IMPROVING THE PERFORMANCE OF EQUALIZATION AND A FAST START-UP TECHNIQUE FOR COMMUNICATION SYSTEMS

Hua Ye, Wanlei Zhou
School of Information Technology
Deakin University
221 Burwood HWY, Burwood.
VIC. 3125. Australia.

Keywords: least-mean-square (LMS), discrete cosine transform (DCT), recursive least squares (RLS), filtered-X LMS, H_2.

Abstract: This paper explores the potential of several popular equalization techniques and proposes new approaches to overcome their disadvantages. Such as the conventional least-mean-square (LMS) algorithm, the recursive least-squares (RLS) algorithm, the filtered-X LMS algorithm and their development. An H_2 optimal initialization has been proposed to overcome the slow convergence problem while keeping the simplicity algorithms. The effectiveness of the methods proposed in this paper has been verified.

1 INTRODUCTION

The least-mean-square (LMS) based adaptive algorithm have been successfully applied in many communication equalization practices. The importance of the LMS algorithm is largely due to two unique attributes(R.D.Gitlin et al., 1992):

- Simplicity of implementation
- Model-independent and therefore robust performance

The main limitation of the LMS algorithm is its relatively slow rate of convergence. Two principal factors affect the convergence behaviour of the LMS algorithm: the step-size parameter μ, and the eigenvalues of the correlation matrix R of the tap-input vector.

The recursive least-square (RLS) algorithm is derived as a natural extension of the method of least square algorithm. The derivation was based on a lemma in matrix algebra known as the matrix inversion lemma. (E.A. Lee et al., 1994). (S. Haykin, 1996).

The fundamental difference between the RLS algorithm and the LMS algorithm can be stated as follows: The step-size parameter μ in the LMS algorithm is replaced in RLS algorithm by $\phi^{-1}(n)$, that is, the inverse of the correlation matrix of the input vector U(n). This modification has a profound impact on the convergence behavior of the RLS algorithm in a stationary environment, as summarized here (David S. Bayard, 1997)-(Steven L. Gay, 1993):

1. The rate of convergence of the RLS algorithm is typically an order of magnitude faster than that of the LMS algorithm.

2. The rate of convergence of the RLS algorithm is invariant to the eigenvalue spread (i.e., condition number) of the ensemble-averaged correlation matrix R of the input vector U(n).

1. The excess mean-squared error $J_{ex}(x)$ of the RLS algorithm converges to zero as the number of iterations, *n,* approaches infinity.

The computational load of the conventional RLS algorithm is prohibited in real time applications. The recursive least-squares (RLS) algorithm is characterized by a fast rate of convergence that is relatively insensitive to the eigenvalue spread

Hua Y. and Zhou W. (2004).
IMPROVING THE PERFORMANCE OF EQUALIZATION AND A FAST START-UP TECHNIQUE FOR COMMUNICATION SYSTEMS.
In *Proceedings of the First International Conference on E-Business and Telecommunication Networks*, pages 129-135
DOI: 10.5220/0001382101290135

of the underlying correlation matrix of the input data, and a negligible misadjustment, although its computational complexity is increased(J.M. Cioffi , 1984)-(BOUCHARD M., 2000).

Though the LMS algorithm does not actually converge to the least--mean--square solution that optimal H_2 model matching solution achieves, they are very close if the adaptive step size μ is small enough. Interestingly, not much effort is needed to find the filter $\hat{P}(z)$ as the filtered-X LMS algorithm still converges so long as the estimate of the channel $P(z)$ has less than $\pm 90^0$ phase shift and unlimited amplitude distortion. The H_∞ robust performance analysis of the LMS algorithm conducted by Hassibi, *et al.* reveals that sum of the squared errors is always upper bounded by the combined effects of the initial weight uncertainty ($\hat{W}(0)$ -- W_0) and the noise $\upsilon(i)$. This evidence strongly supports that the H_2 optimal initialisation presented in this thesis can confine the error to a low level right from the beginning and hence improve the convergence rate dramatically.
A big benefit of this approach is that it makes the adaptive process a virtual fine-tuning process if a reasonable initialization is obtained, which avoids experiencing a possibly long adaptation process in transit to the fine-tuning period. The advantage will be more clearly illustrated by a high eigenvalue spread case. Extensive simulation experiment has shown that, in many cases, the adaptive process starts from an acceptable performance, and it does not need any remedy like Discrete Cosine Transform (DCT) or Discrete Fourier Transform (DFT) even in the case with a very high input signal eigenvalue spread where the conventional LMS algorithm may fail and traditionally a remedy like DCT and DFT technique is required.
The conventional filtered-X LMS is modified and introduced for the purpose of equalization. Generic integration of the filtered-X structure, LMS algorithm, RLS algorithm and optimal H_2 initialization is conducted to meet all paramount criteria of simplicity, robust and fast convergence for equalization of high-speed, distorted communication channels.
Finally, various techniques proposed in this thesis are tested using a popular communication channel example, under both slight non-stationary and sever non-stationary conditions. Comparisons are made with other conventional methods. Significant performance improvement has been observed by Mont Carlo. The effectiveness of the methods proposed in this thesis has been verified.

2 EVALUATION

We present the experiment results of three adaptive equalization algorithms:
least-mean-square (LMS) algorithm, discrete cosine transform-least mean square (DCT-LMS) algorithm, and recursive least square (RLS) algorithm. Based on the experiments, we obtained that the convergence rate of LMS is slow; the convergence rate of RLS is great faster while the computational price is expensive; the performance of that two parameters of DCT-LMS are between the previous two algorithms, but still not good enough. Therefore we will propose an algorithm based on H_2 in a coming paper to solve the problems.
It is well known that high data rate transmission through dispersive communication channels is limited by the inter-symbol interference (ISI). Equalization is an effective way to reduce the effects of ISI by cancelling the channel distortion. However, dynamic, random and time-varying characteristics of communication channels make this task very challenging. High speed of data transmission demands a low computational burden. Hence, simplicity and robust performance play a crucial role in equalizer design. Due to its good robust performance and computational simplicity, least-mean-square (LMS) based algorithms have received a wide attention and been adopted in most applications (E.A. Lee et al., 1994), but one major disadvantage of the LMS algorithm is its very slow convergence rate, especially in high condition number case .
To solve this problem, a variety of improved algorithm have been proposed in the literature. Although their actual implementations and properties may be different but the underlying principle remains the same: trying to orthogonalize as much as possible the input

autocorrelation matrix and to follow a steepest-descent path on the transformed error function.
Therefore, we extend the least square algorithm to a recursive algorithm for the design of adaptive transversal filter. An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant of time when the algorithm is initiated. The resulting rate of convergence is therefore typically an order of magnitude faster than the simple LMS algorithm. This improvement in performance, however, is achieved at the expense of a large increasing in computational complexity.
The RLS algorithm implements recursively an exact least squares solution (Steven L. Gay, 1993). At each time, RLS estimates the autocorrelation matrix of the inputs and cross correlation between inputs and desired outputs based on all past data, and updates the weight vector using the so-called matrix inversion lemma. The DFT/LMS and DCT/LMS algorithms are composed of three simple stages (Steven L. Gay, 1993). First, the tap-delayed inputs are preprocessed by a discrete Fourier or cosine transform. The transformed signals are then normalized by the square root of their power. The resulting equal power signals are input to an adaptive linear combiner whose weights are adjusted using the LMS algorithm. With these two algorithms, the orthogonalizing step is data independent; only the power normalization step is data dependent. Because of the simplicity of their components, these algorithms retain the robustness and computational low cost while improving its convergence speed.
Although the structure of the filtered-X LMS adaptive equalization scheme is a little bit different from that of the basic LMS adaptive equalization scheme, the control adjustment process is the same: adjusting the FIR model of the equalizer to minimize the least mean square error e_k.

3 A FAST START-UP TECHNIQUE

Though the LMS algorithm does not actually converge to the least--mean--square solution that optimal H_2 model matching solution achieves, they are very close if the adaptive step size μ is small enough. Interestingly, not much effort is needed to find the filter $\hat{P}(z)$ as the filtered-X LMS algorithm still converges so long as the estimate of the channel $P(z)$ has less than $\pm 90^0$ phase shift and unlimited amplitude distortion. The H_∞ robust performance analysis of the LMS algorithm conducted by Hassibi, *et al.* reveals that sum of the squared errors is always upper bounded by the combined effects of the initial weight uncertainty ($\hat{W}(0) - W_0$) and the noise $\upsilon(i)$. This evidence strongly supports that the H_2 optimal initialisation presented in this thesis can confine the error to a low level right from the beginning and hence improve the convergence rate dramatically.
A big benefit of this approach is that it makes the adaptive process a virtual fine-tuning process if a reasonable initialization is obtained, which avoids experiencing a possibly long adaptation process in transit to the fine-tuning period. The advantage will be more clearly illustrated by a high eigenvalue spread case. Extensive simulation experiment has shown that, in many cases, the adaptive process starts from an acceptable performance, and it does not need any remedy like Discrete Cosine Transform (DCT) or Discrete Fourier Transform (DFT) even in the case with a very high input signal eigenvalue spread where the conventional LMS algorithm may fail and traditionally a remedy like DCT and DFT technique is required.
The conventional filtered-X LMS is modified and introduced for the purpose of equalization. Generic integration of the filtered-X structure, LMS algorithm, RLS algorithm and optimal H_2 initialization is conducted to meet all paramount criteria of simplicity, robust and fast convergence for equalization of high-speed, distorted communication channels.
Finally, various techniques proposed in this thesis are tested using a popular communication channel example, under both slight non-stationary and sever non-stationary conditions. Comparisons are made with other conventional methods. Significant performance improvement has been observed by Mont Carlo.

The effectiveness of the methods proposed in this thesis has been verified.

This experiment has verified a well known fact that the conventional adaptive LMS algorithm can track slight non-stationary environments such as slowly varying parameters. Now, a more severe non-stationary situation is tested by abruptly increasing the channel impulse response coefficients by 35% of its nominal value. Fig. 1 shows the simulation result.

The conventional adaptive LMS algorithm begins to diverge while the filtered-X LMS algorithm with or without optimal initialization still maintains a good robust performance. The conventional RLS algorithm still has an acceptable performance, which matches the observation that when the time variation of the channel is not small, the RLS algorithm will have a tracking advantage over the LMS algorithm . The filtered-X RLS algorithm has a better robust performance. The robust performance enhancement by the introduction of the filtered-X structure is obvious and significant.

From a computational point of view, optimal initialization needs an additional effort to solve an H_2 optimal model matching or H_2 filtering problem. Since this procedure is a non-iterative solution and can be done off-line, it does not increase the computational burden in online operation. The only extra online computational burden concerned comes from the extra filter that is involved in every adaptive step. However, that structure increases only a computation of one simple algebraic convolution. This poses no serious problem in computation at all.

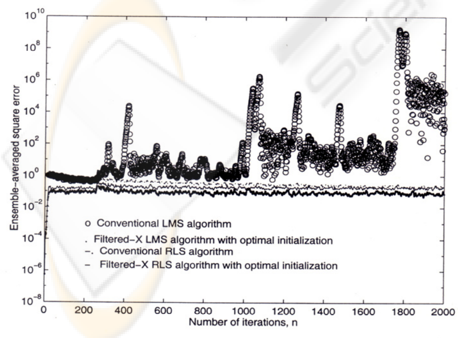

Figure 1: Learning curves of the various adaptive algorithms experiencing a abrupt increase of impulse response of the channel by 35%

4 INTEGRATION METHOD

Therefore, the H_2 optimal solution is actually the limit of the best solution for the filtered-X LMS adaptive equalization algorithm. Similar to the case of the basic LMS adaptive equalization scheme, the filtered-X LMS adaptive equalization scheme can not, in general, achieve this limit. However, its optimal solution is still expected to be close to that point if the adaptive size is small. Therefore, the optimal H_2 initialization method proposed still applies here. Why not simply use the H_2 optimal model matching filter as the final equalizer? This is because of the presence of the model uncertainty and other unexpected disturbances. The optimal solution obtained in offline computation may not be optimal when the filter is implemented in the real world system because of model uncertainty and other unexpected disturbance. For the optimal initialization, a poorly identified system model may give rise to a low quality model matching solution.

However, due to robustness of filtered-X LMS adaptive equalization scheme, this solution may still be well within the convergence region. By extensive simulations and experiments, it is observed that method proposed here can also cope with wide eigenvalue spread of the input without having to use Discrete Cosine Transformation (DCT) that was conventionally required. This is an advantage in real-time operation environment where computation burden is a critical factor.

The focus of this work is on improving the equalization performance of the powerful LMS and RLS adaptive algorithms while minimizing the increase of the related computational complexity. Since these algorithms are very popular in real world applications, the attemption is significant.

Channel equalization is an effective signal processing technique that compensates for channel-induced signal impairment and the resultant inter-symbol interference (ISI) in communications system. Many sophisticated techniques have been proposed for equalization, most of successful real world applications are still dominated by techniques that are related to several popular algorithms, such as the adaptive LMS algorithm, the filtered-X LMS algorithm and the RLS algorithm. For high-speed commercial communication systems, simplicity, robust and fast convergence rate are critical criteria for the design of a good equalizer. The adaptive LMS algorithm, the filtered-X LMS algorithm, and the RLS algorithm meet some of these criteria. Unfortunately, none of them, alone, satisfies all these criteria. Therefore, research on exploring the potential of these techniques while overcoming their disadvantages is important and necessary, which is exactly what has been conducted in this paper.

(1). The practical importance of LMS algorithm is largely due to simplicity of implementation and its robust performance and its main limitation is relatively slow rate of convergence. The RLS algorithm is characterized by a fast rate of convergence that is relatively insensitive to the eigenvalue spread of the underlying correlation matrix of the input data, and a negligible misadjustment. Although it is computational complexity.

(2). The conventional filtered-X LMS is modified and introduced for the purpose of equalization. The famous filtered-X LMS algorithm has found very successful applications in the field of active noise and vibration control. It has inherited the elegant simplicity of the conventional LMS algorithm, and is very robust. For approach in analyzing the performance of the filtered-X LMS algorithm, a heuristic method based on linear time-invariant operator theory has been provided to analyze the robust performance of the filtered-X structure. It indicates that the extra filter could enhance the stability margin of the corresponding non filtered-X structure. In this thesis, a generic integration of the filtered-X structure, LMS algorithm, RLS algorithm and optimal H_2 initialization has been conducted to meet all paramount criteria of simplicity, robust and fast convergence for equalization of high-speed communication channels.

(3). To overcome the slow convergence problem while keeping the simplicity of the LMS based algorithms, an H_2 optimal initialization is proposed. Though the LMS algorithm does not actually converge to the least-mean-square solution that optimal H_2 model matching solution achieves, they are very close if the adaptive step size μ is small enough. Interestingly, not much effort is needed to find the filter $\hat{P}(z)$ as the filtered-X LMS algorithm still converges so long as the estimate of the channel P(z) has less than $\pm 90^0$ phase shift and unlimited amplitude distortion [21]. The H_∞ robust performance analysis of the LMS algorithm conducted by Hassibi, *et al*. reveals that the sum of the squared errors is always upper bounded by the combined effects of the initial weight uncertainty $\left(\hat{W}(0)-W_0\right)$ and the noise *v*(*i*). This evidence strongly supports that the H_2 optimal initialization presented in this thesis can confine the error to a low level right from the beginning and hence improved the convergence rate dramatically.

A big benefit of this approach is that it makes the adaptive process a virtual fine-tuning process if a reasonable initialization is obtained, which avoids experiencing a possibly long adaptation process in transit to fine-tuning period. The advantage will be more clearly illustrated by a high eigenvalue spread case. As it is well known that the conventional LMS converges very slowly or even fails to converge with a no matter how small adaptive step size due to high input signal eigenvalue spread. H_2 optimal model matching solution is independent of this input signal eigenvalue spread, and hence could avoid this trouble. Moreover, this idea can be combined with other speed-up techniques such as Discrete Cosine Transform (DCT) and Discrete Fourier Transform (DFT) as well as various adaptive algorithms. Extensive simulation experiment has shown that, in many cases, the adaptive process starts from an acceptable performance, elated ven in the case with a very high input signal eigenvalue spread.

Another approach proposed here is that it generally does not require detailed knowledge of the external signal which is a great advantage in practice. Since there exist many powerful tools solving H_2 filtering problem, including explicit solution, the method proposed in this thesis is very promising.

(4). A popular communication channel example is used to test the proposed techniques, under both slight non-stationary and severe non-stationary conditions. The level of channel distortion is deliberately raised to a level that is much higher than any published result with system condition number as high as nearly 390. Furthermore, it is assumed that each tap weight of the channel undergoes an independent stationary stochastic process with each parameter fluctuating around its nominal value with a uniform probability distribution over the interval $\left(h_k - 8\%h_k, h_k + 8\%h_k\right)$, in addition to the white noise disturbance of variance 0.001 at the channel output. Mont Carlo simulation experiment of 1000 independent trials is conducted to obtain an ensemble-averaged learning curve. All adaptive algorithms have shown a good robust performance against the time varying, random Gaussian impulse response coefficient fluctuations specified above. The filtered-X LMS with the optimal H_2 initialization has been shown to have the fastest convergence rate and best performance.

(5). A more severe non-stationary situation was tested by abruptly increasing the channel impulse response coefficient by 35% of its nominal value. The conventional adaptive LMS algorithm begins to diverge while the filtered-X LMS algorithm with or without optimal initialization still maintains a good robust performance. The conventional RLS algorithm has an acceptable performance, which matches the observation that when the time variation of the channel is not small, the RLS algorithm will have a tracking advantage over the LMS algorithm (R.D.Gitlin et al., 1992). The filtered-X RLS algorithm has a better robust performance. The performance improvement by using the proposed techniques is significant and hence, the effectiveness of the new method has been verified.

5 CONCLUSIONS

The contributions of this paper are: we are compared the LMS with DCT-LMS and RLS for adaptive equalizer first, then we will be conducted on how to speed up the convergence rate of LMS based algorithm while keeping the increased in-line computational burden as low as passible, we will overcome the slow convergence problem while keeping the simplicity of the LMS based algorithm, and the H_2 Optimal initialization has been applied in adaptive equalizer for communication systems.

There still exists many open problems. For instance, the analysis of the stability margin of the filtered-X LMS was conducted in a heuristic manner. Can we extend this to a general case such as a discrete time MIMO case? What about the filtered-X RLS algorithm? Can we apply the ideas to other adaptive equalization techniques such as decision-feedback equalization, etc.? What happens if we use the H_∞ optimal initialization instead of the H_2 optimal initialization? Another very active area of equalization is wireless communication where the phenomenon of fast multiple-path fading (Rayleigh fading) is very challenging. As indicated in the simulation, rapid and not so small channel variations can cause the conventional LMS algorithm to diverge. It will be interesting and challenging, therefore, to apply the new techniques presented here to those areas in the future.

REFERENCES

R.D.Gitlin, J.F. Hayes, and S.B. Weinstein, *Data communication principles,* Plenum Press, New York, 1992.

E.A. Lee and D.G. Messerschmitt, Digital communication, Second Edition, Kluwer Academic Publishers, 1994.

S. Haykin, Adaptive filter theory, Third Edition Edition, Prentice Hall Information and System Sciences Series, 1996.

David S. Bayard, "LTI representation of adaptive systems with tap delay-line regressors under sinusoidal excitation," "Necessary and sufficient conditions for LTI representations of adaptive systems with sinusoidal regressors," Proceedings of the American Control Conference, Albuquer, New Mexico, June 1997, pp. 1647-1651, pp. 1642-1646.

Steven L. Gay, "A fast converging, low complexity adaptive filtering algorithm," Proceedings of 1993 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics, New Paltz, NY, USA, 1993, pp.4-7.

S.Elliott and P. Nelson. "Active noise control," *IEEE Signal Processing Magazine,* Oct. 1993.

Markus Rupp and Ali H. Sayed, "Robust FXLMS algorithms with improved convergence performance," *IEEE Trans. on Speech, Audio Processing,* vol.6, no.1, Jan.1998, pp.78-85.

E.A. Wan, Adjoint LMS: an efficient alternative to the Filtered-X LMS and multiple error LMS algorithms. IEEE International Conference on Acoustics, Speech, and Signal Processing, 1996, ICASSP-96, vol.3, pp.1842-1845.

Markus Rupp, "Saving complexity of modified filtered-X LMS and delayed update LMS algorithm," *IEEE Trans. on Circuits and System-II: Analog and Digital Signal Processing,* vol.44, no.1, Jan. 1997, pp.57-60.

Steven L. Gay, "A fast converging, low complexity adaptive filtering algorithm," *Proceedings of 1993 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics,* New Paltz, NY, USA, 1993, pp.4-7.

J.M. Cioffi and T. Kailath, "Fast recursive least-squares transversal filters for adaptive filtering," *IEEE Trans. on Acoustics, Speech, and Signal Processing,* vol. ASSP-32, pp. 304-338, April 1984.

F.T.M. Slock, "Reconciling fast RLS lattice and QR algorithms," *1990 International Conference on Acoustics, Speech and Signal Processing,* vol.3, pp.1591-1594, 1990, New York, USA.

M.Bouchard, S.Quednau, "Multichannel RLS Algorithms and Fast-Transversal-Filter Algorithms for Active Noise Control and Sound Reduction System," *IEEE Trans. Speech and Audio Processing*, vol.8, no.5, Sep. 2000

BOUCHARD, M. and QUEDNAU, S. " Multichannel Recursive-Least-Squares Algorithms and Fast-Transversal-Filter Algorithms for Active Noise Control and Sound Reproduction Systems," *IEEE Transactions on Speech and Audio Processing*, vol. 8. No. 5, September 2000.