A MULTI-MODEL APPROACH FOR BILINEAR GENERALIZED PREDICTIVE CONTROL

Anderson Luiz de Oliveira Cavalcanti
Informatic and Industry Academic Department, CEFET, Natal/RN, Brazil

André Laurindo Maitelli
Department of Computation and Automation, UFRN, Campus Universitário S/N, Natal/RN, Brazil

Adhemar de Barros Fontes
Department of Electrical Engineering,UFBA, Rua Aristides Novis, 04, Salvador/BA, Brazil

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Abstract: This paper presents a contribution in multivariable predictive control. A new approach of multi-model based control is presented. The controller used is the quasilinear multivariable generalized predictive control (QMGPC). A metric based in 2-norm is presented in order to build a global model using local models. Simulation results in a distillation column, with a comparative analysis, are presented.

1 INTRODUCTION

The multi-model approach has been presented as an alternative method to be applied is systems that operate in a long range (Aslan *et al.*, 2004). When a process operates in a long range, due to non-linearities, usually the parametric variation of its models is large. For this reason, usually, a controller based in just one model has poor performance in these kind of process.

The basic idea of multi-model approach is to identify a set of models (one for each operating regime in a chosen trajectory) and to interpolate these models (through an interpolation function). Other approach calculates a suitable control effort as a wheighting sum of each control effort (in each designed controller for each operating regime).

Some approches use space state models like (Azimadeh *et al.*, 1998) and (Foss *et al.*, 1995). In (Azimadeh *et al.*, 1998) a set of linear space state models is chosen in a given trajectory. In (Foss *et al.*, 1995) a set on nonlinear space state models is chosen (and a nonlinear predictive controller is designed).

A closed loop metric, that guarantee the global stability, is proposed in (Aslan *et al.*, 2004). In that case, a set of PI controllers is projected and, for each instant, the distance from the current point in a chosen trajectory to a tabled operating regime is calculated.

In this paper, a similar idea to (Foss *et al.*, 1995) is proposed. In this case, a set of local bilinear models is identified. The global model is build with a wheigthing sum of the identified local models. The wheigthing factor is calculated based in a proposed metric. This metric consists of use a 2-norm to measure the distance from the current point (in a chosen monotonic trajectory) and a tabled operating regime. A case study in a debutanized distillation column is presented in order to show an application of the proposed controller.

The next step of this research is the stability and robustness analisys (to presents a stable algorithm proposal).

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2 MULTIVARIABLE MULTI-MODEL

The designed controller is based in quasilinear multivariable generalized predictive control (QMGPC). This controller is based in multivariable bilinear NARIMAX (Non Linear, Auto-Regressive, Moving Average, with exogenous input) models.

The basic idea of QMGPC algorithm is calculate a control effort sequence, based in the minimization of a multi-step objective function, in a defined prediction horizon.

2.1 Multivariable Multi-Model

The multivariable multi-model bilinear NARIMAX model with p-inputs and q-outputs is given by:

$$\begin{aligned} &A^{(k)}(q^{-1})\Delta_q(q^{-1})y(k) = B^{(k)}(q^{-1})\Delta_p(q^{-1})u(k-1) + \\ &D_e^{(k)}(q^{-1})D[u(k-1)]D_d^{(k)}(q^{-1})\Delta_q(q^{-1})y(k-1) + \\ &C^{(k)}(q^{-1})e(k) \end{aligned} \tag{1}$$

where $y(k) \in R^q$ is the process output vector, $u(k) \in R^p$ is the process input vector and $e(k) \in R^q$ is the gaussian white noise with zero mean and covariance $diag(\sigma^2)$. The matrices $A^{(k)}(q^{-1})$, $B^{(k)}(q^{-1})$ and $C^{(k)}(q^{-1})$ are polynomials matrices in shift operator q^{-1} and are defined by:

$$A^{(k)}(q^{-1}) = I_{q\times q} + A_1^{(k)}q^{-1} + \cdots + A_{na}^{(k)}q^{-na} \tag{2}$$

$$B^{(k)}(q^{-1}) = B_0^{(k)} + B_1^{(k)}q^{-1} + \cdots + B_{nb}^{(k)}q^{-nb} \tag{3}$$

$$C^{(k)}(q^{-1}) = I_{p\times p} + C_1^{(k)}q^{-1} + \cdots + C_{nc}^{(k)}q^{-nc} \tag{4}$$

$$D_d^{(k)}(q^{-1}) = D_{d,0}^{(k)} + D_{d,1}^{(k)}q^{-1} + \cdots + D_{d,nd_d}^{(k)}q^{-nd_d} \tag{5}$$

$$D_e^{(k)}(q^{-1}) = D_{e,0}^{(k)} + D_{e,1}^{(k)}q^{-1} + \cdots + D_{e,nd_e}^{(k)}q^{-nd_e} \tag{6}$$

where:
$A^{(k)}(q^{-1}) \in R^{q\times q}$, $B^{(k)}(q^{-1}) \in R^{q\times p}$, $C^{(k)}(q^{-1}) \in R^{q\times q}$, $D_e^{(k)}(q^{-1}) \in R^{q\times p}$ and $D_d^{(k)}(q^{-1}) \in R^{p\times q}$. The matrix $D[u(k-1)]$ is defined as:

$$D[u(k-1)] = \begin{bmatrix} u_1(k-1) & 0 & \cdots & 0 \\ 0 & u_2(k-1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & u_p(k-1) \end{bmatrix} \tag{7}$$

The generic polynomial matrix $P^{(k)}(q^{-1})$ in (1) represents this matrix in instant *k*.

The first step to build (1) is decompose the system's operating range into a number of operating regimes that completely cover the chosen trajectory. Second, for each operating regime, a local model structure must be developed as showed in (Foss *et al.*, 1995). In this case, the model structure is chosen by using the Akaike criterion.

The last step is to identify the parameter's model for each local model. The estimation algorithm is the Multivariable Recursive Least Squares (MRLS).

2.2 Building the Global Model

The global model is built as a weighting sum of the bilinear models in each chosen operating regime. The generic polynomial matrix $P^{(k)}(q^{-1})$ is built as:

$$P^{(k)}(q^{-1}) = \sum_{i=1}^{NOR} P_{(i)}(q^{-1})w_{i,k} \tag{8}$$

where $P_{(i)}(q^{-1})$ is the polynomial matrix in i^{th} operating regime, $w_{i,k}$ is the i^{th} weighting factor calculated in instant *k*, NOR is the number of operating regimes. The computation of $w_{i,k}$ is showed in the section 3 of this paper.

2.3 The Quasilinear Multivariable Multi-Model

The nonlinear model presented in (1) is quasilinearized to be used in QMGPC (Quasilinear Multivariable Generalized Predictive Control). The multivariable quasilinear multi-model must be obtained by rewriting the expression (1) of the following form:

$$\begin{aligned} &A^{(k)}(q^{-1},u)\Delta_q(q^{-1})y(k) = \\ &B(q^{-1})^{(k)}\Delta_p(q^{-1})u(k-1) + C^{(k)}(q^{-1})e(k) \end{aligned} \tag{9}$$

where:

$$\begin{aligned} &A^{(k)}(q^{-1},u) = A^{(k)}(q^{-1}) - \\ &D_e^{(k)}(q^{-1})D[u(k-1)]D_d^{(k)}(q^{-1}) \end{aligned} \tag{10}$$

The polynomial matrix $A^{(k)}(q^{-1},u)$ is calculated considering its parameters as constant in prediction horizon. The polynomial matrix $A^{(k)}(q^{-1})$ is considered diagonal in this paper.

2.4 The Predictor

The output prediction *i-step* ahead may be calculated multiplying the expression (1) for q^i as in the following expression:

$$\begin{aligned}&\widetilde{A}^{(k)}(q^{-1},u)y(k+i)=\\&B^{(k)}(q^{-1})\Delta_p(q^{-1})u(k+i-1)+C^{(k)}(q^{-1})e(k+i)\end{aligned} \tag{11}$$

where $\widetilde{A}^{(k)}(q^{-1},u)=A^{(k)}(q^{-1},u)\Delta_q(q^{-1})$.

In this case, the polynomial matrix $C(q^{-1})=I_{p\times p}$ is uncorrelated (white noise). Considering the following Diophantine equation:

$$I_{p\times p}=E_i^{(k)}(q^{-1},u)\widetilde{A}^{(k)}(q^{-1},u)+q^{-i}F_i^{(k)}(q^{-1},u) \tag{12}$$

where:

$$E_i^{(k)}(q^{-1},u)=E_{i,o}^{(k)}(u)+\cdots+E_{i,i-1}^{(k)}(u)q^{-(i-1)} \tag{13}$$

$$F_i^{(k)}(q^{-1},u)=F_{i,o}^{(k)}(u)+\cdots+F_{i,na}^{(k)}(u)q^{-na} \tag{14}$$

Pre-multiplying (11), with $C(q^{-1})=I_{p\times p}$, for $E_i(q^{-1},u)$ we obtain:

$$\begin{aligned}&E_i^{(k)}(q^{-1},u)\widetilde{A}^{(k)}(q^{-1},u)y(k+i)=\\&E_i^{(k)}(q^{-1},u)B^{(k)}(q^{-1})\Delta_p(q^{-1})u(k+i-1)+\\&E_i^{(k)}(q^{-1},u)e(k+i)\end{aligned} \tag{15}$$

Rewriting (12) of the following form:

$$E_i(q^{-1},u)\widetilde{A}(q^{-1},u)=I_{p\times p}-q^{-i}F_i(q^{-1},u) \tag{16}$$

Substituting (16) in (15) we obtain:

$$\begin{aligned}&y(k+i)=F_i^{(k)}(q^{-1},u)y(k)+E_i^{(k)}(q^{-1},u)e(k+i)\\&E_i^{(k)}(q^{-1},u)B^{(k)}(q^{-1})\Delta_p(q^{-1})u(k+i-1)+\end{aligned} \tag{17}$$

As the degree of $E_i^{(k)}(q^{-1},u)$ is $i-1$, then the sub-optimal prediction of $y(k+i)$ is:

$$\begin{aligned}&\hat{y}(k+i)=F_i^{(k)}(q^{-1},u)y(k)+\\&E_i^{(k)}(q^{-1},u)B^{(k)}(q^{-1})\Delta_p(q^{-1})u(k+i-1)\end{aligned} \tag{18}$$

Make:

$$E_i^{(k)}(q^{-1},u)B^{(k)}(q^{-1})=H_i^{(k)}(q^{-1},u)+q^{-i}H_{ipa}^{(k)}(q^{-1},u) \tag{19}$$

As the degree of $H_i(q^{-1},u)$ is less than $i-1$, the predictor may be written as:

$$\begin{aligned}&\hat{y}(k+i)=F_i^{(k)}(q^{-1},u)y(k)+H_{ipa}^{(k)}(q^{-1},u)\Delta_p(q^{-1})u(k-1)+\\&H_i^{(k)}(q^{-1},u)\Delta_p(q^{-1})u(k+i-1)\end{aligned} \tag{20}$$

The last term of (20) considers the future inputs (forced response) and the two first terms consider only past inputs (free response). Define:

$$\hat{y}(k+i)=H_i^{(k)}(q^{-1},u)\Delta_p(q^{-1})u(k+i-1)+Y_{li}^{(k)} \tag{21}$$

where:

$$Y_{li}^{(k)}=F_i^{(k)}(q^{-1},u)y(k)+H_{ipa}^{(k)}(q^{-1},u)\Delta_p(q^{-1})u(k-1) \tag{22}$$

2.5 The Objetive Function

The objective function is given by:

$$J=\sum_{i=N_1}^{NY}\left\|r(k+i)-\hat{y}(k+i)\right\|^2_{R_{(k)}}+\sum_{i=1}^{NU}\left\|\Delta u(k+i-1)\right\|^2_{Q_{(k)}} \tag{23}$$

where N_1 is minimum prediction horizon, NY is prediction horizon, NU is the control horizon, $R_{(k)}$ and $Q_{(k)}$ are weighting matrices of error signal and control effort in instant k in the chosen trajectory, respectively, $\hat{y}(k+i)$ is the sub-optimum i-step ahead predicted output, $r(k+i)$ is the future reference trajectory.

2.6 The Control Law

The control effort is obtained, without constraints, by the minimization of the objective function. This minimization is obtained by the calculation of its gradient (making it equals zero), of the following form:

$$\frac{\partial J}{\partial u}=0 \tag{24}$$

Consider the predictions set:

$$y_{N_{1y}}=H^{(k)}{}_{N_{1yu}}u_{NU}+Y_{lN_1y}{}^{(k)} \tag{25}$$

where:

$$y_{N_{1y}} = \left[\hat{y}(k+N_1) \quad \hat{y}(k+N_1+1) \quad \cdots \quad \hat{y}(k+NY)\right]^T \tag{26}$$

$$H_{N_{1yu}}{}^{(k)} = \begin{bmatrix} H^{(k)}{}_{N_1-1} & H^{(k)}{}_{N_1-2} & \cdots & H^{(k)}{}_{N_1-NU} \\ H^{(k)}{}_{N_1} & H^{(k)}{}_{N_1-1} & \cdots & H^{(k)}{}_{N_1+1-NU} \\ \vdots & \vdots & \ddots & \vdots \\ H^{(k)}{}_{NY-1} & H^{(k)}{}_{NY-2} & \cdots & H^{(k)}{}_{NY-NU} \end{bmatrix} \tag{27}$$

$$u_{NU} = \begin{bmatrix} \Delta_p(q^{-1})u(k) \\ \Delta_p(q^{-1})u(k+1) \\ \vdots \\ \Delta_p(q^{-1})u(k+NU-1) \end{bmatrix} \tag{28}$$

$$y_{lN_{1y}} = \begin{bmatrix} Y_{lN_1} \\ Y_{lN_1+1} \\ \vdots \\ Y_{lNY} \end{bmatrix} \tag{29}$$

The objective function (23) may be rewritten of the following form:

$$J=(H_{N_{1yu}}u_{NU}+y_{lN_{1y}})^T\overline{R}_{(k)}(H_{N_{1yu}}u_{NU}+y_{lN_{1y}})+u_{NU}^T\overline{Q}_{(k)}u_{NU} \tag{30}$$

where:

$$\overline{R}^{(k)} = diag[R_1{}^{(k)},\cdots,R_{q\times NY}{}^{(k)}] \tag{31}$$

$$\overline{Q}^{(k)} = diag[Q_1{}^{(k)},\cdots,Q^{(k)}{}_{p\times NU}] \tag{32}$$

The computation of an element $x_i^{(k)}$ of $\overline{R}^{(k)}$ and $\overline{Q}^{(k)}$ is given by:

$$x_i^{(k)} = \sum_{j=1}^{NOR} x_{i,j} w_{j,k} \tag{33}$$

where $x_{i,j}$ is the i^{th} element of weigthing matrix ($\overline{R}^{(k)}$ or $\overline{Q}^{(k)}$) for the j^{th} operating regime and $w_{i,k}$ is the i^{th} weighting factor calculated in instant k.

The minimization of (30) produces the following control law:

$$u = (H_{N_{1yu}}^T H_{N_{1yu}} + \overline{Q})^{-1} H_{N_{1yu}}^T \overline{R}(r - y_{lN_{1y}}) \tag{34}$$

Because of the receding control horizon, only the first p rows of (34) are computed.

3 THE PROPOSED METRIC

The proposed metric is based in a 2-norm. Norms, in general, gives a notion of distance in a vectorial space. In multivariable case, in a process with p-inputs and q-outputs, the output is $y(k) \in R^q$ and the input is $u(k) \in R^p$. In a known trajectory of process output, the distance of the process's output from the first operating regime to the last operating regime is given by:

$$d_{1,NOR} = \left\| y_{NOR} - y_1 \right\|_2 \tag{35}$$

where y_{NOR} is the process's output in last operating regime and y_1 is the process's output in the first operating regime.

To measure the distance from the current process's output (in instant k) to the i^{th} operating regime, we can use the expression:

$$\delta_{i,k} = \frac{d_{1,NOR}}{\left\| y(k) - y_i \right\|_2}; \quad i = 1,\cdots,NOR \tag{36}$$

where $y(k)$ is the process's output in instant k and y_i is the process's output to the i^{th} operating regime.

The weighting factor for the i^{th} operating regime in instant k is given by:

$$w_{i,k} = \frac{\delta_{i,k}}{\sum_{j=1}^{NOR} \delta_{j,k}}; \quad i = 1,\cdots,NOR \tag{37}$$

4 APPLICATION OF THE MULTI-MODEL APPROACH

4.1 Distillation Column

In this paper, an application in a debutanizer distillation column is showed. Debutaziner distillation column is usually used to remove the light components from the gasoline stream to produce Liquefied Petroleum Gas (LPG). The column is showed in Figure 1.

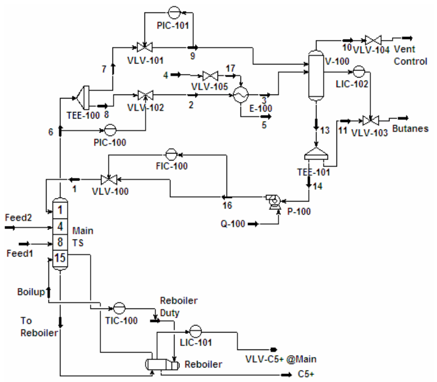

Figure 1: Distillation Column simulated in Hysys Software.

The most common control strategy is to manipulate the reflux flow rate and the temperature in column's bottom and, to control the concentrations of any product in *butanes* stream and in C5+ stream as showed in (Almeida, *et al.*, 2000) and (Fontes, et al., 2006). The chosen process variables are: concentration of i-pentane in butanes stream (y_1) and concentration of i-butene in C5+ stream (y_2).

The reflux flow rate (u_1) is manipulated through the FIC-100 controller and the temperature of column's bottom (u_2) is manipulated through the TIC-100 controller. The reflux flow rate is given in m^3/h and the temperature of column's bottom is given in ^oC.

In this case study, three operation regime were chosen, as showed in Table 1. The identified bilinear models were obtained using the multivariable recursive least squares algorithm and the model's structure has been chosen by using the Akaike criterion. In all models, the chosen sample rate is 4 minutes.

The trajectory of y_1 is monotonically increasing and the trajectory of y_2 is monotonically decreasing.

Table 1: Chosen Operating Regimes.

Operation Regime	Input	Output (Mass Fractions)
1	$u_1 = 40\ m^3/h$	$y_1 = 0.014413$
	$u_2 = 147\ ^oC$	$y_2 = 0.001339$
2	$u_1 = 37\ m^3/h$	$y_1 = 0.017581$
	$u_2 = 147.5\ ^oC$	$y_2 = 0.001161$
3	$u_1 = 34\ m^3/h$	$y_1 = 0.021994$
	$u_2 = 148\ ^oC$	$y_2 = 0.001004$

The operating regimes must be chosen based in a knowledge of the process.

4.2 Results

In this simulation, the process is in the 3rd operating regime and a deviation in reference is applied in the proposed controller. With this reference deviation, the process will come to close to the 1st operating regime. The proposed quasilinear multi-model is compared with quasilinear single-model (using the model of the 3rd operating regime). Figures 2 and 3 show the output comparison.

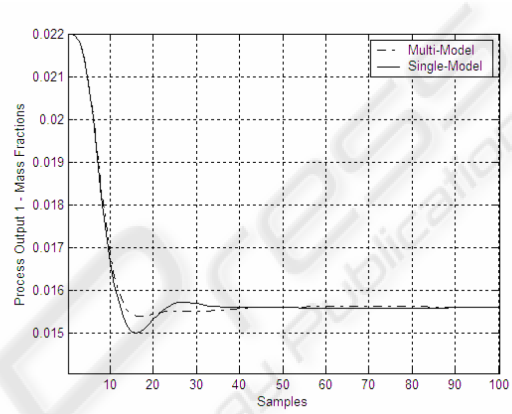

Figure 2: Process Output 1. Comparison between single-model and multi-model approach.

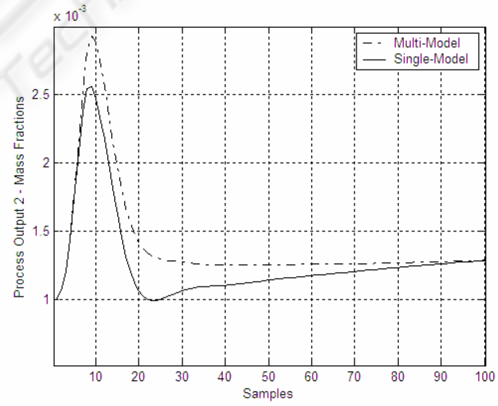

Figure 3: Process Output 2. Comparison between single-model and multi-model approach.

Figures 4 and 5 show the control effort comparison between the quasilinear single-model and multi-model approaches.

The figures show the better performance of multi-model approach when compared with single-model approach.

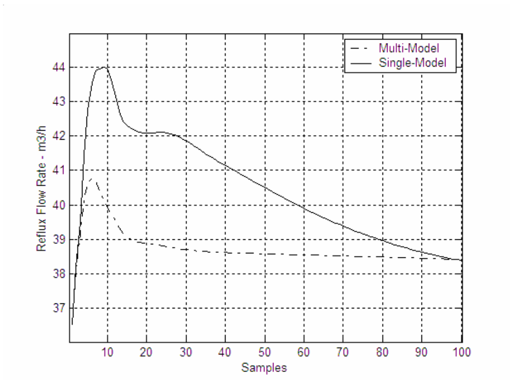

Figure 4: Reflux Flow rate. Comparison between single-model and multi-model approach.

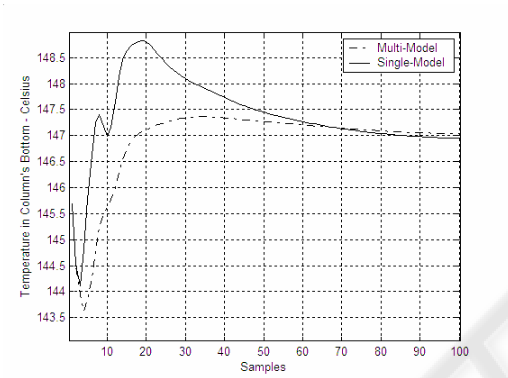

Figure 5: Temperature in column's bottom. Comparison between single-model and multi-model approach.

In order to quantitatively asses the performance of multi-model quasilinear GPC, some indices like showed in (Goodhart, *et al.*, 1994) are calculated. Theses indices may be extended to multivariable case, of the following form:

$$\varepsilon_{1,i} = \sum \left|u_i(k)\right| / N \qquad (38)$$

where $i = 1,\cdots,p$ and N is the amount of control effort applied in the process to achieve the desired response. The index showed in (38) is the account of total control effort to achieve a given response. The variance of controlled actuators is:

$$\varepsilon_{2,i} = \sum (u_i(k) - \varepsilon_{1,i})^2 / N \qquad (39)$$

The deviation of the process of integral of absolute error (IAE) is:

$$\varepsilon_{3,j} = \sum \left|r_j(k) - y_j\right| / N \qquad (40)$$

where $j = 1,\cdots,q$.

The overall measure of effectiveness is defined as:

$$\varepsilon_j = \sum_{i=1}^{p} (\alpha_i \varepsilon_{1,i} + \beta_i \varepsilon_{2,i}) + \rho_j \varepsilon_{3,j} \qquad (41)$$

where $j = 1,\cdots,q$. The factors α_i , β_i and ρ_j are weightings chosen to reflect the actual financial cost of energy usage, actuator wear and product quality, respectively. In this case, we consider $\alpha_i = 0.1$, $\beta_i = 0.15$ and $\rho_j = 0.5$.

Table 2: Comparison of Performance indices between Quasilinear single-model and Quasilinear multi-model with *N*=100.

I/O	Model	ε_1	ε_2	ε_3	ε
1	Single	40.47	2.61	499.46	269.00
	Multi	38.72	0.31	486.20	261.80
2	Single	147.38	0.63	242.40	140.47
	Multi	146.88	0.62	197.71	117.56

Table 2 shows the performance of quasilinear multi-model approach in terms of less energy usage, less actuator wear and better product quality in relation to quasilinear single-model performance.

5 CONCLUSIONS

The multi-model approach is a good alternantive of controller to systems that operate in a large operation range. The indices has shown that this approach presents better performance in relation of quasilinear single model.

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