DYNAMIC TOOLS TO CONTROL COMPLEX SYSTEMS

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Abstract: The idea of complex sequencing is strongly related to scheduling and real time systems. It appears when a resource has to be shared by more than one user or when a job must be handled by some concurrent entities. The originality of our research is mainly to build a dynamic link between a scheduling solution and a simulation approach based on Petri Nets (PN). This paper addresses with the modeling of an electroplating line with max plus algebra theory and Petri Nets. As a schedule representing a sequence that contains a series of jobs, we can obtain a corresponding polynomial by using Lagrange interpolation. Then, we can use these polynomials to ensure a real time connection between a schedule procedure and a Petri nets simulation. The use of this approach will lead us to deal with different disturbances. Indeed, for any disturbance we should calculate a new schedule and a new polynomial will be found. This polynomial will be assigned to Petri nets Model without modifying its structure.

1 INTRODUCTION

The scheduling approach is realized by using max plus algebra theory. It is based on a model that we have previously developed. We first look for a mathematical model to find a cyclic schedule for the hoist Scheduling Problem (HSP) (Bloch, 1999). Therefore, concurrence appears for resource utilization and only deterministic discrete event systems can be described in the max algebra (Gaubert, 1992). A relaxation is then proposed to find a first solution. Then some heuristics (rules) are introduced to respect resource constraints and an admissible solution (schedule) is proposed.

The second approach is based on Petri nets (PN) their practical use for discrete event systems are well known in the literature. Nevertheless, more powerful tools are necessary to deal with some complex systems. The functions attached to arcs in Colored Petri nets (CPN) sometimes seem to be more difficult to understand. We propose here to use Z/pZ Petri nets to model some complex sequencing. A mathematical structure is defined to represent the relation between the colors (Mabed, 2003).

This algebraic structure is an isomorphism between a set of colors and a finite field Z/pZ.

2 PETRI NETS AND EVENT GRAPH

A PN is a bipartite directed graph with two types of nodes: the places and the transitions. These nodes are joined by directed arcs. A transition X_i is enabled if and only if each of its input places contains at least one token (assuming that the weight of each arc is equal to 1).

It is also possible to assign a time to each place or transition. In that case, the Petri net is called respectively P-timed or T-timed Petri nets. Such a time represents the time taken by the related transition to fire.

In a P-Timed Petri Nets (P-TPN), a firing is initiated by removing token from each of the input places after a period of time equals to the time assigned to the place. Then, one token appears in each output place at the instant when the firing terminates. The reader is referred to (Mabed, 2002) for details.

An event graph is a Petri net such that each place has exactly one input transition and one output transition and such that every arc is 1-weighted. In this way only synchronization constraints (logical AND) can be represented, whereas alternative choices (corresponding to

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logical OR) cannot be represented. Figure (1) gives an example of a P-timed event graph. Transition X_1 (resp X_2) will be fired if there is a token in both places P_1 and P_6 (resp P_3 and P_5). In P_1 and P_3, token must spend a period of time equals respectively to 1 and 2. This is the time assigned to these tokens in these places before leaving.

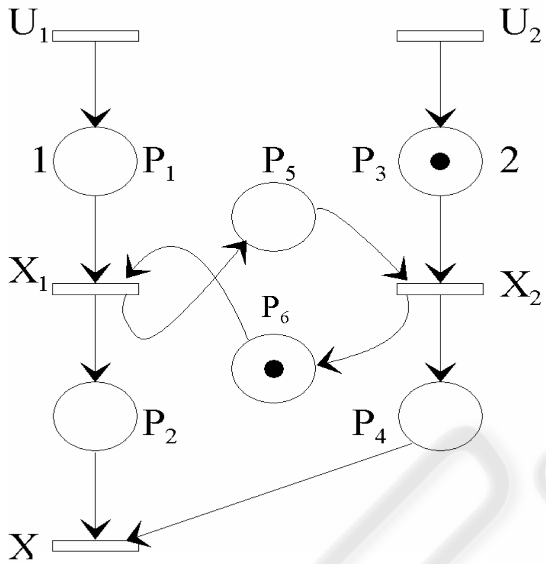

Figure 1:An event graph

3 MAX PLUS THEORY

The dioid IRmax = (IR∪{-∞},max,+) is an algebraic structure with two operations, max as addition (denoted by ⊕) and + as multiplication (denoted by ⊗ or simply omitted). e= -∞ plays the role of the null element for ⊕ (such that e⊕a=a) and ε=0 plays the role of the unit element for ⊗ (such that a⊗ε=ε⊗a=a). We refer the reader to (Cohen, 1989) (Spacek, 1998) to learn the main properties of this structure.

The combinatorial properties of dioids (i.e. associativity, commutativity of ⊕ and ⊗ and distibutivity of ⊗ over ⊕), allow matrix manipulations in a conventional way. It is no hard to see that the following equations must hold for the above example in figure 1.

$X_i(k)$ represents the date of the k^{th} firing of transition X_i.

$X_1(k) \geq U_1(k) + 1$ and $X_1(k) \geq X_2(k-1)$

$X_2(k) \geq U_2(k-1) + 2$ and $X_2(k) \geq X_1(k)$

$X_3(k) \geq X_1(k)$ and $X_3(k) \geq X_2(k)$

or

$X_1(k) \geq \max (X_2(k-1),U_1(k) + 1)$

$X_2(k) \geq \max (X_1(k), U_2(k-1) + 2)$

$X_3(k) \geq \max(X_1(k), X_2(k))$

For instance $X_2(k)$ can be fired if the transitions $X_1(k)$ and $U_2(k-1)$ have been fired and if the minimum duration allowed to the places has been respected.

$$X(k) \geq \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ e & \varepsilon & \varepsilon \\ e & e & \varepsilon \end{pmatrix} X(k) \oplus \begin{pmatrix} \varepsilon & e & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \end{pmatrix} X(k-1)$$

$$\oplus \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & \varepsilon \\ \varepsilon & \varepsilon \end{pmatrix} U(k) \oplus \begin{pmatrix} \varepsilon & \varepsilon \\ \varepsilon & 2 \\ \varepsilon & \varepsilon \end{pmatrix} U(k-1)$$

We can easily write :

$$X(k) \geq \bigoplus_{i=0}^{v} A(i)\, X(k-i) \oplus \bigoplus_{j=0}^{w} B(j)\, U(k-j)$$

We can solve this problem via rational methods familiar in language theory (introducing "star" operation $A^* = e \oplus A \oplus A^2 \oplus \ldots \oplus A^n$)

$$X(k) \geq \bigoplus_{i=1}^{v} \overline{A}(i) X(k-i) \oplus \bigoplus_{j=0}^{w} \overline{B}(j) U(k-j)$$

With $\overline{A}(i) = (A(0))^* A(i)$ and $\overline{B}(j) = (A(0))^* B(j)$

4 DESCRIPTION OF THE PROBLEM

Production line fed with one or more hoists are used in a large number of cyclic industrial applications. The moving devices (hoists) transport products from one station to another, according to a sequence order. This problem is characterized by the nature of the processing times, which are variables of the model. Each of them has to take a value in a given interval $[t_i, T_i]$. The result of a schedule is firstly the sequence of the movements and secondly the values of all processing times.

We consider a production line with 13 different stations and one hoist. These stations are arranged in a row and are fed with one hoist (Baptiste, 1993) (Bloch, 1999).

The first station represented by tank (0), is a shared station for both loading and unloading purpose. A soak operation is performed in each other station see figure 2. The hoist is programmed to perform inter-process moves of the products. Each one consists of four simple hoist operations:

– Raise a product from a process tank;
– Pause over the tank to allow drip-off;
– Transport the product to the next process tank
– Lower the product in that process tank

After performing an inter-process move, the hoist travels to another process tank for the next scheduled move. The following assumptions will be respected throughout this presentation:

– The processing time in each tank is bounded between a minimal and a maximum duration $[t_i, T_i]$: i = 0 to 12;
– Each tank can receive only one product at the same time;
– No buffer exists between tanks;
– We deal with mono-product case: each product must respect the same sequence S=0,1,2, 3, …, 11, 12, 0.

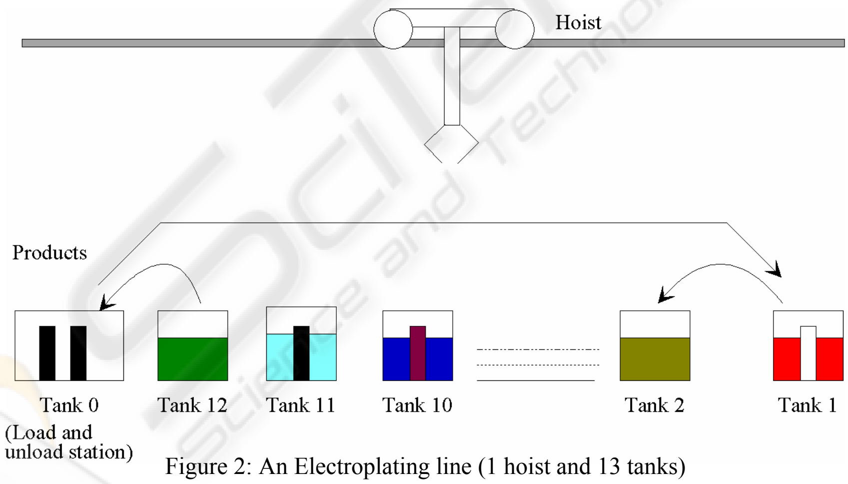

Figure 2: An Electroplating line (1 hoist and 13 tanks)

5 SCHEDULING

Let us study the mono-products modeling case (one hoist, tanks with unit capacity, same processing for all the products, n products on the line).Each product will be successively treated in several tanks and then deposed into the load \ unloading station (tank(0)). There are two major statuses: product transfer from a tank to another one and processing. We will consider the notations given below.

– $X_i(k)$: the starting time to enable transition Xi for the kth time;
– $r_{(i, i+1)}$: the in charge hoist traveling time from tank i to tank (i+1);
– t_i , T_i : minimum and maximum soak time limits in tank i ;
– P_j : Place number j
– $Dv_{(i, i+1)}$: the empty hoist traveling time from tank i to tank (i+1) ;
– $U_i(k)$: command vector representing the availability of the hoist;

5.1 P-Timed Event Graph

Two labels are given with each place. The first one (t) is the minimum time that a token entering the place must spend. The second label (T) is the maximum time that this token can spend in this place. Each tank has its own given bounded time intervals. If a token enters a place of Timed Colored Petri Nets (TCPN), it must spend a minimum time t_i in this place and leave it at a maximum time T_i at the latest. Each time a tank is emptied, it is immediately available for the next product on the line. The last operation of each product sequence is to unload it. We represent this problem by an event graph figure 3.

The first tank (token(0)) filling the unloading and loading tank, we will assume that there is no event before this.

In figure 3 each time a transition is fired, it will mean either a product is being transported from a tank to another or that it is processed. We suppose that the hoist is always available (otherwise concurrence will appears for resource utilisation)

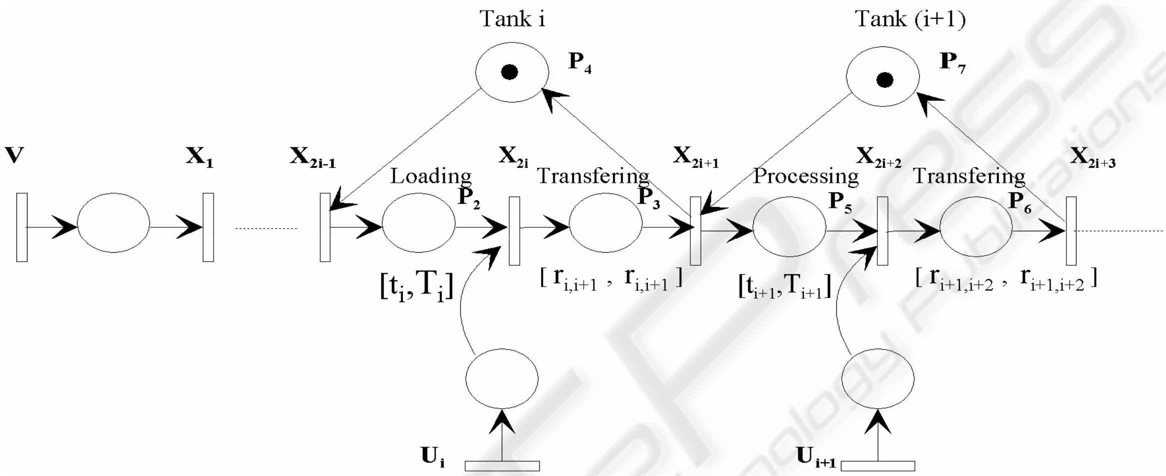

Figure 3: Hoist scheduling P-Timed event graph

5.2 Mathematical Model

According to the previous example, we can represent the event graph presented on figure 3 by the following equations:

$X_1(k) \geq X_3(k-1) \oplus (X_2(k) \otimes (-T_1)) \oplus V(k)$
$X_2(k) \geq U_1(k) \oplus (X_1(k) \otimes t_1) \oplus (X_3(k) \otimes (-r_{1,2}))$
$X_3(k) \geq X_5(k-1) \oplus (X_4(k) \otimes (-T_2)) \oplus (X_2(k) \otimes r_{1,2})$
$X_4(k) \geq U_2(k) \oplus (X_3(k) \otimes t_2) \oplus (X_5(k) \otimes (-r_{2,3}))$
.
.
$X_{26}(k) \geq U_{13}(k) \oplus (X_{25}(k) \otimes t_{13}) \oplus (X_{27}(k) \otimes (-r_{13,1}))$
$X_{27}(k) \geq X_{26}(k) \otimes r_{13,1}$

$$X(k) \geq (A_0 \otimes X(k)) \oplus (A_1 \otimes X(k-1)) \oplus (B \otimes U(k)) \oplus (b \otimes V(k))$$

This system can be also written with matrix notations:

and gives the vector :

$$X^*(k) = \overline{A_1} \otimes X(k-1) \oplus \overline{B} \otimes U(k) \oplus \overline{b} \otimes V(k)$$

These vectors (k=1, 2,…,n) are represented by a matrix. In each column, we can find the starting time of transport or processing for each product k. Nevertheless, this solution must be performed according to the hoist constraints (ie. only one hoist is available in the line). The hoist availability constraint is the most restrictive one as the hoist corresponds to a critical resource of the problem. Each move of product k from the tank where its i^{th} treatment occurs to the tank where its $i+1^{th}$ one occurs, takes place in the following interval $[X_i(k), X_i(k)+ r_{(i,\ i+1)}]$. As all the movements are realized by the same hoist, we must express that all these intervals are disjoined. The objective here is to find an admissible schedule from the solution proposed by the (max, +) theory and then improve it to find an optimal or a near optimal solution.

For all couples of transport operations of the product k (resp k+1) from tank (i) (resp j) to tank (i+1) (resp. j+1) either transport operation of k preceeds transport operation of k+1 (case 1) or transport operation of k+1 preceeds transport operation of k (case 2).

This kind of constraints are called disjunctions, it corresponds to two mutually exclusive constraints.

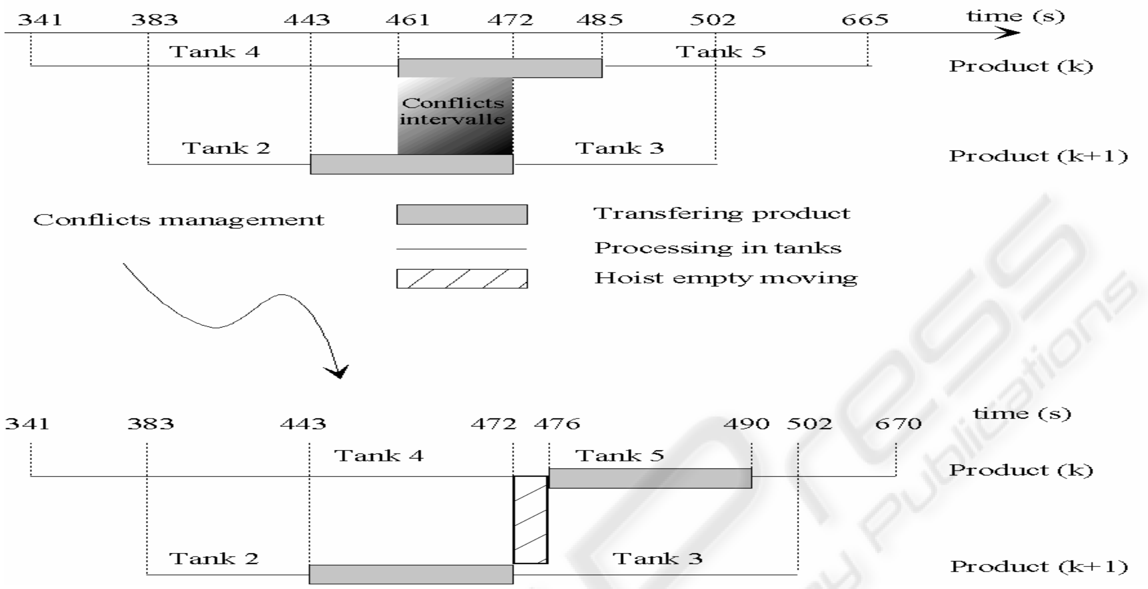

Figure 4: How to deal with conflicts (case 2)

As we can see on the example detailed on figure 4, we have two products k and (k+1) and two operations request the same resource (hoist) during a period of time:

– Move product k from tank 4 to tank 5, during the interval [461,485]

– Move product k+1 from tank 2 to tank 3, during the interval [443,472]

To deal with this conflict, we have to choose between two cases as seen before. In figure 4 we choose the case 2: transport operation of k+1 preceeds transport operation of k. It means that a delay will affect the others tasks for this product. The question is: how to choose how a conflict can be treated? The transcription of our model into a computable version is simply the translation of all linear equations. When a solution is found, we check that the hoist constraints are respected, if not we try to resolve this conflict by an adapted heuristic. We can notice that each conflict corresponds to two alternatives and needs to make a choice during the resolution.

In fact, when the resolution procedure reaches a choice point it divides the search space into two branches. Exploring all these solutions will take us a long time. The situation has led us to develop a new robust method governed by stochastic methods. These methods (not developed in this paper) serve as a meta-strategy to choose the best way to deal with the management of the hoist conflicts. The objective of this work is to explain how to build a dynamic link between a scheduling solution and a simulation approach based on PN.

Before modeling the electroplating line we should present how we can translate this schedule to a polynomial. As a schedule represents a sequence that contains a series of jobs, we can obtain a corresponding polynomial by using Lagrange interpolation. Then, this polynomial will be used to valuate PN arcs.

5.3 Example

Let us consider a simple sequence S representing a schedule:

$S = (S_0, S_1, \ldots, S_{(p-1)})$

We can obtain the corresponding polynomial P(X) by defining a finite ring structure with p distinct elements {0, 1, 2, …, p-1} isomorphic with Z/pZ (p: prime). On this set, we define a polynomial function P(X) as an automorphism of Z/pZ.

$$P(X) = \sum_{i=0}^{p-1} \mathbf{a_i X^i}$$

A polynomial in Z/pZ[X] is completely given by its coefficients A_0 to $A_{(p-1)}$. We already prove that a matrix $[L^p]$ exists there, making the correspondence between the p elements of S.

${}^t[A_0, A_1, \ldots, A_{(p-1)}]=[L^p].{}^t[S_0\, S_1 \ldots S_{(p-1)}]$

$[L_k^p(X)]= [1\ X\ X^2 \ldots X^{(p-1)}].\ [L^p].\ [I_k]$

Where I^k is a column matrix whose elements: equal 1 in k^{th} row and = 0 elsewhere.

$[L_k^p(X)]$ is a (p-1) degree polynomial verifying the conditions:

$[L_k^p(k)]=1$ and $[L_k^p(X)]= 0,\ \forall\ X\neq k$

This polynomial is the interpolation Lagrange polynomial, which can be written, if considering the first theorem of Fermat :

$[L_k^p(X)]= 1+(p-1)(X+p-k)^{(p-1)}$

When we develop $(X+p-k)^{(p-1)}$ by Newton's binomial in Z/pZ, we obtain $[L^p]$ matrix:

$$[L^p]=\begin{matrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & (p-1)^{(p-2)} & (p-2)^{(p-2)} & \ldots & 1 \\ 0 & (p-1)(p-1)^{(p-3)} & (p-1)(p-2)^{(p-3)} & \ldots & p-1 \\ 0 & (p-1)^{(p-4)} & (p-2)^{(p-4)} & \ldots & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ p-1 & p-1 & p-1 & \ldots & p-1 \end{matrix}$$

Let us apply this method to the sequencing vector S=(0,6,1,4,2,5,3), for p=7 the matrix $[L^7]$ is :

$$[L^7]=\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 3 & 2 & 5 & 4 & 1 \\ 0 & 6 & 5 & 3 & 3 & 5 & 6 \\ 0 & 6 & 6 & 1 & 6 & 1 & 1 \\ 0 & 6 & 3 & 5 & 5 & 3 & 6 \\ 0 & 6 & 5 & 4 & 3 & 2 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 \end{matrix}$$

$P(X)= [1\ X\ X^2 \ldots X^6].[L^7].[F]$

with $[F]={}^t[P(0)\ P(1)\ P(2)\ P(3)\ P(4)\ P(5)\ P(6)]$ thus equal to ${}^t[6\ 4\ 5\ 0\ 2\ 3\ 1]$.

We obtain $P(X) =6+6.X+6.X^2+X^4+6.X^5$

and easily verify that P(0)=6, P(6)=1, P(1)=4, P(4)=2, P(2)=5, P(5)=3, P(3)=0;

As said before if the length of the sequence does not correspond to a prime number (n<p), we go back to the previous case and consider (p-m) fictitious tasks that will not be used.

6 MODELING :Z/PZ TPN REACTIVE MODEL

A complex system needs to be dynamically controlled. Even, if the polynomials introduced in this presentation show a clear evolution of the standard functions used in traditional TCPN, a characteristic does remain specific to all these polynomials. It consists of the definition of the color set. In Z/pZ TPN, there is only a global color set and all the polynomials take its values in it. This is why a circumstantial readjustment is always possible (Mabed, 2003).The first tank (token (0)) filling the unloading and loading tank, we will assume that there is no event before this. In this Petri Nets, tokens represent tanks. Each time a transition is fired, it means either a product is being transported from a tank to another, or that it is processed.

We are interested here in modeling an electroplating line with Z/pZ Timed Petri Nets (TPN). It illustrates that these nets can be easily applied to various industrial problems. Modifying a sequence leads to change the Z/pZ TPN parameters (i.e. polynomial) while the net structure remains the same.

NB: For a simple use of Petri Nets parameters we replace T_i (the maximum duration in tank i) by δ_i, T_i will represent the transition i.

Petri nets have been traditionally employed in simulating approaches based on discrete event systems. Here we create a dynamic link between two fields:

– Scheduling approach: based on several stochastic or deterministic methods.
– Simulation approach based on Petri nets.

We present in figure 5 a global model for the HSP problem. This model was built in two sections: first, we create a PN model and secondly we assign polynomials to this PN. When we elaborate this model, several difficulties were encountered due to the two associated stations (loading, unloading) represented by tank (0). The first operation is to load the product, which is represented by a processing (color (0) in place P_3). After a minimal duration of soak the product can leave this tank (represented by color (0)) to another one (represented by the successor of (0)+1=(1)) according to the set definition E={0,1,2,3,4,5,…,10,11,12}. Each product will be successively treated in several tanks and then deposed into the load \ unloading station (tank(0)).

There are two major statuses:

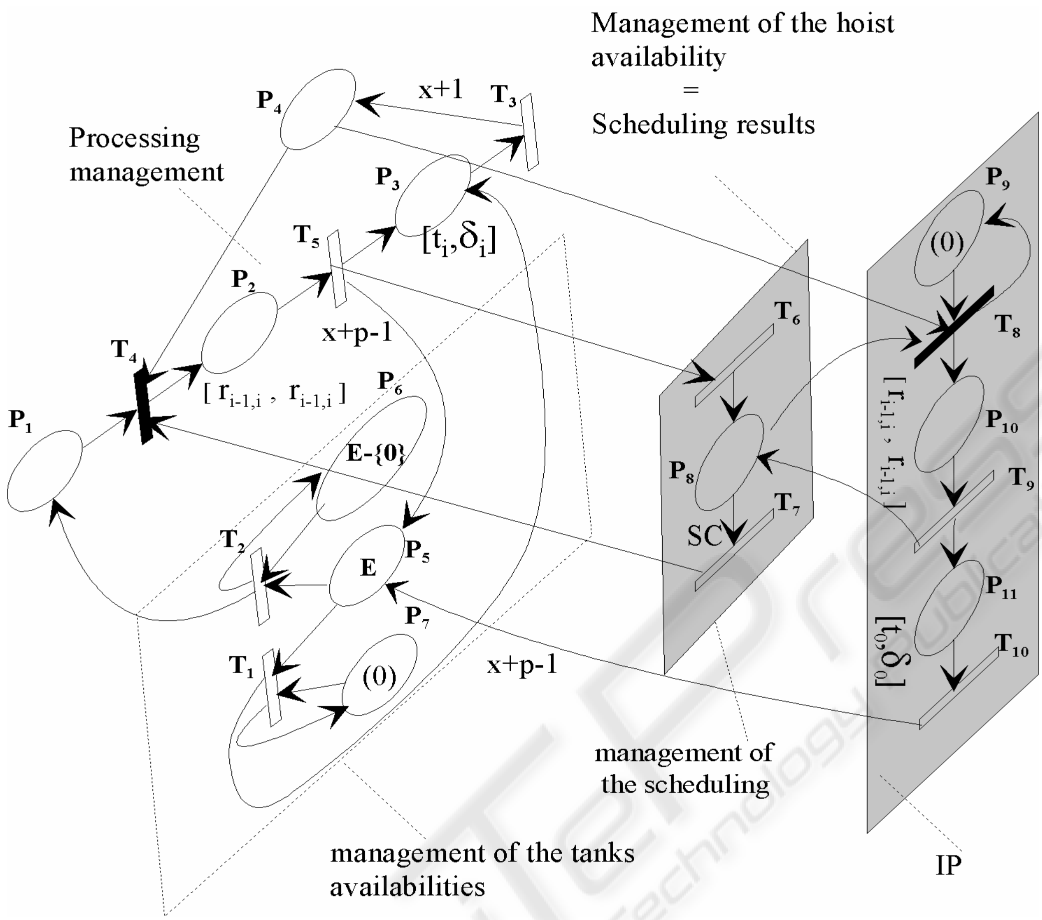

Figure 5: Z/pZ TPN Reactive Model

– Product transfer from a tank to another one: The sequence representing the schedule has been realized by the calculus of a polynomial **SC**. This

These operations will continue until we use the color (12), its successor will be (0) and the transition T_8 will be fired to unload the product. Each evolution or disturbance lead this set unchanged. Only the parameters are to be changed. The model presented below allows a dynamic management.

7 CONCLUSION

The originality of this study is mainly to apply both: max plus algebra theory and Z/pZ TPN for modeling and evaluating an industrial problem. We have seen in this paper that a model can be dynamically controlled by combining existing tools.

We first use max plus algebra theory to find a schedule for an industrial application known as a

polynomial characterizes the results of max plus approach.

– Processing

hoist scheduling problem. Using the analogy between timed event graphs and conventional linear systems lead us to propose a schedule approach. The critical point in this work is to improve the results in order to find an optimal solution. The use of stochastic methods to explore more solutions can be the best way to reach this objective.

After this, we present a model by using Z/pZ Timed Petri Nets in order to control this system. A structure of field Z/pZ is proposed to allow the processing of colors succession with mathematics. This mathematical structure allows an easy modeling of sequences by polynomials and symbolic calculation.

Furthermore, mathematical support may enable us to consider various problem of optimization, particularly those dealing with the resolution of scheduling problems, one of our points of interest.

We also note that this approach offers many advantages to deal with complex models. Developing these polynomials to evaluate models facilitates modeling any kind of systems. This new tool can be used as basis for further studies on scheduling.

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