

A COMPARISON BETWEEN THE PROPORTIONAL KEEN APPROXIMATOR AND THE NEURAL NETWORKS LEARNING METHODS

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Abstract: The Proportional Keen Approximation method is a young learning method using the linear approximation to learn hypothesis. In the paper this methodology will be compared with another well-established learning method i.e. the Artificial Neural Networks. The aim of this comparison is to learn about the strengths and the weaknesses of these learning methods regarding different properties of their learning process. The comparison is made using two different comparison methods. In the first method the algorithm and the known behavioural model of these methods are analysed. Later, using this analysis, these methods are compared. In the second approach, a reference dataset that contains some of the most problematic features in the learning process is selected. Using the selected dataset the differences between two learning methods are numerically analysed and a comparison is made.

1 INTRODUCTION

In a research aimed to compensate for the errors due to the flexibility and the backlash in mechanical systems (Kabiri, 2002), and after unsuccessful implementation of the Artificial Neural Networks (ANN), attention was focused on finding a more suitable method. By the ANN, the author means the Back Propagation (BP) model of the ANN. The aim for that research was to compensate for the positioning errors of a robotic arm by considering it as a Black Box (Mingzhong, 1997). After much research using linear approximation methodology a novel machine learning method was developed and implemented (Kabiri, 1998 & 1999). This approach provides the adaptability feature for the controller but requires a large population of samples. The need for simplicity in the data set might be the reason why previously the researchers were only using this approach for the joints of the manipulator (Chen, 1986). Main problems with the ANN were its slow training and difficulty in training it with complicated patterns in the dataset (Kabiri, 1998)(Sima, 1996). Once the population of the dataset increases, these problems will get worse. The Proportional Keen Approximator (PKA) has no training stage as in the ANN and at the most problematic areas for the ANN it can work without any problems. On the other hand

this method has some limitations of its own, some of which might be handled in future e.g. requires more memory than the ANN and needs tabulated sampled dataset. As it will be described in the following section the early version of the PKA was introduced in the Cartesian co-ordinate system (Kabiri, 1998). The basic cell had two inputs and one output. Later a new version with three inputs and one output was introduced. Currently the work is being continued to extend the PKA into the Spherical and Cylindrical co-ordinate systems that will broaden its scope of application. The PKA in the Cartesian co-ordinate system is implementing the linear approximation to interpolate between the sample points. The PKA is a Memory-Based Learning (MBL) system and requires large memory capacity (Schaal, 1994).

At the following sections the reader will first learn about the PKA methodology. The properties of these methods are compared both in numerical and descriptive ways. Using a sampled dataset derived from an equation, the ANN and the PKA methodologies are compared numerically against each other and results are presented.

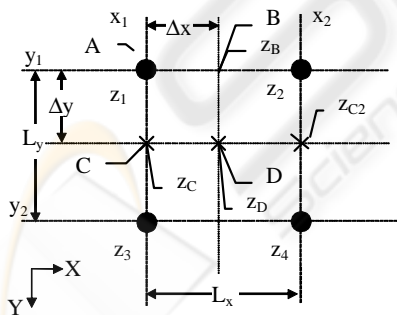
2 2D-PKA IN CARTESIAN CO-ORDINATES SYSTEM

This Approximator is designed for 2 input and 1 output configuration (2D). Later it was expanded to 3 input and one output configuration (3D). In the 2D configuration as it is illustrated in Figure 1, for every ordered pair of (x, y) in the function domain, output of the system can be calculated considering three conditions:

Condition 1. Both of the x and y are available in the sampled data set area. In this condition, the output will be the actual given data sample. Therefore, the accuracy is matched by default 100% e.g. A (x₁, y₁, z₁) in Figure 1.

Condition 2. One of the input components (x or y) overlays one of the Rows or Columns in the given data sample. If the given point has either of the x or y components equal to one of the components of the sampled data, this condition will be triggered. For example, if the ordered pair of (x, y) is considered as input and z considered as the data component that determines the amount of error in that location. The output is calculated with respect to two adjacent neighbours of the given point. Only values of the two unequal components (in the sample and the given data pairs) are used for the calculations. For example, let x₁ = x₂ for the pairs (x₁, y₁) and (x₂, y₂). Then for calculating the output only the y components are used. e.g.

B (x₁+Δx, y₁, z_B) when x₁ = x₂ and



r and c are Row and Column
 z₁, z₂, z₃, and z₄ are values of the sampled points where
 $z_B = z_1 + ((z_2 - z_1) * \Delta x) / L_x$,
 $z_C = z_1 + ((z_3 - z_1) * \Delta y) / L_y$,
 $z_{C2} = z_2 + ((z_4 - z_2) * \Delta y) / L_y$, and
 $z_D = z_C + ((z_C - z_{C2}) * \Delta x) / L_x$; are calculated values.

Figure 1: Calculation Method for the 2D-PKA Approximator.

C (x₁, y₁+Δy, z_C) when y₁ = y₂.

Condition 3. None of the input co-ordinates are equal to any of the Rows or Columns in the Approximator. In this condition the method will be applied to one of the components (x or y) first. Then the result of the calculation is used for the proportional approximation on the other pair of components (same as condition 2). In this condition the output value is calculated by D (x₁+Δx, y₁+Δy, z_C) i.e. output = z_C in Figure 1.

3 METHODOLOGY OF 3D-PKA

The main difficulty for understanding the 3D-PKA is the visualisation of the method that is going to be explained at the following. The key point to understand the idea of expanding the PKA into the 3rd dimension is to consider each 2D-PKA as a plane surface. Here instead of points, flat surfaces are considered and the rest is similar to the 2D-PKA (Kabiri, 1999). Figure 2 illustrates the concept. A point can either be on the top or the bottom layer, or it can be between the two layers. In the first two cases the estimated value is the returned value from the 2D-PKA cell (either the top or the bottom layer that holds the point) i.e. Condition 1. However for the third case the estimated value is the result of a linear approximation between the results out of both top and bottom 2D-PKA layers i.e. Condition 2. It should be noted that as the number of dimensions increases, the required computation power for the method increases as well. The same is with the memory requirements for the 3D-PKA.

4 PROPERTIES OF THE PKA

In the ANN the number of links and nodes in the network depends on the complexity of the training data set. Therefore the execution speed for the ANN is directly related to the complexity of the training data set. However, only one PKA cell is required to learn the pattern regardless of the complexity of the sampled data set. This means that in the PKA method, speed is independent from the complexity of the sampled data set. Since the number of operations in this method is smaller and due to the simplicity of these operations¹, the PKA in the forward execution needs less calculation power than the other alternative methods such as the ANN (particularly in 3D). In the worst case scenario

(condition 3) every 2D-PKA cell needs 3 addition, 6 subtraction, 3 multiplication and 3 division operation times plus the time needed for the memory address calculation and data fetching (delays due to memory related operation are assume to be constant). However, for the ANN this time varies with the size of the network (considering an operation time of one +or- operation time for every node and an operation time of one *or/ operation for each link). Therefore the PKA in the forward execution is expected to be faster than medium or large size ANN. Of course the actual difference depends on the specific ANN and needs to be quantified for each individual case. As an example, a fully connected and biased ANN with the 2:11:15:1 structure can be considered for comparison where 229 multiplication operations and 202 (2-input addition) operations are required (Kabiri, 1999).

Approximator has three levels of precision. As explained above, accuracy in the first condition is 100%. The second and the third conditions respectively have less accuracy. The amount of error in the two latter conditions depends on the closeness of the graph between two sample points to the straight line. The amount of error in those conditions also relates to the distance between two sample points. This means that the higher the resolution, the more accurate is the approximation. There is no training stage for this approach and it is ready as soon as all the samples become accessible to the Approximator (Loaded into the memory). Despite the complexity of the pattern introduced to the PKA, one PKA cell is sufficient for learning the pattern. Another feature for this method is the possibility to update the knowledge of the PKA while using it. This is because the database generated by the method (tabular data), enables us to accurately locate any part of the knowledge and change it.

5 RESULTS

In order to perform a numerical comparison between the PKA and the ANN methodologies an experiment is prepared. In this experiment the data derived from an equation is used as a frame of reference and the methodologies were compared with respect to this frame of reference. In the following, the experiment is described and results are presented.

5.1 System Variables

As it is described in section 4 the accuracy for the PKA in sampled points is 100%. Therefore, the

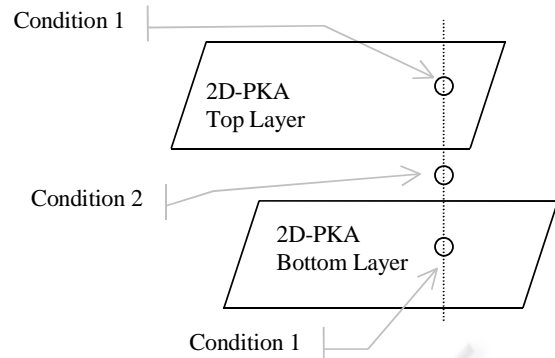


Figure 2: Methodology for the 3D-PKA Calculations.

Maximum Error and the Mean Square Error (MSE) for the PKA at the sampled points are zero. In order to compare the PKA with the ANN method the following experiment was designed and implemented (Kabiri, 1999).

This experiment is aimed to help both in comparing the important features in the hypothesis surfaces of these methods and their resultant error values. These methods (ANN and PKA) will be compared with respect to the error values resulting in the sampled points, interim points and finally all together. In order to do the comparison, function

$$SINC(u, v) = \frac{100.0 * \sin(\sqrt{u^2 + v^2})}{\sqrt{u^2 + v^2}}$$

was selected as the frame of reference for the comparison, because it is sufficiently complex to include a range of angular variations from very sharp to gently curved patterns. The properties of this function are analysed to provide a better form of comparison. The original domain of the $SINC(u, v)$ function is $u, v \in [-\pi/2 \pi/2]$. The input domains for the function are moved to $x \in [110.0 \ 245.0]$ and $y \in [-90.0 \ 90.0]$ in the scale of millimetres. The maximum value for the function is 100.0mm. Using this function, a sample dataset was produced in equal sampling steps of 3.0mm for the x -axis and 9.0mm for the y -axis. The ANN was trained using the generated dataset. As the PKA method has no training stage, it uses this dataset for the generalisation.

In order to generate the generalised dataset, the sampling step for the x and y -axes were respectively altered to 1.0mm and 3.0mm. In other words the resolutions for the axes are increased by the factor of 3. The total number of samples introduced to both methods is 966 samples. As the result of the generalisation, the population of the sampled dataset increases to 8296 samples. The following describes

1. Simpler operation means shorter execution time for the operation.

the training parameters of the ANN.

The ANN consists from a fully connected and biased with 2 layers network structure of 2:11:15:1. The training for the ANN was carried out on a Pentium-400 based PC. Learning rate was initially equal to 0.9 and later it was changed to 0.7, 0.5, 0.3, 0.2 and 0.1. Momentum factor for the training was 0.9 and was later changed to 0.8 and finally 0.7. The training was many times forced to restart again. This was because of different reasons such as need for more nodes in the network, over training or lack of progress in the training. The training process took around 10 days. In the final reported training, the total number of elapsed epochs was 1382755 epochs. Total training time was 285899.0 seconds (3 days, 7 hours, 24 minutes and 59 seconds). The final MSE using normalised data on the output was 0.00027 and the final MSE using de-normalised (in order to make it comparable with the PKA values) data on the output was 6.04.

Table 1 compares two methods. In this table by the term “the whole generalised points” the author means the sampled points plus the generalised interim points. Figure 4-a to Figure 4-c respectively depict the output from the reference equation, PKA and ANN. In Figure 3-a to Figure 3-c the aforementioned dataset is generalised using respectively the reference equation, PKA and ANN.

In analysing the data it is important to understand that in any estimation the size of the sampling step plays an important role in the final resulting estimation error. If the speed of change in the dataset is very high then a small sampling step is required. This is some how the case in this example. The ratio of the rise in the height of the graph is sometimes too fast for the applied sampling step size. Figure 5 depicts this concept.

As it can be seen in Figure 5-a, in the PKA method the error values are higher at the centre of the function domain than the rest of the points. In the PKA method, the error is not evenly distributed in the function domain. In this example in the PKA method, although the maximum error seems to be higher than the ANN method (Figure 5-b), most of

the points are well below the MSE value.

Figure 5-b depicts the distribution of the error values in the ANN method. This figure shows that the ANN method has a more evenly distributed error pattern than the PKA method. However most of the error values in this method are close to its MSE.

5.2 Analysis of the Result

Analysing the result, first the differences can be divided into two categories training stage and the execution stage. Later both of these categories will be divided into smaller subsets such as difference in execution speed, accuracy and some differences due to the properties of the PKA methodology.

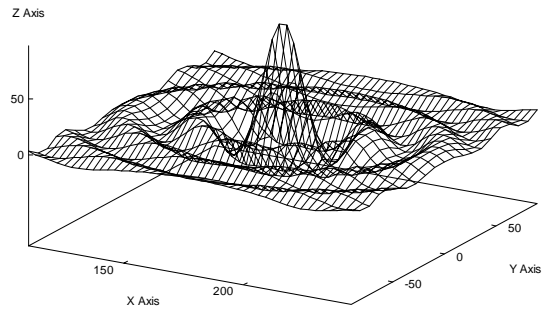
In the training stage as it is for the ANN, the difference is huge. No training for the PKA in comparison to the 3 days training for the ANN is a clear advantage for the PKA methodology. Contrary to the ANN that the required time for the training can be different from one dataset to another, the time required for the PKA to learn knowledge (the training stage) is always the same. This property plus ability to update the knowledge during the execution are two of the main advantages of the PKA. On the other hand PKA requires a large memory to hold the samples that makes it a memory-bound process, the fully connected and biased ANN using the BP model is mostly a CPU-bound process. Currently cost of the CPU time and the Memory module are falling.

Sensitivity to the complexity in the training pattern and sample population affects the training stage for the ANN and in the supervised training it might be necessary to reorganise the network and restart the training. However, in the PKA none of these problems will be encountered. After the training for the ANN using extra sample points the result has to be verified, but in the PKA due to the consistency in the linear approximation this is not necessary. In the reported experiment maximum error in the PKA method was larger than the ANN method. This difference is due to the large

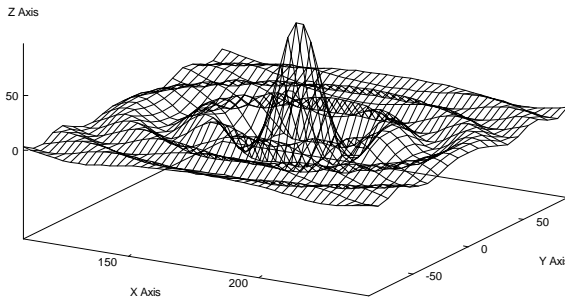
Table 1: A numerical comparison between the PKA and the ANN. All the values are in millimetres. (MSE is calculated using de-normalised data values). (*) Means that: Where output should be 91.98 it was 78.95

	The ANN	The PKA
Maximum ³ error for the sampled points.	1.3	0.0
MSE for the sampled points.	6.0	0.0
Maximum error (difference) for the interim points.	8.7	13.0 (*)
MSE for the interim points.	9.0	4.1
Maximum error (difference) for the whole generalised points.	8.7	13.0 (*)
MSE for the whole generalised points.	8.0	3.6

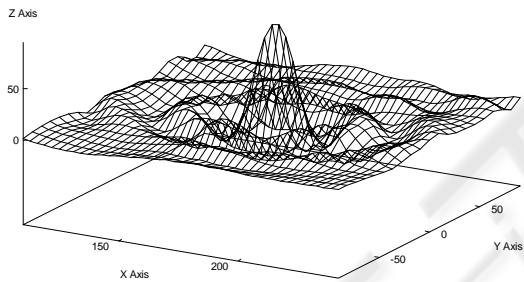
3. Maximum error is a relative error (between the reference value and the generalised value).



a) The original sampled dataset (X*1, Y*1).



b) The output from the PKA with no generalisation.



c) The output from the ANN with no generalisation.

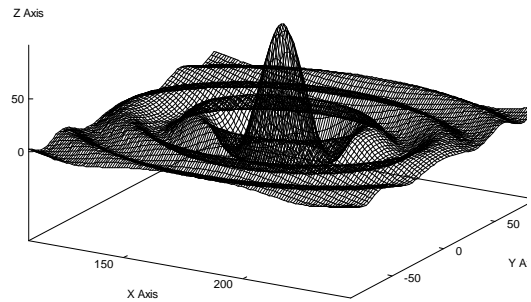
Figure 4: A comparison between the Original sampled dataset and the resulted output from the both PKA and ANN methods without any generalisation (X*1, Y*1).

The z-axis represents the output.

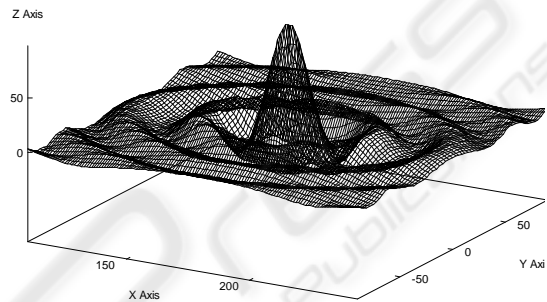
$$SINC(u, v) = \frac{100.0 * \sin(\sqrt{u^2 + v^2})}{\sqrt{u^2 + v^2}}$$

non-linearity in the hypothesis space at that point and low sampling step.

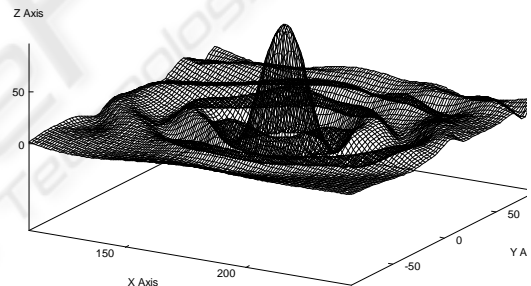
This result may change for other curves and it is an uncertain region for both methods. Nevertheless this problem is the subject for further research to implement the PKA method in the Spherical or Cylindrical co-ordinate systems. In training the ANN any randomly collected sample data is acceptable, but the PKA method can only work with tabulated dataset collected with equal sampling steps. This is a major drawback for this method, but the reader has to note that in the industry tabulated data sampling is the most common method. The



a) The original sampled dataset with 3 times higher sample resolution in either of the axes.



b) The output from the PKA with 3 times higher sample resolution in either of the axes.



c) The output from the ANN with 3 times higher sample resolution in either of the axes.

Figure 3: A comparison between the original sampled dataset and the resulted output from both PKA and ANN methods with generalisation (X*3, Y*3). The z-axis represents the output.

requirement for the tabulated data makes the PKA not suitable for the biological kind of research activities that may need to use randomly collected data. Possibility of using the random sampled data for the PKA is also a research subject that is under investigation.

The execution speed of the ANN is dependent on the organization of the network and the number of nodes and links in the network. However, the execution time for the PKA has a maximum delay condition (Condition 3, section 2 for the 2D-PKA and section 3); this is because only one cell is sufficient to learn any complex pattern in the dataset.

Execution time for large and medium size ANN is longer than the PKA (section 4).

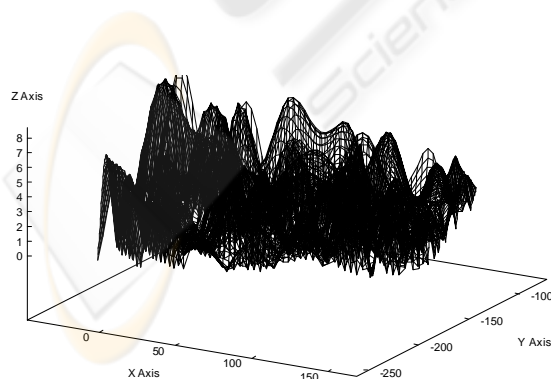
6 CONCLUSIONS

In comparing the PKA and the ANN methodologies, before deriving any conclusions the reader should be reminded that in this paper a young (4 years old) method i.e. PKA has been compared with a well established method such as the ANN. Therefore, expectation should be as far as a 4 years old methodology and one should just consider this comparison as an introduction to potentials of a new methodology (PKA) and in no way as a mean of disregarding the ANN. For the PKA there is still a long way to go to reach maturity in concept as it has been achieved for the ANN.

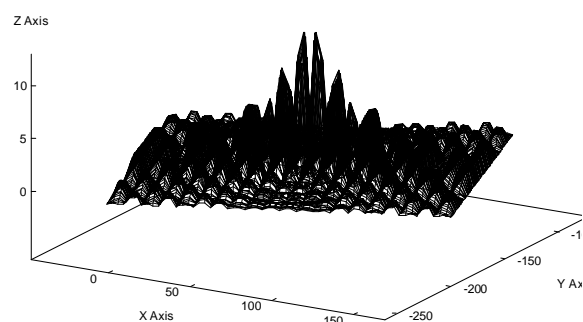
This paper highlights the differences between the PKA and the ANN methodologies and compares some limitations of one versus advantages of the other. In this report only a 2D-PKA in Cartesian coordinates is compared with back propagation type of the ANN with 2 inputs and 1 output. In both methodologies sampling resolution plays a major role in the accuracy of the estimation. In some areas such as the training process, execution speed and accuracy the 2D-PKA shows some advantages over the ANN. On the other hand the ANN has some of its own properties such as the ability to learn randomly collected dataset, which the 2D-PKA is not yet capable of performing. As a young methodology the PKA still needs more research to be done to extend its methodology and to find more about its properties. New generations of the PKA in Cylindrical and Spherical co-ordinate systems are expected to provide us with new features and better accuracy for the estimation of the curved patterns.

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The distribution of the error for the ANN



The distribution of the error for the PKA

Figure 5: A comparison for the resultant error between the original sampled dataset and the resulted output from the both PKA and ANN methods with generalisation ($X*3, Y*3$). The z-axis represents the difference between the output from the reference equation (SINC) and the output produced by each method.