k)\ \varphi_i^m(k)]^T$, the start and end points are used to predict only *visible* segment lines. Since the landmarks are supposed to be stationary, their dynamic description is simply $\mathbf{x}_m(k+1) = \mathbf{x}_m(k)$, with $\mathbf{x}_m(k) = [\mathbf{x}_{m,1}^T(k)\ \ldots\ \mathbf{x}_{m,n}^T(k)]^T$.

In SLAM, the robot and the map states are treated together $\mathbf{x}(k) = [\mathbf{x}_r^T(k)\ \mathbf{x}_m^T(k)]^T$, its covariance is,

$$\mathbf{P}(k) = \begin{bmatrix} \mathbf{P}_{rr}(k) & \mathbf{P}_{rm}(k) \\ \mathbf{P}_{rm}^T(k) & \mathbf{P}_{mm}(k) \end{bmatrix}$$

The prediction of the state and its covariance is,

$$\hat{\mathbf{x}}(k+1|k) = \begin{bmatrix} \mathbf{f}_r(\hat{\mathbf{x}}_r(k)) \\ \hat{\mathbf{x}}_m(k) \end{bmatrix}$$

$$\mathbf{P}_{rr}(k+1|k) = \mathbf{F}_r(k)\mathbf{P}_{rr}(k)\mathbf{F}_r^T(k) + \mathbf{\Gamma}_r(k)\mathbf{Q}(k)\mathbf{\Gamma}_r^T(k)$$
$$\mathbf{P}_{rm}(k+1|k) = \mathbf{F}_r(k)\mathbf{P}_{rm}(k)$$
$$\mathbf{P}_{mm}(k+1|k) = \mathbf{P}_{mm}(k)$$

where $\mathbf{F}_r(k)$ and $\mathbf{\Gamma}_r(k)$ are the Jacobians of $\mathbf{f}_r(\mathbf{x}_r(k))$ and $\gamma_r(\mathbf{x}_r(k), \mathbf{w}(k))$, respectively.

2.2 Measurement Model

In our application, the robot is an industrial forklift with tricycle configuration. We have sensed the two front wheels with incremental encoders, the rear wheel with an absolute encoder and we have installed laser rangers at the front and rear of the vehicle (Mora et al., 2003). The measurement equations are particularized for these sensors, however it is possible to extend the model to other sensors such as GPS, vision, compass, sonar, etc.

Measurement equations of encoders are as follows:

$$\mathbf{y}^{inc} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbf{h}^{inc}(\mathbf{x}_r) + \mathbf{u}^{inc} = \begin{bmatrix} v - l\omega \\ v + l\omega \end{bmatrix} + \begin{bmatrix} u_1^{inc} \\ u_2^{inc} \end{bmatrix} \quad (1)$$

$$\mathbf{y}^{abs} = \beta = \mathbf{h}^{abs}(\mathbf{x}_r) + \mathbf{u}^{abs} = \frac{-M\omega}{v} + u^{abs} \quad (2)$$

where $\mathbf{u}^{inc}$ and $\mathbf{u}^{abs}$ are measurement noises with covariances $\mathbf{R}^{inc}$ and $\mathbf{R}^{abs}$, respectively. For simplicity we suppress index k where no confusion can arise.

For each detected line, we the following sensor model (Jensfelt and Christensen, 2001),

$$\mathbf{y}_i^{las} = \begin{bmatrix} \rho_i^{las} \\ \varphi_i^{las} \end{bmatrix} = \mathbf{h}_i^{las}(\mathbf{x}_r, \mathbf{x}_{m,i}) + \mathbf{u}_i^{las} = \quad (3)$$
$$= \begin{bmatrix} \rho_i^m - \sqrt{x^2+y^2}\cos(\xi - \varphi_i^m) \\ \varphi_i^m - \theta \end{bmatrix} + \begin{bmatrix} u_{\rho,i}^{las} \\ u_{\varphi,i}^{las} \end{bmatrix}$$

where $\xi = \arctan(y, x)$ while $\mathbf{u}_i^{las}$ is the measurement noise with covariance $\mathbf{R}_i^{las}$.

Detected landmarks are those which successfully pass the Data Association Test (DAT), described below. When the DAT fails, the detected feature is included in a temporary map $\mathcal{M}'$, where the i-th element $\mathbf{x}_{tm,i}$ is defined by the line parameters with respect to the global frame. Until the feature is sufficiently stable, it is not included in $\mathcal{M}$. However, DAT may fail despite the fact data belong to the same feature. In that case, in order to avoid repeated landmarks, a test between the map and the temporary map is also performed. Only successfully associated elements are included as measurements,

$$\mathbf{y}_i^{tm} = \begin{bmatrix} \rho_i^{tm} \\ \varphi_i^{tm} \end{bmatrix} = \mathbf{h}_i^{tm}(\mathbf{x}_{m,i}) + \mathbf{u}_i^{tm} = \begin{bmatrix} \rho_i^m \\ \varphi_i^m \end{bmatrix} + \begin{bmatrix} u_{\rho,i}^{tm} \\ u_{\varphi,i}^{tm} \end{bmatrix} \quad (4)$$

where $\mathbf{u}_i^{tm}$ is the noise with covariance $\mathbf{R}_i^{tm}$.

The Jacobians of equations (1)-(4) are respectively denoted as $\mathbf{H}^{inc}(k)$, $\mathbf{H}^{abs}(k)$, $\mathbf{H}^{las}(k)$ and $\mathbf{H}^{tm}(k)$.

2.3 Multi-rate Fusion

Assume a general multi-rate sampling with periods associated to incremental encoders (T^{inc}), absolute encoder (T^{abs}), laser measurements (T^{las}) and the measurements associated with the temporary map (T^{tm}). The base period is the great common divisor of all sampling rates, $T =$

$\gcd(T^{inc}, T^{abs}, T^{las}, T^{tm})$. We also define periodicity ratios $r^{inc} = T^{inc}/T$, $r^{abs} = T^{abs}/T$, $r^{las} = T^{las}/T$, $r^{tm} = T^{tm}/T$.

Sensor measurements depend on functions $\delta^{inc}(k)$, $\delta^{abs}(k)$, $\delta^{las}(k)$, $\delta^{tm}(k)$, indicating whether their respective measurements are valid ($\delta(k) = 1$) or not ($\delta(k) = 0$). Thus, we can easily: discard erroneous incremental encoder measurements if we detect wheel slippage; not consider the absolute encoder measurement when the vehicle is stopped, since its measurement equation $\mathbf{h}^{abs}(\mathbf{x}(k))$ is undetermined for $v = 0$; not include features which do not pass the DAT.

Multi-rate fusion is performed by containing available measurements in rows on the output vector $\mathbf{y}(k)$,

$$\mathbf{y}^{inc}(k) \subseteq \mathbf{y}(k) \text{ iff } \mathrm{mod}(k, r^{inc}) = 0 \text{ and } \delta^{inc}(k) = 1$$
$$\mathbf{y}^{abs}(k) \subseteq \mathbf{y}(k) \text{ iff } \mathrm{mod}(k, r^{abs}) = 0 \text{ and } \delta^{abs}(k) = 1$$
$$\mathbf{y}^{las}(k) \subseteq \mathbf{y}(k) \text{ iff } \mathrm{mod}(k, r^{las}) = 0 \text{ and } \delta^{las}(k) = 1$$
$$\mathbf{y}^{tm}(k) \subseteq \mathbf{y}(k) \text{ iff } \mathrm{mod}(k, r^{tm}) = 0 \text{ and } \delta^{tm}(k) = 1$$

Valid measurements are also combined by rows in the Jacobian matrix $\mathbf{H}(k)$. Noise covariance matrix, $\mathbf{R}(k)$, is a diagonal matrix of valid measurement covariances.

The state update is as usual in EKF:

$$\mathbf{S}(k+1) = \mathbf{H}(k+1)\mathbf{P}(k+1|k)\mathbf{H}^T(k+1) + \mathbf{R}(k+1)$$
$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}^T(k+1)\mathbf{S}^{-1}(k+1)$$
$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}(k+1)(\mathbf{y}(k+1) - \mathbf{h}(\hat{\mathbf{x}}(k+1|k)))$$
$$\mathbf{P}(k+1) = (\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}(k+1))\mathbf{P}(k+1|k)$$

3 IMPLEMENTED ALGORITHM

The overall diagram can be seen in figure 2, where white blocks are executed at a high sampling rate, gray ones at a low sampling rate and dotted blocks act as interfaces between high and low sampling rates. Usually, object path planning and obstacle avoidance algorithms require high computational power and should be executed at lower sampling rates than those required by the position controller. In this sense, the block MR-HOH (multi-rate high order holds) is used to extrapolate reference points to the control at high sampling rate from points provided at a low sampling rate (Armesto and Tornero, 2003).

3.1 Feature Extraction

For feature extraction, we have implemented the RIEPFA (Recursive Iterative End Point Fit Algorithm) (Duda and Hart, 1973). The algorithm performs the segmentation of a laser scan providing points belonging to the same line segment. Parameters of segment lines are estimated with the Orthogonal Least-Squares method. (Deriche et al., 1992) give

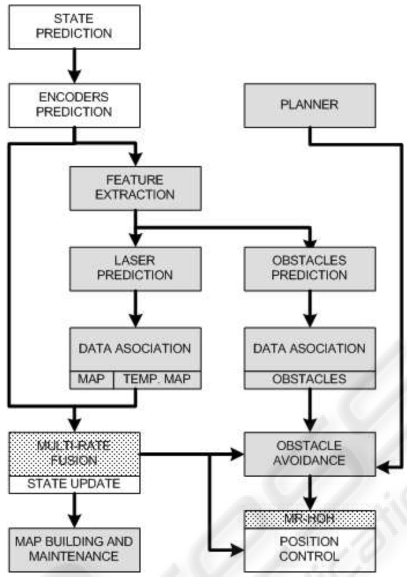


Figure 2: Bock diagram for SLAM with multi-rate fusion, including obstacle avoidance and position control

the exact formulation for computing line parameter covariances, assuming that data are affected by Cartesian and polar noise.

3.2 Data Association

The objective of data association is to associate measurements to the features in the map. In this work, data association is based on the Mahalanobis distance in the innovation space (Bar-Shalom and Fortman, 1988). The innovation matrix of a feature i is given by:

$$\mathbf{S}_i^{las} = \mathbf{H}_i^{las} \begin{bmatrix} \mathbf{P}_{rr} & \mathbf{P}_{ri} \\ \mathbf{P}_{ir} & \mathbf{P}_{ii} \end{bmatrix} (\mathbf{H}_i^{las})^T + \mathbf{R}_i^{las}$$

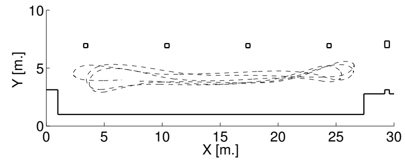
The innovation is $\mathbf{z}_i^{las} = \mathbf{y}_i^{las} - \mathbf{h}_i^{las}(\mathbf{x}_r, \mathbf{x}_{m,i})$ and the Mahalanobis distance $(\mathbf{z}_i^{las})^T(\mathbf{S}_i^{las})^{-1}\mathbf{z}_i^{las} \leq \eta$, where $\eta = 9.0$ represents 98.9% of a 2D Gaussian curve. Additional gating conditions have been considered, such as the norm $||\mathbf{z}_i^{las}||$, should be lower than a threshold, $\mathbf{z}_{norm}$, and segments separated by more than a distance, l_{max}, are treated separately.

3.3 Map Building and Maintenance

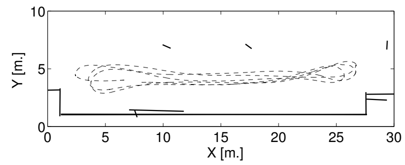
In this section, we describe how a new feature m_{N+1} is incorporated into the system vector state,

$$\mathbf{x}_{m,N+1} = \mathbf{x}_{tm,i}, \quad \mathbf{P}_{N+1N+1} = \mathbf{P}_{tm,ii}$$

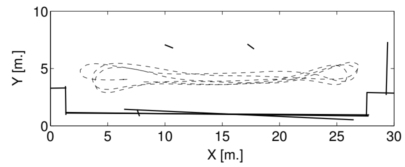
where $\mathbf{x}_{tm,i}$ is the feature in the temporary map $\mathcal{M}'$ once it has already been validated. The feature covariance in the temporary map is assigned to the new feature covariance in the map. For simplicity, the cross-



(a) *Ground-truth* robot trace



(b) SLAM with multi-rate EKF



(c) SLAM with single-rate EKF

Figure 3: Robot trace estimation and map built with the different Kalman filter approaches.

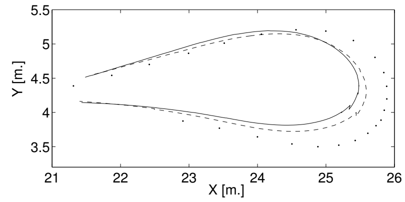


Figure 4: Trace comparison between *ground-truth* (solid line), MR-EKF (dashed line) and SR-EKF (dotted line).

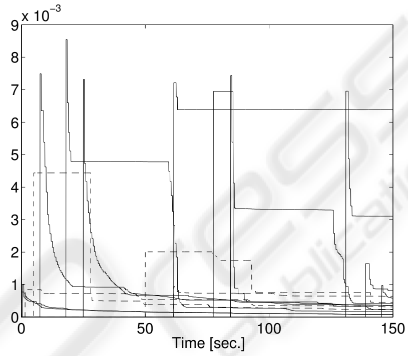


Figure 5: Standard deviation of estimated landmarks (σ_m^2). Solid lines (walls) and dashed lines (columns).

covariances of the new feature with the robot state and the rest of the features is initialized to zero.

4 EXPERIMENTAL RESULTS

Data has been taken from a parking lot, where encoder measurements were sampled every 50ms and the laser scans every 500ms for 150 sec. *Ground-truth* estimation has been performed using a detailed parking layout, see figure 3(a). In addition, the vehicle trace and map estimation using the proposed multi-rate filter is depicted in figure 3(b), where it can be appreciated that the trace estimation is very similar to *ground-truth* estimation and the estimated map contains all the main features of the parking lot.

In order to compare the benefits of using the multi-rate filter, the single-rate estimation is also performed, where all measurements were sampled at 500ms. Figure 4 shows the trace estimation of the three filters, during the first turn of the trajectory (time between 14.5-26.5sec.). It should be noted that, depending on the tuning parameters, we have found many numerical cases where the multi-rate sampling stabilized the estimation, which was not stable at single-rate.

Landmark standard deviation with the multi-rate filter are shown in figure 5. Landmark covariance is not represented until incorporated to the map.

5 CONCLUSIONS

In this paper, a multi-rate simultaneous localization and mapping based on Extended Kalman filter has been presented. Real-time control applications can clearly benefit from multi-rate SLAM algorithm, because vehicle and map estimation are computed at the fast sampling rate of the vehicle control. The algorithm is based on well-known techniques for feature extraction, data association and map building. The algorithm should be seen as part of a complete autonomous robot navigation system.

Experimental results have shown that the estimation is successfully performed, with the trace of the robot and the map very close to the *ground-truth* estimation. In addition, MR-EKF performance is much improved with respect to SR-EKF.

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