Unsupervised Learning of a Finite Discrete Mixture Model Based on the Multinomial Dirichlet Distribution: Application to Texture Modeling

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Abstract. This paper presents a new finite mixture model based on the Multinomial Dirichlet distribution (MDD). For the estimation of the parameters of this mixture we propose an unsupervised algorithm based on the Maximum Likelihood (ML) and Fisher scoring methods. This mixture is used to produce a new texture model. Experimental results concern texture images summarizing and are reported on the *Vistex* texture image database from the MIT Media Lab.

1 Introduction

Scientific pursuits and human activity in general generate data. These data may be incomplete, redundant or erroneous. Statistical pattern recognition methods are particularly useful in understanding the patterns present in such data. The finite mixture densities appears as the fundamental model in areas of statistical pattern recognition. A mixture model consists of multiple probability density function (PDFs). The PDFs are called components densities of the mixture and are chosen to be Gaussian in the majority of cases. The Gaussian mixture is not the best choice in all applications, however, and it will fail to discover *true* structure where the partitions are clearly non-Gaussian [14]. In [4] we have shown that the Dirichlet distribution can be a very good choice to overcome the disadvantages of the Gaussian. In this paper, we present another distribution based in the Dirichlet and the multinomial distributions which we call the MDD (Multinomial Dirichlet Distribution). We prove, through a novel texture model, that the MDD is very efficient to model discrete data (vectors of counts) for them the Gaussian is not an appropriate choice. In order to solve the problem of MDD mixture parameters estimation, we propose an algorithm based on the Fisher scoring method and we use an entropy-based criterion to determine the number of components.
The paper is organized as follows. The next section describes the MDD mixture in details. In section 3, we propose a method for estimating the parameters of this mixture. In section 4, we present a way of initializing the parameters and give the complete estimation algorithm. Section 5 is devoted to experimental results. We end the paper with some concluding remarks.

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2 The Multinomial Dirichlet Mixture

Let $\boldsymbol{c} = (c_1, \ldots, c_{dim})$ denotes a vector of counts (ex. the frequency of a given dim features in a document). The vector $\boldsymbol{c}$ follows a Multinomial distribution with parameter vector $\boldsymbol{P} = (P_1, \ldots, P_{dim})$ given by:

$$p(\boldsymbol{c}|\boldsymbol{P}) = \frac{|\boldsymbol{c}|!}{c_1!c_2!\ldots c_{dim}!} \prod_{k=1}^{dim} P_k^{c_k} \tag{1}$$

The conjugate prior for $\boldsymbol{P}$ is the dirichlet distribution:

$$p(\boldsymbol{P}) \sim D(\alpha_1, \ldots, \alpha_{dim}) = \frac{\Gamma(|\boldsymbol{\alpha}|)}{\prod_{k=1}^{dim} \Gamma(\alpha_k)} \prod_{k=1}^{dim} P_k^{\alpha_k - 1} \tag{2}$$

where $|\boldsymbol{\alpha}| = \sum_{k=1}^{dim} \alpha_k$, $P_k > 0$ and $\sum_{k=1}^{dim} P_k = 1$. The Beta distribution is the special case when $k = 2$. Given a Dirichlet prior, the joint density is:

$$p(\boldsymbol{c}, \boldsymbol{P}|\boldsymbol{\alpha}) = \frac{\Gamma(|\boldsymbol{c}|+1)\Gamma(|\boldsymbol{\alpha}|)}{\prod_{k=1}^{dim} \Gamma(\alpha_k)\Gamma(c_k+1)} \prod_{k=1}^{dim} P_k^{\alpha_k + c_k - 1} \tag{3}$$

then [4]:

$$p(\boldsymbol{c}|\boldsymbol{\alpha}) = \frac{\Gamma(|\boldsymbol{c}|+1)\Gamma(|\boldsymbol{\alpha}|)}{\Gamma(|\boldsymbol{c}|+|\boldsymbol{\alpha}|)} \prod_{k=1}^{dim} \frac{\Gamma(c_k+\alpha_k)}{\Gamma(\alpha_k)\Gamma(c_k+1)} \tag{4}$$

We call this density the MDD (Multinomial Dirichlet Distribution).
A MDD mixture with M components is defined as :

$$p(\boldsymbol{c}/\Theta) = \sum_{j=1}^{M} p(\boldsymbol{c}/j, \Theta_j) P(j) \tag{5}$$

where the $P(j)$ $(0 < P(j) < 1$ and $\sum_{j=1}^{dim} P(j) = 1)$ are the mixing proportions and $p(\boldsymbol{c}/j, \Theta_j)$ is the MDD. The symbol Θ refers to the entire set of parameters to be estimated:

$$\Theta = (\boldsymbol{\alpha_1}, \ldots, \boldsymbol{\alpha_M}, P(1), \ldots, P(M))$$

where $\boldsymbol{\alpha_j}$ is the parameter vector for the j^{th} population. In the following developments, we use the notation $\Theta_j = (\boldsymbol{\alpha_j}, P(j))$ for j = 1 ... M.

3 Maximum Likelihood Estimation

The problem of estimating the parameters which determine a mixture has been the subject of diverse studies [5]. During the last two decades, the method of maximum likelihood (ML) has become the most common approach to this problem. Of the variety of iterative methods which have been suggested as alternatives to optimize the parameters of a mixture, the one most widely used is Expectation Maximization (EM). EM was originally proposed by Dempster et al. [6] for estimating the Maximum Likelihood Estimator (MLE) of stochastic models. This algorithm gives us an iterative procedure and the practical form is usually simple. But, it suffers from the following drawback: the need to specify the number of components each time. In order to overcome this problem, criterion functions have been proposed, such as the Akaike Information Criterion (AIC) [8], Minimum Description Length (MDL) [9] and Schwartz's Bayesian Inference Criterion (BIC) [7]. A maximum likelihood estimate associated with a sample of observations is a choice of parameters which maximizes the probability density function of the sample. Thus, with ML estimation, the problem of determining Θ becomes:

$$max_{\Theta} p(\boldsymbol{c}/\Theta) \tag{6}$$

with the constraints: $\sum_{j=1}^{M} P(j) = 1$ and $P(j) > 0 \quad \forall j \in [1, M]$. These constraints permit us to take into consideration *a priori* probabilities $P(j)$. Using Lagrange multipliers, we maximize the following function:

$$\Phi(\boldsymbol{c}, \Theta, \Lambda) = ln(p(\boldsymbol{c}/\Theta)) + \Lambda(1 - \sum_{i=1}^{M} P(i)) + \mu \sum_{j=1}^{M} P(j) ln(P(j)) \tag{7}$$

where Λ is the Lagrange multiplier. For convenience, we have replaced the function $p(\boldsymbol{c}/\Theta)$ in Eq. 6 by the function $ln(p(\boldsymbol{c}/\Theta))$. If we assume that we have N random vectors $\boldsymbol{c_i}$ which are independent, we can write:

$$p(\boldsymbol{c}/\Theta) = \prod_{i=1}^{N} p(\boldsymbol{c_i}/\Theta) \tag{8}$$

$$p(\boldsymbol{c_i}/\Theta) = \sum_{j=1}^{M} p(\boldsymbol{c_i}/j, \Theta_j) P(j) \tag{9}$$

Replacing equations 8 and 9, we obtain:

$$\begin{aligned} \Phi(\boldsymbol{c}, \Theta, \Lambda) = & \textstyle\sum_{i=1}^{N} ln(\sum_{j=1}^{M} p(\boldsymbol{c_i}/j, \Theta_j) P(j)) \\ & + \Lambda(1 - \textstyle\sum_{j=1}^{M} P(j)) + \mu \sum_{j=1}^{M} P(j) ln(P(j)) \end{aligned} \tag{10}$$

In order to automatically find the number of components needed to model the mixture, we use an entropy-based criterion previously used in the case of Gaussian mixtures [10]. Thus, the first term in Eq. 10 is the log-likelihood function, and it assumes its global maximum value when each component represents only one of the feature vectors. The last term (entropy) reaches its maximum when all of the feature vectors are modeled by a single component, i.e., when $P(j1) = 1$ for some $j1$ and $P(j) = 0, \forall j, j \neq j1$. The algorithm starts with an over-specified number of components in the mixture, and as it proceeds, components compete to model the data. The choice of μ is critical to the effective performance of the algorithm, since it specifies the tradeoff between the required likelihood of the data and the number of components to be found. We choose μ to be the ratio of the first term to the last term in Eq. 10 of each iteration t, i.e.,

$$\mu(t) = \frac{\sum_{i=1}^{N} ln(\sum_{j=1}^{M} p^{t-1}(\boldsymbol{c_i}/j, \Theta_j)P^{t-1}(j))}{\sum_{j=1}^{M} P^{t-1}(j)ln(P^{t-1}(j))} \tag{11}$$

We will now try to resolve this optimization problem. To do so, we must determine the solution to the following equations: $\frac{\partial}{\partial\Theta}\Phi(\boldsymbol{c},\Theta,\Lambda) = 0$ and $\frac{\partial}{\partial\Lambda}\Phi(\boldsymbol{c},\Theta,\Lambda) = 0$ Calculating the derivative with respect to Θ_j:

$$\frac{\partial}{\partial\Theta_j}\Phi(\boldsymbol{c},\Theta,\Lambda) = \sum_{i=1}^{N} p(j/\boldsymbol{c_i},\Theta_j)\frac{\partial}{\partial\Theta_j}ln(p(\boldsymbol{c_i}/j,\Theta_j)) \tag{12}$$

where $p(j/\boldsymbol{c_i},\Theta_j)$ is the a posterior probability.
Since $p(\boldsymbol{c_i}/j,\boldsymbol{\alpha_j})$ is independent of $P(j)$, straight forward manipulations yield:

$$P(j)^{new} = \frac{\sum_{i=1}^{N} p^{old}(j/\boldsymbol{c_i},\boldsymbol{\alpha_j}) + \mu(p(j)^{old}(1 + ln(p(j)^{old})))}{N + \mu\sum_{j=1}^{M}(p(j)^{old}(1 + ln(p(j)^{old})))} \tag{13}$$

In order to estimate the $\boldsymbol{\alpha}$ parameters we will use Fisher's scoring method. This approach is a variant of the Newton-Raphson method. The scoring method is based on the first, second and mixed derivatives of the log-likelihood function. Thus, we have computed these derivatives [4]. During iterations, the α_{jl} can become negative. In order to overcome this problem, we reparametrize, setting $\alpha_{jl} = e^{\beta_{jl}}$, where β_{jl} is an unconstrained real number. Given a set of initial estimates, Fisher's scoring method can now be used. The iterative scheme of the Fisher method is given by the following equation [11]:

$$\hat{\boldsymbol{\beta}}_j^{new} = \hat{\boldsymbol{\beta}}_j^{old} + V^{old} \times \frac{\partial\phi}{\partial\hat{\beta}_j}^{old} \tag{14}$$

where j is the class number. The variance-covariance matrix V is obtained as the inverse of the Fisher's information matrix:

$$\mathbf{I} = I_{l_1 l_2} = -E[\frac{\partial^2}{\partial\beta_{jl_1}\partial\beta_{jl_2}}\Phi(\boldsymbol{X},\Theta,\Lambda)] \tag{15}$$

An interesting geometric interpretation of this iterative scheme is given in [4]

4 Algorithm

In order to make our algorithm less sensitive to local maxima, we have used some initialization schemes including the Fuzzy C-means [13] and the method of moments (MM) [4]. Thus, our initialization method can be summed up as follows:
INITIALIZATION Algorithm

1. Apply the Fuzzy C-means to obtain the elements, covariance matrix and mean of each component.

2. Apply the MM for each component j to obtain the vector of parameters $\boldsymbol{\alpha_j}$.
3. Assign the data to clusters, assuming that the current model is correct.
4. Update the $P(j)$ using this equation:

$$P(j) = \frac{\text{Number of elements in class j}}{N} \tag{16}$$

5. If the current model and the new model are sufficiently close to each other, terminate, else go to 2.

With this initialization method in hand, our algorithm for estimating a Dirichlet mixture can be summarized as follows:
MDD MIXTURE ESTIMATION Algorithm

1. INPUT: dim-dimensional data c_i, $i = 1, \ldots, N$ and an over-specified number of clusters M.
2. INITIALIZATION Algorithm.
3. Update the $\boldsymbol{\alpha_j}$ using Eq. 14, $j = 1, \ldots, M$.
4. Update the $P(j)$ using Eq. 13, $j = 1, \ldots, M$.
5. If $p(j) < \epsilon$ discard component j, go to 3.
6. If the convergence test is passed, terminate, else go to 3.

The convergence tests could involve testing the stabilization of the $\boldsymbol{\beta_j}$ or the value of the maximum likelihood function.

5 Application: Texture Modeling

Texture analysis and modeling is an important component in image processing and is fundamental to many applications areas including industrial automation, remote sensing, and biomedical image processing. Texture is essentially a neighborhood property. Recent work by Tuceryan and Jain provides a comprehensive survey of most existing structural and statistical approaches to texture [1].
One of the most established method for texture modeling is the Spatial Gray Level method which was proposed by Haralick [2]. It's based on the estimation of the joint probability functions of two picture elements in some given relative position (cooccurrence matrices). The cooccurrence matrices are mostly used as intermediate feature and dimensionality reduction is performed in computing features of the types described in [3]. The main drawbacks to using cooccurrence matrices is the large memory requirement for storing them and the difficulty to use them for statistical approaches because of their numerical nature.
In the following, we will use $\{I(x,y), 0 \leq x \leq K-1, 0 \leq y \leq L-1\}$ to denote an $K \times L$ image with G grey levels. The $G \times G$ cooccurrence matrix $C(d_1, d_2)$ for a displacement vector $\boldsymbol{d} = (d_1, d_2)$ is defined as follows. The entry (i,j) of $C(d_1, d_2)$ is the number of occurrences of the pair of gray levels i and j which are a distance $\boldsymbol{d}$ apart. Formally, it is given as:

$$c(i,j;\boldsymbol{d}) = Card\{(r,s) : I(r,s) = i, I(r+d_1, s+d_2) = j\} \tag{17}$$

Where $Card\{\}$ refers to the number of elements of a set. It's possible to characterize a particular texture sample by a collection of cooccurrence matrix that have been estimated for different displacement d_1 and d_2. In fact suppose that we have dim displacement vector $\boldsymbol{d}$, then for each entry (i, j) we will have a vector of dim elements given as follows:

$$\boldsymbol{c}(i, j; \boldsymbol{d}_1, \ldots, \boldsymbol{d}_{dim}) = (c_1(i, j; \boldsymbol{d}_1), \ldots, c_{dim}(i, j; \boldsymbol{d}_{dim})) \tag{18}$$

As $\boldsymbol{c}$ is a vector of frequencies, the dim cooccurrence matrices can be modeled by a mixture of MDD. With this model in hand, we use it for two experiments: texture images summarizing and retrieval.

5.1 Texture Images Summarizing

Automatic texture images summarizing is an important problem in image processing applications. In the texture images summarizing problem, an image is known to contain data from one of a finite number of texture classes, and it is desired to assign the image to the correct class on the basis of measurements made over the entire image. This application is very important especially in the case of content-based image retrieval. Summarizing the database simplifies the task of retrieval by restricting the search for similar images to a smaller domain of the database. Summarization is also very efficient for browsing. Knowing the categories of images in a given database allows the user to find the images he is looking for more quickly [12]. A block diagram outlining our approach to solve the texture summarizing problem is shown in Fig. 1. The inputs to the classifier are images from a finite number of texture classes. These images are separated into the unknown or test set of images, whose texture class is unknown, and the training set of images, whose texture class is known. The training set is necessary to adapt the classifier to each possible texture class before the unknown set is applied to the classifier. All the input images are passed through the cooccurrence matrices computing stage and then the mixture's parameters estimation stage in which these cooccurrence matrices are modeled as an MDD mixture (we use $G = 32$). After this stage every texture class is represented by a MDD mixture. Finally, the classification stage uses mixtures estimated on the unknown images to determinate which texture class is present. In fact, the estimated mixture is compared to the trained ones by using the $\mathcal{L}_2$ distance for the densities defined by:

$$D(e, t) = \int (p(\boldsymbol{c}|e) - p(\boldsymbol{c}|t))^2 dc \tag{19}$$

Where e and t index respectively an MDD mixture of a texture image to be classified ($p(\boldsymbol{c}|e)$) and an MDD mixture obtained from training and which represents a texture class ($p(\boldsymbol{c}|t)$). Then, the classification will be performed by using this rule: the image e is assigned to class t_1 if $D(e, t1) < D(e, t) \forall t \neq t1$.

For the results in this paper, we use the *Vistex* texture database obtained from the MIT Media Lab. In our experimental framework, each of the 512 × 512 images from the *Vistex* database was divided into 64 × 64 images. Since each 512 × 512 "mother

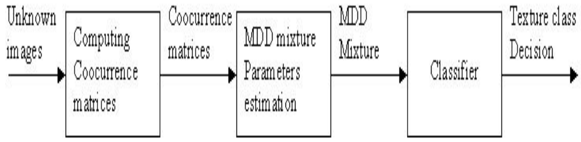
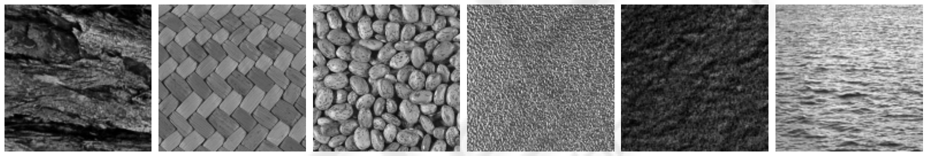


Fig. 1. Block diagram of image texture summarizing application.

image" contributes 64 images to our database, ideally all the 64 images should be classified in the same class. In the experiment, six homogeneous texture groups, "bark", "fabric", "food", "metal", "water" and "sand" were used to create a new database. A database with 1920 images of size 64×64 pixels was obtained. Four images from the bark, fabric and metal texture groups were used to obtain 256 images for each of these categories, and 6 images from the water, food and sand were used to obtain 384 images for this category. Examples of images from each of the categories are shown in Fig. 2.



(a) (b) (c) (d) (e) (f)

Fig. 2. Sample images from each group. (a) Bark, (b) Fabric, (c)Food, (d) Metal, (e) Sand, (f) Water.

After computing the cooccurrence matrices, each category of images will be modeled by a MDD mixture. Table 1 gives the parameters of these mixtures when we consider 8 displacements.

The confusion matrix of the texture images classification is given in table 2. In this confusion matrix, the cell (i, j) represents the number of images from category i which are classified as category j. The number of images misclassified was: 38 images, which represents an accuracy of 98.02 percent. For comparison, we have used Gaussian mixture instead of the MDD one. Table 3 shows the confusion matrix when Gaussian mixtures is used (81 misclassified image, i.e an accuracy of 95.79). From the tables, we see that the performance of the MDD mixture is better.

After the database was summarized, we conducted another experiment designed to retrieve images similar to a query.

5.2 Texture Images Retrieval

To retrieve images that are similar to a query two-step sequence is followed. First, the $\mathcal{L}_2$ distance was used to find the two categories closest to the query by comparing

Category	MDD Mixture parameters									
	class	P(j)	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Bark	class1	0.153	006.959	006.789	006.943	006.770	006.924	006.909	006.981	006.988
	class2	0.241	011.032	012.354	011.799	011.743	011.622	012.353	011.198	011.982
	class3	0.133	004.283	006.602	6.709	006.871	005.152	006.645	007.542	007.331
	class4	0.201	024.685	015.902	016.022	015.908	021.489	015.920	013.616	013.620
	class5	0.141	005.853	006.475	006.323	006.474	005.998	006.342	006.404	006.455
	class6	0.131	005.658	005.941	006.023	005.964	005.795	006.035	006.244	006.206
Fabric	class1	0.258	025.386	022.075	023.681	022.517	026.823	025.745	025.192	024.311
	class2	0.261	112.721	120.025	119.351	109.494	105.727	105.704	113.218	117.024
	class3	0.266	556.039	595.921	573.386	583.624	512.379	518.183	522.132	532.666
	class4	0.215	011.525	011.477	011.380	011.455	012.298	12.331	12.229	11.879
Food	class1	0.205	022.871	022.037	020.157	022.649	023.620	023.145	022.324	021.280
	class2	0.165	004.914	007.579	006.025	006.969	008.663	010.282	012.798	011.390
	class3	0.144	040.978	037.249	042.325	037.690	032.248	030.280	028.814	032.241
	class4	0.164	004.684	007.368	005.662	006.805	008.725	010.188	012.726	011.234
	class5	0.146	096.283	084.071	107.244	085.898	066.255	064.771	057.927	071.078
	class6	0.176	012.889	013.675	012.205	013.282	015.050	015.165	015.388	014.079
Metal	class1	0.233	019.143	016.351	020.814	018.257	014.195	013.714	014.422	013.947
	class2	0.121	006.077	005.706	006.141	006.019	005.650	005.508	005.599	005.557
	class3	0.239	014.560	015.379	013.888	015.914	016.619	016.503	016.439	017.173
	class4	0.242	017.159	016.779	016.589	016.944	017.009	016.151	016.517	016.376
	class5	0.165	005.848	007.433	005.629	005.579	007.619	008.433	007.985	007.971
Sand	class1	0.287	007.062	008.344	007.069	007.607	008.798	008.944	009.106	008.709
	class2	0.337	013.811	014.133	013.802	013.945	014.409	014.645	014.781	14.419
	class3	0.376	128.296	114.760	129.365	122.650	110.878	106.602	104.713	109.643
Water	class1	0.504	846.694	832.655	853.937	857.329	797.566	804.157	805.096	822.290
	class2	0.496	135.545	135.492	135.237	135.450	136.719	136.355	136.490	135.512

Table 1. Estimation of the parameters of the MDD Mixture for the different categories when we consider 8 displacements.

the query to each representative images of the different categories. The same distance measure was used in the second step to determine the similarity between the query and the elements within the two closest components. The best n images were then retrieved, where n depending on the experiment. To measure the retrieval rates, each image was used as a query and the number of relevant images among those that were retrieved was noted. Precision and Recall, which are the measures most commonly used by the information retrieval community, was then computed using Eq. 20 and Eq. 21. These measures was then averaged over all the queries.

$$\text{precision} = \frac{\text{number of images retrieved and relevant}}{\text{total number of retrieved images}} \tag{20}$$

$$\text{recall} = \frac{\text{number of images retrieved and relevant}}{\text{total number of relevant images}} \tag{21}$$

	Bark	Fabric	Food	Metal	Sand	Water
Bark	252	0	0	0	4	0
Fabric	0	250	6	0	0	0
Food	0	5	379	0	0	0
Metal	0	0	0	256	0	0
Sand	2	0	0	0	382	0
Water	1	0	0	5	2	376

Table 2. Confusion matrix for image classification by classifier based on the MDD mixture.

	Bark	Fabric	Food	Metal	Sand	Water
Bark	243	0	0	3	8	2
Fabric	0	240	10	0	4	2
Food	0	12	367	4	0	1
Metal	0	0	0	249	2	5
Sand	7	0	0	0	374	3
Water	4	0	0	8	6	366

Table 3. Confusion matrix for image classification by using texture features from cooccurrence matrices and Gaussian mixture.

As each 512×512 images from the *Vistex* gives 64 images to our database, given a query image, ideally all the 64 images should be retrieved and are considered to be relevant. Table 4 and table 5 presents the retrieval rates in term of precision and recall obtained when the two methods were used. The results are shown when 16, 48, 64, 80, 96 and 128 images were retrieved from the database in response to a query.

	No. of retrieved images				
Method	16	48	64	80	96
MDD Mixture	0.93	0.95	0.92	0.80	0.66
Gaussian Mixture	0.87	0.91	0.85	0.77	0.66

Table 4. Precision obtained for the texture database.

6 Conclusion

In this paper, we have introduced a new mixture based on the Dirichlet and the multinomial distributions. We estimated the parameters of this mixture using the maximum likelihood and Fisher scoring methods. The experiments involved a new texture model and its application to texture image databases summarizing for efficient retrieval. A comprehensive performance evaluation of the model is given using a large number of texture images and a comparison to other model. The MDD mixture is currently used

	No. of retrieved images				
Method	16	48	64	80	96
MDD Mixture	0.23	0.71	0.92	0.94	0.96
Gaussian Mixture	0.21	0.68	0.82	0.92	0.93

Table 5. Recall obtained for the texture database.

in a lot of other applications which involve discrete data or cooccurrence matrices such as textmining, Webmining and E-mail segmentation.

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References

1. M. Tuceryan and A. K. Jain, Texture Analysis, *The Handbook of Pattern Recognition and Computer Vision*, pp. 207-248, 1998.
2. R. M. Haralick, K. Shanmugan, and I. Dinstein Textural features for image classification, *IEEE Transactions on Systems, Man and Cybernetics*, vol. 8, pp. 610-621, 1973.
3. M. Unser, Sum and Difference Histograms for Texture Classification, *IEEE Transactions on Pattern Recognition and Machine Intelligence*, vol. 8, No. 1, pp. 118-125, 1986.
4. N. Bouguila, D. Ziou and J. Vaillancourt, Novel Mixtures Based on the Dirichlet Distribution: Application to Data and Image Classification , *IAPR International Conference on Machine Learning and Data Mining, lNCS2734*, pp. 172-181, 2003.
5. R. A. Redner and H. F. Walker, Mixture Densities, Maximum Likelihood and EM Algorithm , *SIAM Review*, vol. 26, No. 2, pp. 195-239, 1984.
6. A. P. Dempster, N. M. Laird and D. B. Rubin, Maximum Likelihood from Incomplete Data via the EM Algorithm , *Journal of the Royal Statistical Society, B*, vol. 39, pp. 1-38, 1977.
7. G. Schwarz, Estimating the Dimension of a Model , *The Annals of Statistics*, vol. 6, No. 2, pp. 461-464, 1978.
8. H. Akaike, A New Look at the Statistical Model Identification , *IEEE Transactions on Automatic Control*, vol. AC-19, No. 6, pp. 716-723, 1974.
9. J. Rissanen, Modeling by Shortest Data Description , *Biometrika*, vol. 14, pp. 465-471, 1978.
10. S. Medasani and R. Krishnapuram, Categorization of Image Databases for Efficient Retrieval Using Robust Mixture Decomposition, *Computer Vision and Image Understanding*, vol. 83, pp. 216-235, 2001.
11. C. R. Rao, *Advanced Statistical Methods in Biomedical Research*, New York: John Wiley and Sons, 1952
12. N. Bouguila, D. Ziou and J. Vaillancourt, Probabilistic Multimedia Summarizing Based on the Generalized Dirichlet Mixture, *Supplementary Proceedings of ICANN/ICONIP 2003*, pp 150-154, 2003.
13. J. C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorithms*, Plenum Press, New York, 1981.
14. A. E. Raftery and J. D. Banfield, Model-Based Gaussian and Non-Gaussian Clustering *Biometrics*, vol. 49, pp. 803-821, 1993.