MODEL REFERENCE CONTROL IN INVENTORY AND SUPPLY CHAIN MANAGEMENT

The implementation of a more suitable cost function

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Abstract: A method of model reference control is investigated in this study in order to present a more suitable method of controlling an inventory or a supply chain. The problem of difficult determining of the cost of change made in the control in supply chain related systems is studied and a solution presented. Both model predictive controller and a model reference controller are implemented in order to simulate results. Advantages of model reference control in supply chain related control are presented. Also a new way of implementing supply chain simulators is presented and used in the simulations.

1 INTRODUCTION

In recent years model predictive control (MPC) has gained a lot of attention in supply chain management and in inventory control. It has been found to be a suitable method to control business related systems and very promising results has been shown in many studies. The main idea in MPC has remained the same in most studies but many variations of the cost function can be found. Basically these cost functions, used in studies concerning MPC in supply chain management, can be separated in two different categories: quadratic and linear cost functions. The use of a linear cost function can be seen appropriate as it can take advantage of actual unit costs determined in the case. On the other hand these costs need to be fairly accurate to result as an effective control. Examples of studies using linear cost functions in supply chain control can be found in (Ydstie, Grossmann *et al.*, 2003) and (Hennet, 2003). In this study we will no longer study the linear form of the cost function but concentrate on the quadratic form. The quadratic form of the cost function is used in, for example, (Tzafestas *et al.*, 1997) and (Rivera *et al.*, 2003). In supply chain management the question is not only about how to control the chain but also about what is being controlled. The traditional quadratic form of the cost function used in MPC has one difficulty when it comes to controlling an inventory or a supply chain. The quadratic form involves penalizing of changes in the controlled variable. Whether this variable is the order rate or the inventory level or some other actual variable in the business, it is always very difficult to determine the actual cost of making a change in this variable. In this study we present an effective way of controlling an inventory with MPC without the problem of determining the cost of changing the controlled variable. The method of model reference control will be demonstrated in inventory control and results presented. The structure of this paper is as follows. In Chapter 2 we will take a closer view on model predictive control and on the theory behind model reference control. In Chapter 3 we present simulations with both model predictive control and model reference control and do some comparisons between those two. Finally we conclude the results from our study in the last chapter, Chapter 4.

2 MODEL PREDICTIVE CONTROL

Model predictive control originated in the late seventies and has become more and more popular ever since. MPC itself is not an actual control strategy, but a very wide range of control methods which make use of a model of the process. MPC was originally developed for the use of process control but has diversified to a number of other areas of control, including supply chain management and

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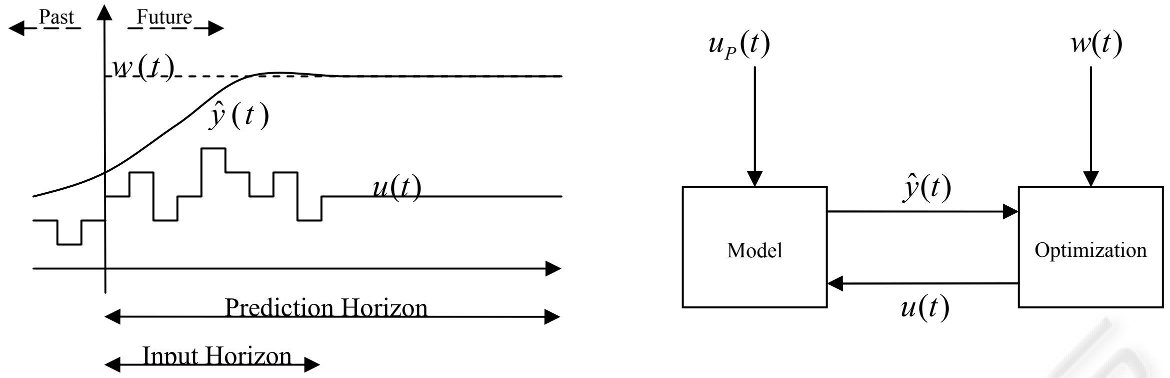

Figure 1: The main idea and the implementation of MPC.

inventory control in which it has gained a lot of attention. Today MPC is the only modern control method to have gained success in real world applications. (Camacho and Bordons 2002), (Maciejowski 2002)

As stated earlier, Model Predictive Control is a set of control algorithms that use optimization to design an optimal control law by minimizing an objective function. The basic form of the objective function can be written as

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} \delta(j)\left[\hat{y}(t+j \mid t) - w(t-j)\right]^2 + \sum_{j=1}^{N_u} \lambda(j)\left[\Delta u(t+j-1)\right]^2 \tag{1}$$

, where N_1 and N_2 are the minimum and maximum cost horizons and N_u is the control horizon. $\delta(j)$ and $\lambda(j)$ can be seen as the unit costs of the control. w(t), ŷ(t) and Δu(t) are the reference trajectory, the predicted outputs and the change between current predicted control signal and previous predicted control signal, respectively. (Camacho and Bordons 2002)

The algorithm consists of two main elements, an optimization tool and a model of the system. The optimizer contains a cost function with constraints and receives a reference trajectory w(t) to which it tries to lead the outputs as presented in Figure 1. The actual forecasting in MPC is done with the model which is used to predict future outputs ŷ(t) on the basis of the previous inputs $u_P(t)$ and future inputs u(t) the optimizer has solved as presented in Figure 1. These forecasts are then used to evaluate the control and a next optimization on the horizon is made. After all the control signals on the horizon are evaluated, only the first control signal is used in the process and the rest of the future control signals are rejected. This is done because on the next optimizing instant, the previous output from the process is already known and therefore a new, more accurate forecast can be made due to new information being available. This is the key point in the receding horizon technique as the prediction gets more accurate on every step of the horizon but also is the source of heavy computing in MPC. The receding horizon technique also allows the algorithm to handle long time delays. (Camacho and Bordons 2002)

2.1 Implementing the cost function

As presented in equation 1, the basic form of a MPC cost function penalizes changes made in control weighted with a certain parameter λ. This kind of damping is not very suitable for controlling an inventory or a supply chain due to the difficulty of determining the parameter λ as it usually is either the cost of change in inventory level or the cost of change in ordering. On the other hand the parameter λ cannot be disregarded as it results as minimum-variance control which most definitely is not the control desired. Another problem with the basic form of MPC used in inventory control is the fact that it penalizes the changes made in ordering and not in inventory levels, which can cause unnecessary variations in the inventory level as will be shown later in this study.

In this study we present a more suitable way to form the cost function used in a model predictive controller. The problematic penalizing of changes in the control is replaced with a similar way to the one presented in, for example, (Lambert, 1987) and used, for example, in (Koivisto *et al.*, 1991). An inverted discrete filter is implemented in the cost function so that the resulting cost function can be written as

$$J = \sum_{i=N_1}^{N_2} \left(y^*(i) - P(q^{-1}) \cdot \hat{y}(i) \right)^2 \qquad (2)$$

, where $y^*(i)$ = Target output,
$\hat{y}(i)$ = Predicted output
$P(q^{-1})$ = Inverted discrete filter

The filter $P(q^{-1})$ used can be written as

$$P\left(q^{-1}\right) = \frac{1 - p_1 q^{-1} - p_2 q^{-2} - \ldots}{1 - p_1 - p_2 - \ldots} \qquad (3)$$

As can be seen, the number of tuneable parameters can be reduced as the simplest form of the cost function consists of only one tuneable parameter, p_1 which is used in the filter. Naturally the reduction of tuneable parameters is a definite improvement it self.

The dampening performed by the model reference control is also an advantage concerning bullwhip effect as over ordering has been found one of the major causes of this problem. (Towill, 1996) When the model reference control is applied to an inventory level controller the most basic form of the cost function results as

$$J = \sum_{i=N_1}^{N_2} \left(I^*(i) - P(q^{-1}) \cdot \hat{I}(i) \right)^2 \qquad (4)$$

, where $I^*(i)$ = Desired inventory level
$\hat{I}(i)$ = Predicted inventory level
$P(q^{-1})$ = Inverted discrete filter as in equation (3)

3 SIMULATIONS

The simulations in this study were made using MATLAB® and Simulink®. The goal in the simulations was to show the advantages of a model reference controller in inventory control compared to a traditional model predictive controller. To construct the simulators a set of universal supply chain blocks was used. The main idea in these blocks is the ability to construct any supply chain desired without programming the whole chain from scratch. The basic structure of a desired chain can be implemented with basic drag and drop operations and actual dynamics can be programmed afterwards. The set of blocks consists of three different elements which are inventory block, production block and a so called dummy supplier block. These blocks are the actual interface for programming each individual element. With these blocks the whole supply chain can be constructed and simulated with a high level of visibility and clarity.

3.1 Simulator implementation tool

The main idea in the universal production block can be seen in Figure 2. The submodules Stock, Control and Demand forecast can all be implemented

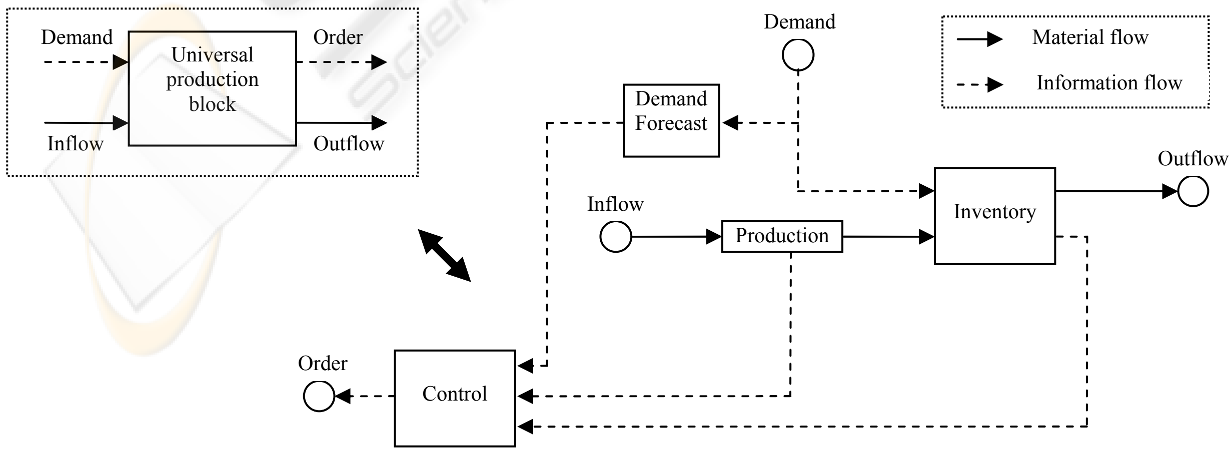

Figure 2: Structure of the universal production block.

uniquely. For example the Inventory block can be constructed to operate linearly in order to test more theoretical control methods or it can even be implemented as realistic as possible to study the performance of a real world supply chain. The inventory element represents an end product inventory of a production plant. Also different control and demand forecasting methods can be tested and tuned via the Control and Demand forecast elements in the block, respectively. The universal production block has naturally a submodule called Production which consists of production dynamics in the simulated factory.

The universal inventory block is basically the universal production block but without the Production submodule. The universal inventory block can be used as a traditional warehouse or as a whole saler or even as a material inventory for a production plant. As it consists of the same control related elements as the universal production block, it can have a control method and a inventory policy of its own independently from the production plant.

The dummy supplier block is very different from the rest of the set. It is used to solve the problem of long supply chains. Usually one does not want to model the whole supply chain as it can consist of tens of companies. Most of the upstream companies are also irrelevant in the simulations from the end products point of view. Therefore it is necessary to replace the companies in the upstream of the chain with a dummy supplier block. This block takes the order from its customer as an input and supplies this order with certain alterations as an output. These alterations can be anything from basic delay to consideration of decay. Once again, this block can be seen as an interface for the programmer who can decide the actual operations within the block.

3.2 Inventory control simulations

To present the advantages in the model reference control used in inventory control, a very simple model was constructed using the universal block set presented earlier. The structure of the simulator can be seen in Figure 3. Both the universal production block and the dummy supplier block are as presented earlier. To keep the model as simple as possible, all delays in the model are constant. Each block in the model consists of a unit delay so that total delay in the model is 3 units. Also no major plant-model mismatch is involved in the controller and no constraints are set. Both controllers also receive identical accurate demand forecasts. With this model we present two simulations with different demand patterns.

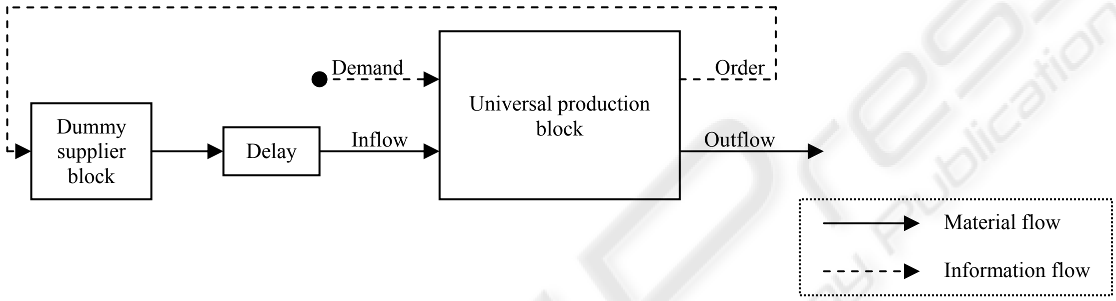

Figure 3: The structure of the simulator used in this study.

For the traditional model predictive controlled inventory the following cost function was implemented

$$J = \sum_{i=N_1}^{N_2} \left(\delta\left(I^*(i) - \hat{I}(i)\right)^2 + \lambda\left(\Delta O(i)\right)^2 \right) \quad (5)$$

, where $I^*(i)$ = Desired inventory level
$\hat{I}(i)$ = Predicted inventory level
$\Delta O(i)$ = Change in order rate
δ, λ = Weight parameters

For model reference controlled inventory we used the cost function presented in equation 4 with the most basic form of the filter so that the only parameter to tune is p_1. As mentioned earlier, the number of tunable parameters in model reference control is reduced by one when compared to the traditional model predictive control. This is obvious when we look at the cost functions presented in this

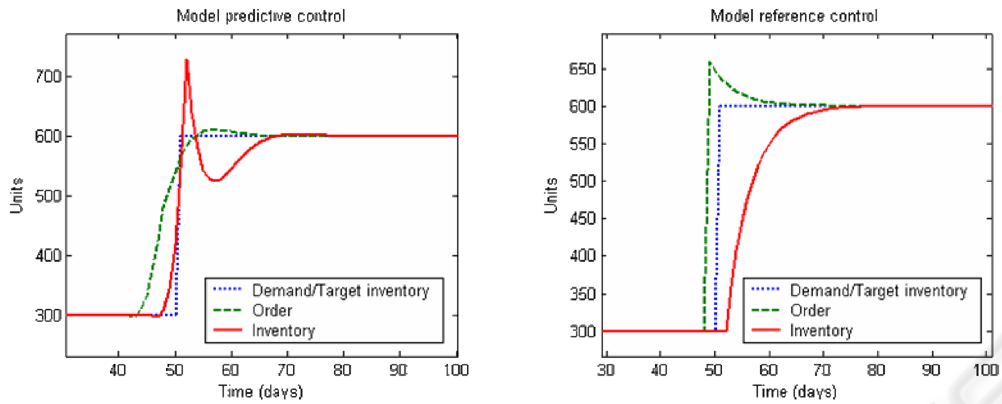

Figure 4: Step responses of an MPC-controlled inventory. In the left-hand figure the cost function is in the form presented in equation 5, and in the right-hand figure the cost function is in the form presented in equation 4.

simulation, as the cost function used in model reference controller has only one parameter p_1 instead of the two parameters, δ and λ included in the model predictive controller.

3.3 Step response simulations

The first simulation was a basic step response test which demonstrated the difference between a model refence controlled inventory and a traditional model predictive controlled inventory. The step was implemented in both demand and target inventory levels at the same time to cause a major change in the system. In other words, target inventory level was set to follow the demand so that every day the company had products in stock worth one day's sale. In this simulation the controller parameters were chosen to be as follows: $\delta = 0.1$, $\lambda = 0.9$ and $p_1 = 0.8$, with the control horizon of 10 days. The results can be seen in Figure 4 where a step in demand has occured at the moment of 50 days. As can be seen, the response of the model reference controller is much more smoother than the step response of a traditional form of the model predictive controller. This is due to the fact that the model reference controller dampens directly the changes made in the inventory level instead of dampening the changes made in ordering. Model predictive controller is forced to start ordering excessively in advance to the step due to the limitations in changing the control action. The more reasonably implemented model reference controller orders exactly the amount needed every instant. It becomes obvious that the penalizing of control actions is not a suitable way to control supply chain related tasks as it causes additional variations in the system. Model reference control is much more efficient in achieving what was beeing pursued in this study – smooth control method which is simple to tune and implement.

3.4 Simulations with a more realistic demand pattern

A more practical simulation was also completed with more realistic demand pattern and forecasting error. The demand involved also noise which made the control task even more realistic. The control horizon was kept at 10 days in both controllers but the parameters δ and λ needed to be retuned as the parameters δ and λ used in the previous simulation resulted as very poor responses. New parameters were chosen as follows: $\delta = 0.3$ and $\lambda = 0.7$. The model reference controller did not need any retuning as it survived both simulations very satisfyingly with same parameter $p_1 = 0.8$. The target inventory level was kept constant in the level of 100 units. This is propably not the most cost efficient way of managing an inventory but was used nevertheless to keep the results easy to understand.

The demand curve and inventory response can be seen in Figure 5. The demand curve is identical in both pictures but as can be seen there were major differences in inventory levels. Inventory levels in the model predictive controlled case showed major variations at the same time as demand rapidly increased. No such variations were found in the model reference case. Once again the penaling of control actions forced the controller to order excessive amounts in order to avoid stock-out.

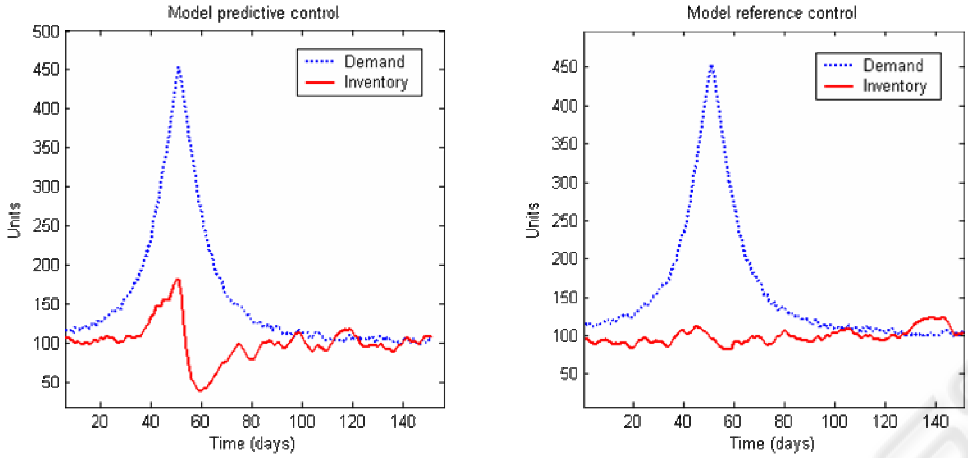

Figure 5: The inventory responses of both controllers to a gaussian-shaped demand pattern.

4 CONCLUSIONS

In this study we presented a solution to the problematic determining of the cost parameter penalizing changes made in the control in model predictive controller used in business related systems. The model reference control was studied and simulations performed to demonstrate the abilities and advanteges of this control method. It has been shown, that the model reference control method is an effective way to control an inventory and most of all that the method allows us to avoid the problematic parameter λ in the equation 1. This reduction of a very problematic parameter is most definitely inevitable if any kind of practical solutions are ever desired. Therefore all future research concerning business related control should concider this. It should also be kept in mind that any reduction of tuneable parameters can be seen as an advantage.

Also we showed in this study that the model reference control is at least as applicable in inventory control as model predictive control if not even better. The simple, yet effective and smooth response model reference control provides suits perfectly to the unstable and variating environment of business related systems. It can also be said that the filter-like behaviour is desireable in order to reduce the bullwhip behaviour, but further research is needed on this field.

A new supply chain simulator interface was also presented and used in the simulations of this study. The set of universal supply chain blocks gives an opportunity of testing and simulating the perforance of different control methods or even different forms of supply chains without reprogramming the basic elements of inventory and production.

REFERENCES

Camacho, E. F. and Bordons, C., 2002, *Model Predictive Control*. Springer-Verlag London Limited

Hennet, J.-C., 2003. A bimodal scheme for multi-stage production and inventory control. In *Automatica, Vol 39, pp. 793-805.*

Koivisto, H., Kimpimäki, P., Koivo, H., 1991. Neural predictive control – a case study. In *1991 IEEE International Symposium on Intelligent Control.* IEEE Press.

Lambert, E. P., 1987. *Process Control Applications of Long-Range Prediction*. PhD Thesis, University of Oxford. Trinity.

Maciejowski, J. M., 2002, *Predictive Control with Constraints*. Pearson Education Limited

Rivera, D. E., Braun, M. W., Flores, M. E., Carlyle, W. M., Kempf, K. G., 2003. A Model predictive control framework for robust management of multi-product, multi-echelon demand networks. In *Annual Reviews in Control, Vol. 27, pp. 229-245.*

Towill, D. R., 1996. Industrial dynamics modeling of supply chains. In *International Journal of Physical Distribution & Logistics Management, Vol. 26, No. 2, pp.23-42.*

Tzafestas, S., Kapsiotis, G., Kyriannakis, E., 1997. Model-based predictive control for generalized production problems. In *Computers in Industry, Vol. 34, pp. 201-210.*

Ydstie, B. E., Grossmann, I. E., Perea-López, E., 2003, A Model predictive control strategy for supply chain optimization. In *Computers and Chemical Engineering, Vol 27, pp. 1202-1218.*