A NOVEL REPETITIVE CONTROL ALGORITHM COMBINING ILC AND DEAD-BEAT CONTROL

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Abstract: In this paper it is explored whether or not a well-known adjoint Iterative Learning Control (ILC) algorithm can be applied to Repetitive Control (RC) problems. It is found that due to the lack of resetting in Repetitive Control, and the non-causal nature of the adjoint algorithm, the implementation requires a truncation procedure that can lead to instability. In order to avoid the truncation procedure, as a novel idea it is proposed that dead-beat control can used to shorten the impulse response of the plant to be so short that the need for truncation is removed. Therefore, convergence is guaranteed, if the adjoint algorithm is applied to the closed-loop plant with a dead-beat controller. The proposed algorithm is validated using real-time experiments on a non-minimum phase spring-mass-damper system. The experimental results show fast convergence to near perfect tracking, demonstrating the applicability of the proposed algorithm to industrial RC problems.

1 INTRODUCTION

Many signals in engineering are periodic, or at least they can be accurately approximated by a periodic signal over a large time interval. This is true, for example, of most signals associated with engines, electrical motors and generators, converters, or machines performing a task over and over again. Hence it is an important control problem to try to track a periodic signal with the output of the plant or try to reject a periodic disturbance acting on a control system.

In order to solve this problem, a relatively new research area called Repetitive Control has emerged in the control community. The idea is to use information from previous periods to modify the control signal so that the overall system would 'learn' to track perfectly a given T-periodic reference signal. The first paper that uses this ideology seems to be (Inouye et al., 1981), where the authors use repetitive control to obtain a desired proton acceleration pattern in a proton synchrotron magnetic power supply. Since then repetitive control has found its way to several practical applications, including robotics (Kaneko and Horowitz, 1997), motors (Kobayashi et al., 1999), rolling processes (Garimella and Srinivasan, 1996) and rotating mechanisms (Fung et al., 2000). However, most of the existing Repetitive Control algorithms are designed in continuous time, and they either don't give perfect tracking or they require that original process is positive real.

In order to overcome these limitations, in this paper a novel approach of combining an adjoint ILC algorithm and a dead-beat controller is proposed. As is shown in this paper, the new algorithm results in asymptotic convergence under mild controllability and observability conditions. In order to evaluate how the algorithm performs with real systems, the algorithm is applied to a *non-minimum* phase mass-damper system. This plant type is, based on past experience, a very challenging one due to the instable nature of the inverse plant model, see (Freeman et al., 2003a) and (Freeman et al., 2003b). The plant also has nonlinearities at high frequencies. Therefore, if the new algorithm is sensitive to modelling errors or nonlinearities, the experimental work carried out in this paper should certainly expose these weaknesses.

The rest of the paper is organized as follows: Section 2 rigorously defines the RC problem, and shows how the Internal Model Principle is related to the RC problem. Section 3 first motivates the use of the adjoint algorithm in the ILC context. After that, it is is shown how dead-beat control can be used to make the adjoint algorithm to be applicable to RC problems as well. Section 4 explains in detail the experiment

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set-up that is used to validate the new algorithm. Section 5 reports and analyses the experimental results. Finally, Section 6 contains directions for future work and concludes the paper.

2 REPETITIVE CONTROL - PROBLEM DEFINITION

As a starting point in discrete-time Repetitive Control (RC) it is assumed that a mathematical model

$$\begin{cases} x(t+1) = \Phi x(t) + \Gamma u(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{1}$$

of the plant in question exists with $x(0) = x_0$, $t \in [0, 1, 2, \ \ldots, \ \infty)$. Furthermore, Φ, Γ, C and D are finite-dimensional matrices of appropriate dimensions. From now on it is assumed that $D = 0$, because in practise it very rare to find a system where the input function $u(t)$ has an immediate effect on the output variable $y(t)$. Furthermore, a reference signal $r(t)$ is given, and it is known that $r(t) = r(t+N)$ for a given N (in other words the actual shape of $r(t)$ is not necessarily known). The control design objective is to find a feedback controller that makes the system (1) to track the reference signal as accurately as possible (i.e. $\lim_{t\to\infty} e(t) = 0$, $e(t) := r(t) - y(t)$), under the assumption that the reference signal $r(t)$ is N-periodic. As was shown by (Francis and Wonhan, 1975), a necessary condition for asymptotic convergence is that a controller

$$[Mu](t) = [Ne](t) \tag{2}$$

where M and N are suitable operators, must have an internal model or the reference signal inside the operator M. Because $r(t)$ is N-periodic, its internal model is $1 - \sigma_N$, where $[\sigma_N v](t) = v(t-N)$ for $v : \mathbb{Z} \to \mathbb{R}$. In the discrete-time case this requirement results in the algorithm structure

$$u(t) = u(t-N) + [Ke](t) \tag{3}$$

and if it is assumed that K can is a causal LTI filter, the algorithm can be written equivalently in the $\mathcal{Z}$-domain as

$$u(z) = z^{-T} u(z) + K(z)e(z) \tag{4}$$

In this case the design problem is to select $K(z)$ which results in accurate tracking but is not prone to uncertainties in the plant model. One particular way of achieving this is shown in the following sections.

3 THE 'ADJOINT' ALGORITHM

3.1 ILC adjoint algorithm

In ILC the state of the system is reset to x_0 when $t = N$, and hence it is sufficient to consider the system (1) over the the finite time-interval $t \in [0, N]$. Due to finite nature of the problem, it can be shown that the state-space equation (1) can be replace with an equivalent matrix representation (see (Hätönen et al., 2003a) for details)

$$y_k = G_e u_k \tag{5}$$

where k is the trial index and G_e is given by

$$G_e = \begin{bmatrix} C\Gamma & 0 & 0 & \ldots & 0 \\ C\Phi\Gamma & C\Gamma & 0 & \ldots & 0 \\ C\Phi^2\Gamma & C\Phi\Gamma & C\Gamma & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C\Phi^{N-1}\Gamma & C\Phi^{N-2}\Gamma & \ldots & \ldots & C\Gamma \end{bmatrix} \tag{6}$$

One possible ILC algorithm to achieve perfect tracking is to use the following 'adjoint' algorithm (see(Hätönen et al., 2003b) for details)

$$u_{k+1} = u_k + \beta G_e^T e_k \tag{7}$$

where G_e^T is the adjoint operator of the matrix G_e. This control results in the error evolution equation

$$e_{k+1} = (I - \beta G_e G_e^T) e_k \tag{8}$$

Taking the inner production between e_{k+1} and (8) results in

$$\|e_{k+1}\|^2 - \|e_k\|^2 = -2\beta \|G_e^T e_k\|^2 + \beta^2 \|G_e G_e^T e_k\|^2 \tag{9}$$

Therefore, if β is taken to be sufficiently small, the algorithm will result in monotonic convergence, i.e. $\|e_{k+1}\| \leq \|e_k\|$. Furthermore, in (Hätönen et al., 2003b) it has been show that the algorithm converges monotonically to zero tracking. The convergence is also robust, i.e. the algorithm can tolerate reasonable modelling uncertainty in the plant model G_e. (Hätönen et al., 2003b) also proposes an automatic mechanism to select β so that monotonic convergence is achieved without extensive tuning on β. In this case β becomes in fact iteration varying, resulting in an adaptive ILC algorithm.

3.2 RC adjoint algorithm and truncation

In order to introduce the necessary mathematical notation, consider an arbitrary sequence $f(k)$ where $k \in \mathbb{N}$. The $\mathcal{Z}$-transform $f(z)$ of $f(k)$ is defined to be

$$f(z) = \sum_{i=0}^{\infty} x(i) z^{-i} \tag{10}$$

where it is assumed that $f(z)$ is converges absolutely in a region $|z| > R$. Dually, $f(k)$ can be recovered from $f(z)$ from the equation

$$f(k) = \frac{1}{2\pi i} \oint_{\Omega} f(z) z^k \frac{dz}{z} \qquad (11)$$

where Ω is a closed contour in the region of convergence of $f(z)$. Finally, if

$$f(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \ldots \qquad (12)$$

then

$$f(z^{-1}) := f_0 + f_1 z + f_2 z^2 + \ldots \qquad (13)$$

Consider now the standard left-shift operator q^{-1} where $(q^{-1}v)(k) = v(k-1)$ for an arbitrary $v(k) \in \mathbb{R}^{\mathbb{Z}}$. Using the q^{-1} operator, the plant equation (1) can be written as

$$y(k) = G(q)u(k) = C(qI - \Phi)^{-1}\Gamma u(k) \qquad (14)$$

where it is assumed that $G(q)$ is controllable, observable and stable, and without loss of generality, that $x(0) = 0$. As a more restrictive assumption, suppose that the plant has a finite impulse-response (FIR), and the length of the impulse-response is less than the length of the period N. In other words, $C\Phi^i\Gamma = 0$ for $i \geq M$, $M < N$. In the following text this assumption is named as the 'FIR assumption'.

Consider now an 'intuitive' Repetitive Control version of the adjoint algorithm, which can be written in the using the q^{-1} operator formalism as

$$u(k) = q^{-N}u(k) + \beta G(q^{-1})q^{-N}e(k) \qquad (15)$$

It is important to realise that even the algorithm contains a 'non-causal element' $G(q^{-1})$, the algorithm (15) is causal, because it can easily be seen from (15) that $u(t) = f(e(s))$ for $t - M \leq s \leq t$.

The multiplication of (15) from the left with the plant model (14) together with some algebraic manipulations (note that $q^{-N}r(k) = r(k)$) results in the error evolution equation

$$e(k) = q^{-N}(1 - \beta G(q)G(q^{-1})e(z) \qquad (16)$$

This equation can be used to establish the the convergence of the algorithm under the FIR assumption on $G(q)$:

Proposition 1 *Assume that the condition*

$$\sup_{\omega[0,2\pi]} |1 - \beta|G(e^{j\omega})|^2| < 1 \qquad (17)$$

is met. In this case the the tracking error $e(t)$ satisfies that $\lim_{t\to\infty} e(t) = 0$.

Remark 1 *This condition can be always met, if $\beta < \sup_{\omega\in[0,2\pi]} |G(e^{j\omega})|^2|$.*

Proof. Note that by restricting the time-axis to be $[0, \infty)$, the error evolution equation can equivalently represented as a autonomous system as

$$\left(1 - q^{-N}(1 - \beta G(q)G(q^{-1})\right) e(q) = 0 \qquad (18)$$

with initial conditions $e(0) = e_0, \ldots, e(N-1) = e_{n-1}$, where the initial conditions are dependent on the 'initial guess' $u(0), \ldots, u(N-1)$. According to the Nyquist stability test (see (Astrom and Wittenmark, 1984)), the poles of the system (18) are inside the unit circle (which guarantees that $\lim_{t\to\infty} e(t) = 0$, if the locus of

$$1 - \beta G(z)G(z^{-1})|_{z=e^{j\omega}} = 1 - \beta|G(e^{j\omega})|^2 \qquad (19)$$

if $z^{-N}(1-\beta G(z)G(z^{-1}))|_{z=e^{j\omega}}$ encircles the critical point $(-1, 0)$ n times, where n is the number right-half poles of $z^{-N}(1 - \beta G(z)G(z^{-1}))$. Due to the FIR property of $G(z)$, $z^{-N}(1 - \beta G(z)G(z^{-1}))$ does not have any poles outside the unit circle, and therefore for stability $z^{-N}(1-\beta G(z)G(z^{-1}))|_{z=e^{j\omega}}$ is not allowed to encircle $(-1, 0)$-point. A sufficient condition for this is

$$\sup_{\omega[0,2\pi]} |(1 - \beta|G(e^{j\omega})|^2)| < 1 \qquad (20)$$

which concludes the proof. □

In summary, if the algorithm satisfies the FIR assumption, the algorithm will drive the tracking error to zero in the limit. However, in practical applications of ILC, it is quite rare that the FIR assumption would hold.

One possible way to approach to problem is to to truncate the impulse response of the plant in the update-law (15), i.e. the elements of the impulse response are set to zero for $t \geq N$. However, as is shown in (Chen and Longman, 2002), in which windowing techniques are used to theoretically eliminate the phase difficulties associated with truncation, this can lead to instability, but (Chen and Longman, 2002) does not establish any criteria for divergence. Therefore, the next subsection analyses the effect of truncation and other modelling uncertainty on the convergence.

3.3 Robustness analysis of the algorithm

Consider not the case when a nominal plant model $G_o(q)$ is is used to approximate the true plant model $G(q)$ (which possibly has an infinite impulse-response, IIR), where $G_o(z)$ satisfies the FIR assumption defined in the previous subsection. In this case the plant model can be written as

$$y(k) = G(q)u(k) = G_o(q)U(q)u(k) \qquad (21)$$

where $U(q)$ is a multiplicative uncertainty that reflects the uncertainties caused by modelling errors and truncation. Again, it is assumed that each of these transfer functions are controllable, observable and stable. In this case the control law (15) with nominal model $G_o(q)$ results in the error evolution equation

$$\begin{aligned} e(k) &= q^{-N}(1 - \beta G(q)G_o(q^{-1})e(k) \\ &= q^{-N}(1 - \beta U(q)G_o(q)G_o(q^{-1})e(k) \end{aligned} \tag{22}$$

The following result shows a sufficient condition for convergence in the presence of multiplicative uncertainty:

Proposition 2 *Assume that the condition*

$$\sup_{\omega[0,2\pi]} |1 - \beta U(e^{j\omega})|G(e^{j\omega})|^2| < 1 \tag{23}$$

is met. In this case the the tracking error $e(t)$ satisfies that $\lim_{t\to\infty} e(t) = 0$.

Proof. The proof is a trivial modification of the proof for Proposition 1. This is due to the fact that the stability assumption on $U(q)$ guarantees no right-half poles are introduced to $z^{-N}(1-\beta U(z)G(z)G(z^{-1}))$. □

The problem, however, is that this proposition does not reveal any information of $U(q)$, i.e. which are are the properties of $U(q)$ that guarantee that the convergence condition in Proposition 2 is met. The next proposition shows that the phase of $U(q)$ is the property that can cause either convergence or divergence:

Proposition 3 *Assume that $U(e^{j\omega})$ satisfies that $Re(U(e^{j\omega})) > 0$ for $\omega \in [0, 2\pi]$. Then there always exists β so that the convergence condition in Proposition 2 is met.*

Proof. Note that

$$\begin{aligned} &|1 - \beta U(e^{j\omega})|G(e^{j\omega})|^2|^2 = \\ &(1 - \beta U(e^{j\omega})|G(e^{j\omega})|^2)^*(1 - \beta U(e^{j\omega})|G(e^{j\omega})|^2) \\ &= 1 - \beta \text{Re}\{U(e^{j\omega})\}|G(e^{j\omega})|^2 + \beta^2|U(e^{j\omega})|^2|G(e^{j\omega})|^4 \end{aligned} \tag{24}$$

where z^* is the complex conjugate of a complex number $z \in \mathbb{Z}$. This shows immediately that if $\text{Re}\{U(e^{j\omega})\} > 0$ for $\omega \in [0, 2\pi]$, β can be be to chosen to be small enough in order to satisfy the convergence condition in Proposition 2. □

Note that the condition $\text{Re}\{U(e^{j\omega})\} > 0$ for $\omega \in [0, 2\pi]$ is equivalent to the condition that the Nyquist-diagram of $U(q)$ lies strictly in right-half plane. This is, on the other hand, is equivalent to the phase of $U(q)$ being inside ± 90 degrees. In summary, if the phase of the nominal model $G_o(q)$ lies inside ± 90 degrees 'tube' around the phase of the true plant $G(q)$, there β can always made sufficiently small so that the algorithm will converge to zero tracking error. Note, however, that $\beta \approx 0$ implies that $u(t) \approx u(t - T)$, which is an indication of a slow convergence rate.

The truncation of $G(q)$, however, can result in a nominal model $G_o(q)$, which does not satisfy the convergence condition on the uncertainty $U(q)$. This is, in particular, probable in situations where the period length N is short but the impulse response of the system is slow and oscillatory. Consequently, in the next section it is proposed that dead-beat control can be used to decrease the potential for divergence.

3.4 RC adjoint algorithm with a dead-beat controller

Consider the plant model in (1) and the following state-feedback control law

$$u(t) = -Kx(t) \tag{25}$$

It is a well-known result that if the system (1) is observable and controllable, the state-feedback law (25) can be used to place the closed-loop poles anywhere inside the unit circle. One possibility is to use K to place each pole into the origin, which is typically referred as the dead-beat control algorithm. The benefit from this particular choice of K is that impulse response will go to zero in n steps, where n is the number of states in the plant model. Therefore, if the number of states n is less than the length of the period, and the adjoint algorithm is applied to the closed-loop system with a dead-beat controller, non truncation is required, and the algorithm will converge asymptotically to zero tracking error.

Remark 2 *Note that the dead-beat controller requires information of the state $x(\cdot)$. This can can be achieved either by directly measuring the states or observing them, for example, with a Kalman observer.*

Remark 3 *The dead-beat controller is known to suffer from high-amplitude control activity, see (Astrom and Wittenmark, 1984). Non-minimum plants result in additional complexities, see Section 5. Therefore, in practice, the feedback gain K should be tuned so that that it adequetely shortens the length of the impulse response but does not result in an excessive control activity.*

4 EXPERIMENTAL SET-UP

The experimental test-bed has previously been used to evaluate a number of RC schemes and consists of a rotary mechanical system of inertias, dampers, torsional springs, a timing belt, pulleys and gears. The non-minimum phase characteristic is achieved by using the arrangement shown in Figure 1 where θ_i and

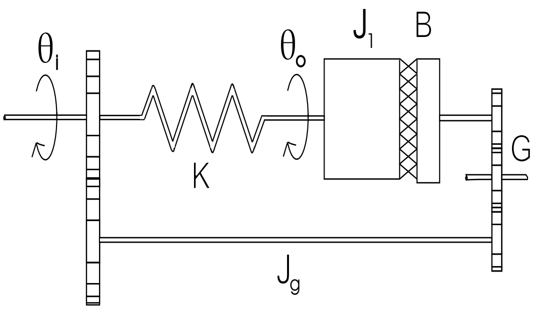

Figure 1: Non-minimum phase section

θ_o are the input and output positions, J_1 and J_g are inertias, B is a damper, K is a spring and G represents the gearing. A further spring-mass-damper system is connected to the input in order to increase the relative degree and complexity of the system. A 1000 pulse/rev encoder records the output shaft position and a standard squirrel cage induction motor supplied by an inverter, operating in Variable Voltage Variable Frequency (VVVF) mode, drives the load. The control scheme has been implemented in DOS to increase the available sampling frequency. The system has been modelled using a LMS algorithm to fit a linear model to a great number of frequency response test results. The resulting continuous time plant transfer function has thus been established as

$$G_{o/l}(s) = e^{-0.06s}\frac{1.202(4-s)}{s(s+9)(s^2+12s+56.25)} \tag{26}$$

A PID loop around the plant is used since this has been found to produce superior results. This also allows the adjoint algorithm to be used with no pole-placement since the closed-loop system, termed G(jw), therefore has a FIR. Provided (17) is satisfied, the convergence at a frequency, w, is dictated by

$$|1-\beta|G(jw)|^2| = 1-\beta|G(jw)|^2 \tag{27}$$

(Longman, 2000), the smaller it is, the faster the convergence. It is desirable that $|G(jw)|$ equals unity at low frequencies since, as these include the fundamental frequency of the demand, this results in β dictating the initial convergence of the algorithm. This is maximum at $\beta = 1$ and reducing β reduces the convergence rate whilst adding robustness. This aids comparison between different plants, $G(jw)$, and it will therefore be assumed that each plant on which the adjoint algorithm is used has been scaled to have a magnitude at low frequencies of unity. If w_m corresponds to the frequency of the greatest magnitude of $|G(jw)|$ then the learning at other frequencies in comparison is scaled by

$$\frac{|G(j\omega)|}{|G(j\omega_m)|} \tag{28}$$

For the greatest rate of convergence over the range of frequencies present in the demand it is therefore necessary to reduce $|G(j\omega_m)| - |G(j\omega)|$ over these frequencies. This means that in the design of the pole-placed system

$$H(z) = \frac{BC}{Iz-(A-BK)} \tag{29}$$

it is favourable that the gain be 'flat' over the system bandwidth. If deadbeat control were used and the poles of $H(z)$ were all at the origin then the contribution to the gain would solely be from the zeros of the original plant. Since this contribution $\rightarrow \infty$ as $w \rightarrow \infty$ it would be necessary that $\beta \rightarrow 0$ for stability. This would result in negligible convergence. This suggests that the emphasis should be on the convergence over the system bandwidth with the condition that the IR is sufficiently truncated to meet the FIR assumption. In the experimental tests a sine-wave and a repeating sequence demand are used with the system. Their period is three seconds, and the repeating sequence is one of the signals shown in Figure 6.

5 ANALYSIS OF EXPERIMENTAL RESULTS

Figure 2 shows cycle error results when using the original plant $G(jw)$ and the adjoint algorithm. As discussed, this has been scaled to have 0 dB gain at low frequencies. 400 cycles have been performed and the total error recorded for each cycle. The normalised error (NE) is simply the total error produced in a period multiplied by a scalar chosen so that a constant zero plant output produces a NE of unity. As β is increased so is the convergence rate, but this is limited by the onset of instability. In the remaining results, β will be fixed at 0.5 to facilitate comparison between the choice of pole locations that will be used.

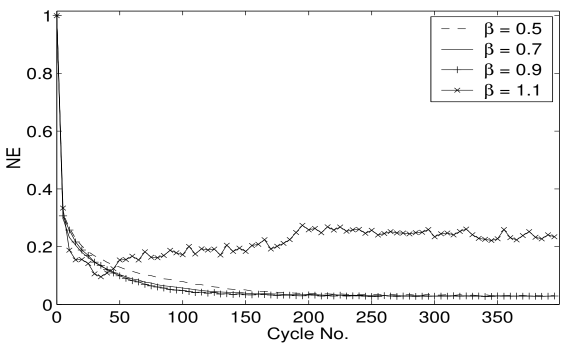

Figure 2: Cycle error results for original plant using repeating sequence demand

Figure 3 shows bode plots of six systems, $H_1(jw) \ldots H_6(jw)$, resulting from applying the state-feedback law (25) to the plant G(jw) for different values of K. Each of these is in turn due to a different choice of closed-loop pole locations. An optimisation routine was used to choose the pole locations in order to arrive at a flat magnitude plot with a gain of 0 dB up until a prescribed cut-off frequency. It was stipulated that $H_1(jw)$ and $H_2(jw)$ have complex conjugate poles, $H_3(jw)$ and $H_4(jw)$ have real poles, and $H_5(jw)$ and $H_6(jw)$ have a combination and favour a sharper cut-off. The Bode plots associated with these choices of pole locations show the range of characteristic available to the designer. No information relating to the phase of the systen has been incorporated into the cost criteria for the optimisation. However the phase has been reduced at low frequencies in each case and each system has an IR that is either less than the period used or requires negligible truncation. Therefore the problem of truncation has been solved as a by-product of the design process described. The systems $H_1(jw) \ldots H_6(jw)$

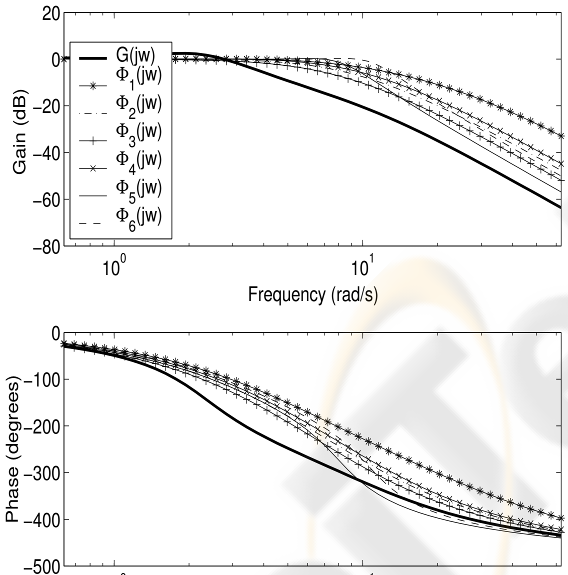

Figure 3: Bode plot of $G(jw)$ and a variety of pole-placed systems

are used with the adjoint algorithm, H_i^T replacing G_e^T in (7). Figure 4 shows the cycle error results for these systems seen using the sine-wave demand. It is clear that all the pole-placed systems lead to improved convergence compared with $G(jw)$ and there is no sign of instability. Figure 5 shows cycle error results using the repeating sequence demand. The increased bandwidth of this demand benefits from the increased learning of the pole-placed systems compared to the

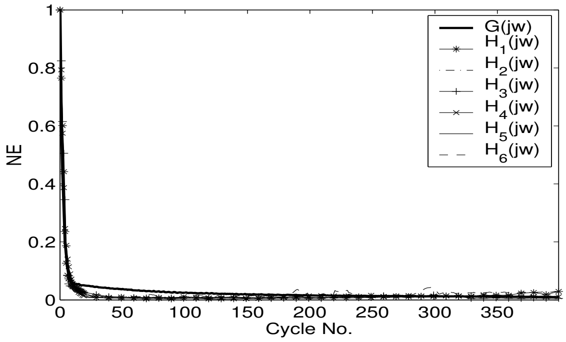

Figure 4: Cycle error for sine-wave demand

original. Only when using $H_6(jw)$, which has the greatest bandwidth, are there signs of instability. It is clear that robustness issues limit the frequencies that can be learnt. Although the figure only shows results from 400 cycles, the tests have been run for 1000 cycles, with most of the pole-placed systems still showing no sign of instability. This is a clear sign that their convergence benefits over using G(jw) are not heavily penalised in terms of lack of robustness. Figure 6

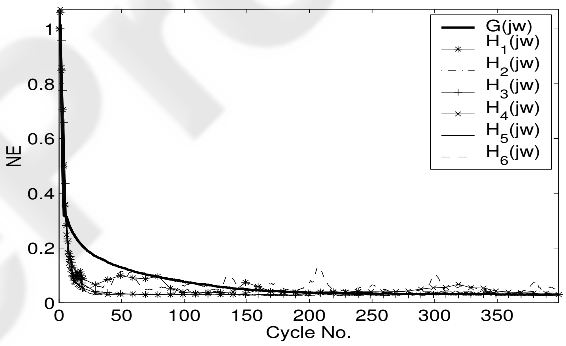

Figure 5: Cycle error for repeating sequence demand

highlights the initial convergence of the plant output to the demand. It shows data from the 1^{st} cycle and every 5^{th} thereafter. Excellent tracking is achieved using the adjoint algorithm and $H_5(jw)$ by the 21^{st} cycle compared to the slow convergence when using $G(jw)$ and the adjoint algorithm.

6 CONCLUSIONS AND FUTURE WORK

This paper has explored the possibility of using a well-know Iterative Learning Control algorithm in the Repetitive Control framework. It has been noted that

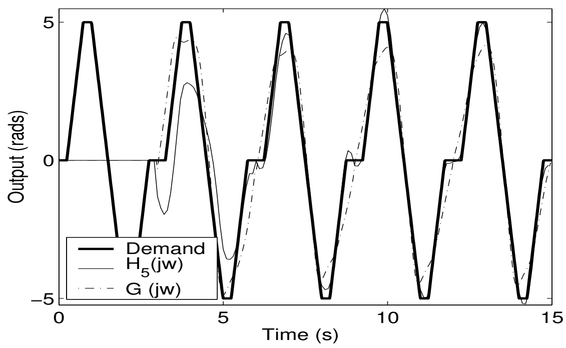

Figure 6: Tracking of repeating sequence demand

due to the non-causal nature of the algorithm, the algorithm can be applied only to systems that have a finite-impulse response (FIR). Furthermore, the impulse response has to go to zero at most in N steps, where N is period of the reference signal. If the plant satisfies these assumptions, the tracking error converges to zero exponentially.

If the plant does not satisfy the FIR assumption, the algorithm can be still applied by using a plant model where the impulse response is truncated after N time steps. It turns out that if the phase of the multiplicative uncertainty, which is caused by truncation, does not exceed ± 90 degrees, the algorithm still converges exponentially to zero.

In the case when the uncertainty condition is not met due to truncation, it is proposed that a dead-beat controller can be used to shorten the length of the impulse response. As a result, the closed loop system should have an impulse response, which is short enough for the algorithm to converge to zero tracking error.

The algorithm has been applied to a non-minimum phase spring-mass-damper system. The experimental results show that the algorithm is capable of producing near perfect tracking after a small number of cycles, demonstrating the algorithm should be applicable to industrial problems.

The crucial point in tuning of the algorithm is the selection of the feedback gain K. The objective is to find a gain K that shortens the length of the impulse response adequetely, produces rapid convergence over the bandwidth and is robust. Currently K is chosen using the 'trial and error' method. Therefore, as a future work, it is important to find systematic design rules that can be used to tune K.

Another interesting future research topic is the tuning of β. In ILC, it is possible to make β to be iteration-varying, and it can be shown that it results in enhances the robustness properties of the algorithm. How to transfer this idea to RC framework is still an open problem for future research.

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