SCALED GRADIENT DESCENT LEARNING RATE
Reinforcement learning with light-seeking robot

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Abstract: Adaptive behaviour through machine learning is challenging in many real-world applications such as robotics. This is because learning has to be rapid enough to be performed in real time and to avoid damage to the robot. Models using linear function approximation are interesting in such tasks because they offer rapid learning and have small memory and processing requirements. Adalines are a simple model for gradient descent learning with linear function approximation. However, the performance of gradient descent learning even with a linear model greatly depends on identifying a good value for the learning rate to use. In this paper it is shown that the learning rate should be scaled as a function of the current input values. A scaled learning rate makes it possible to avoid weight oscillations without slowing down learning. The advantages of using the scaled learning rate are illustrated using a robot that learns to navigate towards a light source. This light-seeking robot performs a Reinforcement Learning task, where the robot collects training samples by exploring the environment, i.e. taking actions and learning from their result by a trial-and-error procedure.

1 INTRODUCTION

The use of machine learning in real-world control applications is challenging. Real-world tasks, such as those using real robots, involve noise coming from sensors, non-deterministic actions and uncontrollable changes in the environment. In robotics, experiments are also longer than simulated ones, so learning must be relatively rapid and possible to perform without causing damage to the robot. Only information that is available from robot sensors can be used for learning. This means that the learning methods have to be able to handle partially missing information and sensor noise, which may be difficult to take into account in simulated environments.

Artificial neural networks (ANN) are a well-known technique for machine learning in noisy environments. In real robotics applications, however, ANN learning may become too slow to be practical, especially if the robot has to explore the environment and collect training samples by itself. Learning by autonomous exploration of the environment by a learning *agent* is often performed using *reinforcement learning* (RL) methods.

Due to these requirements, one-layer linear function approximation ANNs (often called *Adalines* (Widrow & Hoff, 1960)) are an interesting alternative. Their training is much faster than for non-linear ANNs and their convergence properties are also better. Finally, they have small memory- and computing power requirements.

However, when Adaline inputs come from sensors that give values of different magnitude, it becomes difficult to determine what learning rate to use in order to avoid weight oscillation. Furthermore, as shown in the experiments section of this paper, using a fixed learning rate may be problematic also because the optimal learning rate changes depending on the state of the agent and the environment. This is why the use of a *scaled learning rate* is proposed, where the learning rate value is modified according to Adaline input values. The scaled learning rate makes learning steps of similar magnitude independently of the input values. It is also significantly easier to determine a suitable value for the scaled learning rate than it is for a fixed learning rate.

After this introduction, Section 2 gives background information about gradient descent learning and RL. Section 3 defines the scaled

learning rate, followed by experimental results in Section 4. Related work is treated in Section 5, followed by conclusions.

2 GRADIENT DESCENT REINFORCEMENT LEARNING

In gradient descent learning, the free parameters of a model are gradually modified so that the difference between the output given by a model and the corresponding "correct" or *target value* becomes as small as possible for all training samples available. In such *supervised learning*, each training sample consists of input values and the corresponding target values. Real-world training samples typically involve noise, which means that it is not possible to obtain a model that would give the exact target value for all training samples. The goal of learning is rather to minimize a statistical error measure, e.g. the Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{M} \sum_{k=1}^{M} \left(t_j^k - a_j^k\right)^2} \qquad (1)$$

where M is the number of training examples, t_j^k is the target value for output j and training sample k and a_j^k is the model output for output j and training sample k. In RL tasks, each output a_j typically corresponds to one possible action.

RL differs from supervised learning at least in the following ways:

- The agent has to collect the training samples by exploring the environment, which forces it to keep a balance between exploring the environment for new training samples and exploiting what it has learned from the existing ones (the *exploration/exploitation trade-off*). In supervised learning, all training samples are usually pre-collected into a training set, so learning can be performed off-line.
- Target values are only available for the used actions. In supervised learning, target values are typically provided for all actions.
- The target value is not always available directly; it may be available only after the agent has performed several actions. Then we speak about a *delayed reward learning task.*

RL methods usually model the environment as a Markov Decision Process (MDP), where every state of the environment needs to be uniquely identifiable. This is why the model used for RL learning is often a simple "lookup-table", where each environment state corresponds to one row (or column) in the table and the columns (or rows) correspond to possible actions. The values of the table express how "good" each action is in the given state.

Lookup-tables are not suitable for tasks involving sensorial noise or other reasons for the agent not being able to uniquely identify the current state of the environment (such tasks are called *hidden state* tasks). This is one of the reasons for using state generalization techniques instead of lookup-tables. Generalisation in RL is based on the assumption that an action that is good in some state is probably good also in "similar" states. Various classification techniques have been used for identifying similar states. Some kind of ANN is typically used for the generalisation. ANNs can handle any state descriptions, not only discrete ones. Therefore they are well adapted for problems involving continuous-valued state variables and noise, which is usually the case in robotics applications.

2.1 Gradient descent learning with Adalines

The simplest ANN is the linear Adaline (Widrow & Hoff, 1960), where neurons calculate their output value as a weighted sum of their input values

$$a_j(s) = \sum_{i=1}^{N} s_i w_{i,j} \qquad (2)$$

where $w_{i,j}$ is the weight of neuron j associated with the neuron's input i, $a_j(s)$ is the output value of neuron j, s_i is the value of input i and N is the number of inputs. They are trained using the Widrow-Hoff training rule (Widrow & Hoff, 1960)

$$w_{i,j}^{new} = w_{i,j} + \alpha(t_j - a_j)s_i \qquad (3)$$

where α is a learning rate. The Widrow-Hoff learning rule is obtained by inserting equation (2) into the RMSE expression and taking the partial derivative against s_i. It can easily be shown that there is only one optimal solution for the error as a function of the Adaline weights. Therefore gradient descent is guaranteed to converge if the learning rate is selected sufficiently small.

When the back propagation rule for gradient descent in multi-layer ANNs was developed (Rumelhart et al., 1988), it became possible to learn non-linear function approximations and classifications. Unfortunately, learning non-linear functions by gradient descent tends to be slow and to converge to locally optimal solutions.

2.2 Reinforcement learning

RL methods often assume that the environment can be modelled as a MDP. A (finite) MDP is a tuple $M=(S,A,T,R)$, where: S is a finite set of states; $A = \{a_1, ..., a_k\}$ is a set of $k \geq 2$ actions; $T = [P_{sa}(\cdot) \mid s \in S, a \in A\}$ are the next-state transition probabilities, with $P_{sa}(s')$ giving the probability of transitioning to state s' upon taking action a in state s; and R specifies the reward values given in different states $s \in S$. RL methods are based on the notion of *value functions*. Value functions are either *state-values* (i.e. value of a state) or *action-values* (i.e. value of taking an action in a given state). The value of a state $s \in S$ can be defined formally as

$$V^\pi(s) = E_\pi \left\{ \sum_{k=0}^{\infty} \gamma^k r_{t+k+1} \Big| s_t = s \right\} \quad (4)$$

where $V^\pi(s)$ is the state value that corresponds to the expected return when starting in s and following policy π thereafter. The factor r_{t+k+1} is the reward obtained when arriving into states s_{t+1}, s_{t+2} etc. γ^k is a discounting factor that determines to what degree future rewards affect the value of state s.

Action value functions are usually denoted $Q(s,a)$, where $a \in A$. In control applications, the goal of RL is to learn an action-value function that allows the agent to use a policy that is as close to optimal as possible. However, since the action-values are initially unknown, the agent first has to explore the environment in order to learn it.

2.2.1 Exploring the environment for training samples

Although convergence is guaranteed for Widrow-Hoff learning in Adalines, in RL tasks convergence of gradient descent cannot always be guaranteed even for Adalines (Boyan & Moore, 1995). This is because the agent itself has to explore the environment and collect training samples by exploring the environment. If the action selection policy π does not make the agent collect relevant and representative training samples, then learning may fail to converge to a good solution. Therefore, the action selection policy must provide sufficient exploration of the environment to ensure that "good" training samples are collected. At the same time, the goal of training is to improve the performance of the agent, i.e. the action selection policy so that the learned model can be exploited.

Balancing the exploration/exploitation trade-off is one of the most difficult problems in RL for control (Thrun, 1992). A random search policy achieves maximal exploration, while a greedy policy gives maximal exploitation by always taking the action that has the highest action value. A commonly used method for balancing exploration and exploitation is to use *ε-greedy exploration*[1], where the greedy action is selected with probability $(1-\varepsilon)$ and an arbitrary action is selected with probability ε using a uniform probability distribution. This method is an *undirected exploration method* in the sense that it does not use any task-specific information. Another undirected exploration method selects actions according to Boltzmann-distributed probabilities

$$\text{Prob}(a_j) = \frac{\exp(Q(s_i,a_i)/T)}{\sum_k \exp(Q(s_i,a_k)/T)} \quad (5)$$

where T (called *temperature*) adjusts the randomness of action selection. The main difference between this method and ε-greedy exploration is that non-greedy actions are selected with a probability that is proportional to their current value estimate, while ε-greedy exploration selects non-greedy actions randomly.

Directed exploration uses task-specific knowledge for guiding exploration. Many such methods guide the exploration so that the entire state space would be explored in order to learn the value function as well as possible. In real-world tasks exhaustive exploration may be impossible or dangerous. However, a technique called *optimistic initial values* offers a possibility of encouraging exploration of previously un-encountered states mainly in the beginning of exploration. It can be implemented by using initial value function estimates that are bigger than the expected ones. This gives the effect that unused actions have bigger action value estimates than used ones, so unused actions tend to be selected rather than already used actions. When all actions have been used a sufficient number of times, the true value function overrides the initial value function estimates. In this paper, ε-greedy exploration is used for exploration. The effect of using optimistic initial values on learning is also studied.

2.2.2 Delayed reward

When reward is not immediate for every state transition, rewards somehow need to be propagated "backwards" through the state history. Temporal Difference (TD) methods (Sutton, 1988) are currently the most used RL methods for handling delayed reward. TD methods update the value of a state not only based on immediate reward, but also

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[1] Thrun (1992) calls this *semi-uniform distributed exploration*

based on the discounted value of the next state of the agent. Therefore TD methods update the value function on every state transition, not only after transitions that result in direct reward. When reward has been "temporally back propagated" a sufficient number of times, these discounted reward values can be used as target values for gradient descent learning (Barto, Sutton & Watkins, 1990). Such gradient descent learning allows using almost any ANN as the model for RL, but unfortunately the MDP assumption underlying TD methods often gives convergence problems.

Delayed reward tasks are out of the scope of this paper, which is the reason why such tasks are not analysed more deeply here. Good overviews on delayed reward are (Kaelbling, Littman & Moore, 1996) and (Sutton & Barto, 1998). The main goal of this paper is to show how scaling the Adaline learning rate improves learning, which is illustrated using an immediate reward RL task.

3 SCALED LEARNING RATE

In methods based on gradient descent, the learning rate has a great influence on the learning speed and on if learning succeeds at all. In this section it is shown why the learning rate should be scaled as a function of the input values of Adaline-type ANNs. Scaling the learning rate of other ANNs is also discussed.

3.1 Adaline learning rate

If we combine equations (2) and (3), we obtain the following expression for the new output value after updating Adaline weights using the Widrow-Hoff learning rule:

$$a_j^{new}(s) = \sum_{i=1}^{N} s_i w_{i,j}^{new} =$$

$$\sum_{i=1}^{N} s_i \left(w_{i,j} + \alpha(t_j - a_j)s_i \right) =$$

$$\sum_{i=1}^{N} s_i w_{i,j} + \sum_{i=1}^{N} \alpha s_i^2 (t_j - a_j) = \qquad (6)$$

$$a_j + \alpha \sum_{i=1}^{N} s_i^2 (t_j - a_j)$$

where $a_j^{new}(s)$ is the new output value for output j after the weight update when the input values s are presented again. If the learning rate is set to $\alpha = 1$,

then $a_j^{new}(s)$ would be exactly equal to t_j if the expression

$$\sum_{i=1}^{N} s_i^2 (t_j - a_j) \qquad (7)$$

is multiplied by

$$\frac{1}{\sum_{i=1}^{N} s_i^2} \qquad (8)$$

Then, by continuing from equation (6):

$$a_j^{new}(s) =$$

$$a_j + \alpha \sum_{i=1}^{N} s_i^2 (t_j - a_j) \Big/ \sum_{i=1}^{N} s_i^2 = \qquad (9)$$

$$a_j + \alpha(t_j - a_j) = t_j$$

when setting $\alpha = 1$. Multiplying (7) by (8) scales the weight modification in such a way that the new output value will approach the target value with the ratio given by the learning rate, independently of the input values s. In the rest of the paper, the term *scaled learning rate* (slr) will be used for denoting a learning rate that is multiplied by expression (8).

If the value of the (un-scaled) learning rate is greater than the value given by expression (8), then weights are modified so that the new output value will be on the "opposite side" of the target value in the gradient direction. Such overshooting easily leads to uncontrolled weight modifications, where weight values tend to oscillate towards infinity. This kind of weight oscillations usually makes learning fail completely.

The squared sum of input values in expression (8) cannot be allowed to be zero. This can only happen if all input values s_i are zero. However, the *bias* term used in most Adaline implementations avoids this. A bias term is a supplementary input with constant value one, by which expression (2) can represent any linear function, no matter what is the input space dimensionality. Most multi-layer ANNs implicitly also avoid situations where all Adaline input values would be zero at the same time. This is studied more in detail in the following section.

3.2 Non-linear, multi-layer ANNs

In ANNs, neurons are usually organized into *layers*, where the output values of neurons in a layer are independent of each other and can therefore be calculated simultaneously. Figure 1 shows a feed-forward ANN with one input and one output (there

may be an arbitrary number of both). ANN input values are distributed as input values to the *hidden neurons* of the *hidden layer*. Hidden neurons usually have a non-linear output function with values limited to the range [0, 1]. Some non-linear output functions have values limited to the range [-1, 1].
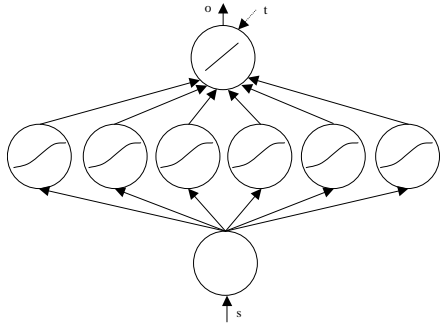

Figure 1: Three-layer feed-forward ANN with sigmoid outputs in hidden layer and linear output layer.

A commonly used output function for hidden neurons is the *sigmoid* function

$$o = f(A) = \frac{1}{1 + e^{-A}} \qquad (10)$$

where o is the output value and A is the weighted sum of the inputs (2). The output neurons of the output layer may be linear or non-linear. If they are linear, they are usually Adalines.

Learning in multi-layer ANNs can be performed in many ways. For ANNs like the one in Figure 1, a gradient descent method called back-propagation is the most used one (Rumelhart et al., 1988). For such learning, two questions arise:

- Is it possible to use a scaled learning rate also for non-linear neurons?
- If the output layer (or some other layer) is an Adaline, is it then useful to use the scaled learning rate?

The answer to the first question is probably "no". The reason for this is that functions like the sigmoid function (10) do not allow arbitrary output values. Therefore, if the target value is outside the range of possible output values, then it is not possible to find weights that would modify the output value so that it would become exactly equal to the target value. Instead, the learning rate of non-linear neurons is usually dynamically adapted depending on an estimation of the steepness of the gradient over several training steps (Haykin, 1999).

The answer to the second question is "maybe". If Adaline inputs are limited to the range [0, 1], then the squared sum in (8) remains limited. Still, in the beginning of training, hidden neuron outputs are generally very small. Then a scaled learning rate

might accelerate output layer learning, while slowing down when hidden neurons become more activated. This could be an interesting direction of future research to investigate.

4 EXPERIMENTAL RESULTS

Experiments were performed using a robot built with the Lego Mindstorms Robotics Invention System (RIS). The RIS offers a cheap, standard and simple platform for performing real-world tests. In addition to Lego building blocks, it includes two electrical motors; two touch sensors and one light sensor. The main block contains a small computer (RCX) with connectors for motors and sensors. Among others, the Java programming language can be used for programming the RCX.

Figure 2: Lego Mindstorms robot. Light sensor is at the top in the front, directed forwards. One touch sensor is installed at the front and another at the rear.

The robot had one motor on each side; touch sensors in the front and in the back and a light sensor directed straight forward mounted in the front (Figure 2). Robots usually have more than one light sensor, which were simulated by turning the robot around and getting light readings from three different directions. One light reading was from the direction straight forward and the two others about 15 degrees left/right, obtained by letting one motor go forward and the other motor backward for 250 milliseconds and then inversing the operation. The light sensor reading from the forward direction after performing an action is directly used as the reward value, thus avoiding hand tuning of the reward function.

Five actions are used, which consist in doing one of the following motor commands for 450 milliseconds: 1) both motors forward, 2/3) one forward, other stopped, 4/5) one forward, other backward. Going straight forward means advancing about 5 cm, actions 2/3 going forward about 2 cm

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and turning about 15 degrees and actions 4/5 turning about 40 degrees without advancing.

The robot starts about 110 centimetres from the lamp, initially directed straight towards it. Reaching a light value of 80 out of 100 signifies that the goal is reached, which means one to fifteen centimetres from the lamp depending on the approach direction and sensor noise. In order to reach the goal light value, the robot has to be very precisely directed straight towards the lamp. The lamp is on the floor level and gives a directed light in a half-sphere in front of it. If the robot hits an obstacle or drives behind the lamp, then it is manually put back to the start position and direction. The test room is an office room with noise due to floor reflections, walls and shelves with different colours etc. The robot itself is also a source of noise due to imprecise motor movements, battery charge etc. However, the light sensor is clearly the biggest source of noise as shown in Figure 3, where light sensor samples are indicated for two different levels of luminosity.
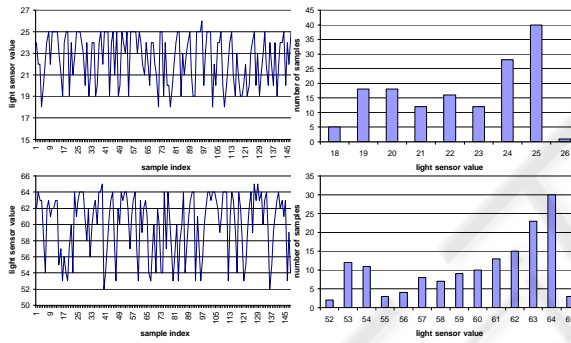

Figure 3: 150 light samples for two different light conditions, taken with 500 millisecond intervals. Average values are 22.5 and 60.0. Raw values are shown to the left, value distribution to the right.

Table 1: Hand-coded weights. One row per action, one column per state variable (light sensor reading).

Action	Left	Middle	Right
Forward	0.1	0.8	0.1
Left/forward	0.6	0.3	0.1
Right/forward	0.1	0.3	0.6
Left	0.6	0.2	0.2
Right	0.2	0.2	0.6

When using an ANN there is one output per action, where the output value corresponds to the action-value estimate of the corresponding action. With five actions and three state variables, a 5x3 weight matrix can represent the weights (no bias input used here). A "hand-coded agent" with pre-defined weights (Table 1) was used in order to prove that an Adaline linear function approximator can solve the

control task and as a reference for judging how good the performance is for learning agents. These weights were determined based on the principle that if the light value is greatest in the middle, then make the forward-going action have the biggest output value. In the same way, if the light value is greater to the left, then favour some left-turning action and vice versa for the right side. The hand-coded agent reached the goal after a 30-episode average of about 17 steps.

Learning agents used the same Adaline architecture as the hand-coded agent. Weights are modified by the Widrow-Hoff training rule (3). All agents used ε-greedy exploration with $\varepsilon = 0.2$, which seemed to be the best value after experimentation. Tests were performed both with weights initialised to small random values in the range [0, 0.1] and with weights having optimistic initial values in the range [0, 1]. Such weights are optimistic because their expected sum per action neuron is 1.5, while weight values after training should converge to values whose sum is close to one. This is because state variable values and reward values are all light sensor readings, so the estimated reward value should be close to at least one of the state variable values. If the RL is successful, then the estimated reward should even be a little bit bigger since the goal by definition of RL is to make the agent move towards states giving higher reward.

An un-scaled learning rate value of 0.0001 was used after a lot of experimentation. This value is a compromise. Far from the lamp, light sensor values are about 10, so expression (8) gives the value $1/(10^2 + 10^2 + 10^2) = 0,00333…$. Close to the lamp, light sensor values approach 80, so the corresponding value would be 0.00005. According to expression (8), the un-scaled learning rate value 0.0001 corresponds to light sensor values around 58, which is already close to the lamp. Therefore, excessive weight modifications probably occur close to the lamp, but then the number of steps remaining to reach the goal is usually so small that weights do not have the time to oscillate.
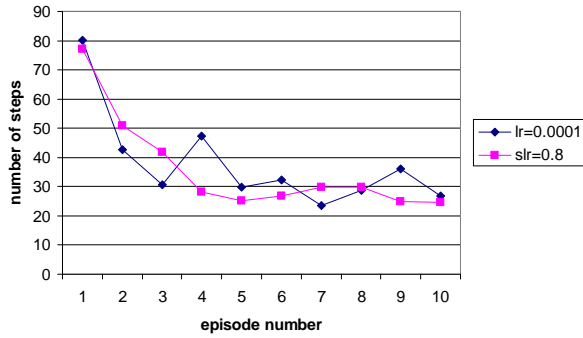
Figure 4: Comparison between static (*lr*) and scaled (*slr*) learning rate. Averages for 10 runs.

Figure 5: Results for different values of scaled learning rate. Averages for 10 runs.

Figure 4 compares the performance of an agent using the un-scaled learning rate 0.0001 and an agent using *slr* = 0.8. With this scaled learning rate, the first episode is slightly faster. Convergence is also much smoother with the scaled learning rate than with the un-scaled learning rate. The statistics shown in Table 2 further emphasize the advantage of using a scaled learning rate. The total number of steps is slightly smaller, but the average length of the last five episodes is clearly lower for the agents using the scaled learning rate. This is most probably because the un-scaled learning rate sometimes causes excessive weight modifications that prevent the agent from converging to optimal weights.

The number of manual resets is a further indication of excessive weight modifications. One bad light sensor reading may be sufficient to make the robot get stuck into using the same action for several steps. If that action happens to be going straight forward, then the robot usually hits a wall after a few steps. Reducing the value of the un-scaled learning rate could reduce this phenomenon, but it would also make learning slower.

Table 2: Statistics for agents using different learning rates. Averages for 10 runs.

Agent	Total	Aver. 5 last	Man. resets
lr=0.0001	378	29.5	30
slr=0.2	458	28.6	18
slr=0.5	355	27.7	10
slr=0.8	359	27.1	10

Figure 5 compares the performance of agents using different values for the scaled learning rate. All graphs are smooth and converge nearly as rapidly. This result shows that the scaled learning rate is tolerant to different values. The meaning of the scaled learning rate is also easy to understand (i.e. "percentage of modification of output value towards target value"), so determining a good value for the scaled learning rate is easier than for the un-scaled learning rate.
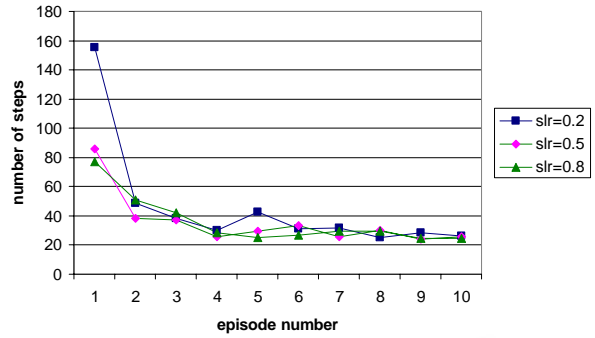
Figure 6 and Table 3 compare the performance of agents whose weights are initialised with random values from the range [0, 0.1) and agents whose weights are initialised with optimistic initial values, i.e. random values from the range [0, 1). Using optimistic initial values clearly gives faster initial exploration. The number of manual resets with optimistic initial values is also lower for agents using *slr* = 0.5 and *slr* = 0.8, but instead it is higher for *lr* = 0.0001 and *slr* = 0.2.

Finally, when setting $\varepsilon = 0$ after 10 episodes, i.e. always taking the greedy action, the trained agents had identical performance as the hand-coded agent. However, one should remember that learning agents could also adapt to changes in the environment or differences in sensor sensibility, for instance.
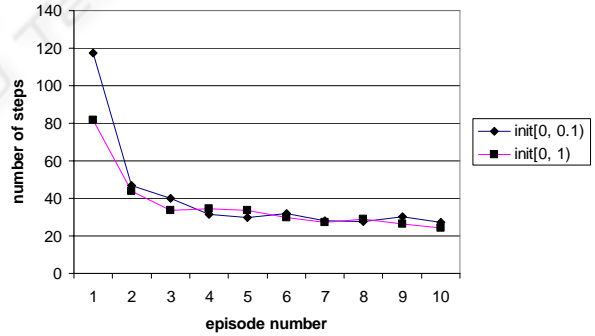

Figure 6: Results for different initial weights. Averages for 20 runs.

Table 3: Statistics for different initial weights. Averages for 20 runs.

Initial weights	Total	Aver. 5 last	Man. resets
[0, 0.1)	411	29.1	33
[0, 1)	364	27.3	35

5 RELATED WORK

The amount of literature on gradient descent learning is abundant. One of the most recent and exhaustive sources on the subject is (Haykin, 1999). Adjusting the Adaline learning rate has been studied previously at least by Luo (1991), who shows that the Adaline learning rate should be reduced during learning in order to avoid "cyclically jumping around" the optimal solution. References in (Luo, 1991) also offer a good overview of research concerning the gradient descent learning rate. However, to the author's knowledge, the concept of scaled learning rate introduced in this paper is new.

RL has been used in many robotic tasks, but most of them have been performed in simulated environments. Only few results have been reported on the use of RL on real robots. The experimental setting used here resembles behavior learning performed by Lin (1991) and Mahadevan & Connell (1992). Behavioral tasks treated by them include wall following, going through a door, docking into a charger (guided by light sensors), finding boxes, pushing boxes and getting un-wedged from stalled states. Some of these behaviors are more challenging than the light-seeking behavior used in this paper, but the simple linear Adaline model used here for state generalization greatly simplifies the learning task compared to previous work. An example of non-RL work on light-seeking robots is Lebeltel et al. (2004).

6 CONCLUSIONS

One of the most important advantages of the scaled learning rate presented in this paper is that it is easy to understand the signification of the values used for it. Evidence is also shown that the scaled learning rate improves learning because it makes the network output values approach the corresponding target values with a similar amount independently of the input values. Experimental results with a real-world light-seeking robot illustrate the improvement in learning results by using the scaled learning rate.

It seems rather surprising that a scaled learning rate has not been used yet, according to the author's best knowledge. One explanation might be that in supervised learning tasks, the training samples are usually available beforehand, which makes it possible to normalize them into suitable values. In real-world RL tasks, with constraints on learning time and the availability of training samples, this may not be possible. Using multi-layer non-linear ANNs also might reduce the utility of scaling the learning rate, as explained in section 3.2.

In addition to the scaled learning rate, the RL exploration/exploitation trade-off is also addressed in the paper. The exploration policy used determines the quality of collected training samples and therefore greatly affects learning speed and the quality of learned solutions. Empirical results are shown mainly on the advantages of using optimistic initial values for the network weights when possible.

Future work includes improving exploration policies and handling delayed reward. Obtaining further results on the use of the scaled learning rate for other than RL tasks would also be useful.

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