SATURATION FAULT-TOLERANT CONTROL FOR LINEAR PARAMETER VARYING SYSTEMS

Ali Abdullah
Kuwait University, Electrical Engineering Department, P. O. Box 5969, Safat-13060, Kuwait

Keywords: Fault diagnosis, fault-tolerant systems, parameter estimation.

Abstract: This paper presents a methodology for designing a fault-tolerant control (FTC) system for linear parameter varying (LPV) systems subject to actuator saturation fault. The FTC system is designed using linear matrix inequality (LMI) and model estimation techniques. The FTC system consists of a nominal control, fault diagnostic, and fault accommodation schemes. These schemes are designed to achieve stability and tracking requirements, estimate a fault, and reduce the fault effect on the system. Simulation studies are used to illustrate the proposed design.

1 INTRODUCTION

In recent years, the field of designing FTC systems has received considerable attention (Blanke et al., 2001; Bodson, 1995; Isermann et al., 2002; Patton, 1997; Rauch, 1994; Stengel, 1991). For the case of actuator fault, most of this research had addressed fault accommodation for system subject to parameter variation or frozen output. Other types of actuator fault have been rarely considered. In this paper, a methodology for designing FTC system for LPV systems subject to a reduction in the actuator saturation limit is presented. The LPV systems are defined as a class of linear time-varying systems whose state space matrices depend on a set of parameters that are bounded and can be measured or estimated online.
In the case of using an analytical approach, the main idea behind fault tolerance is the use of fault diagnostic and accommodation schemes. A fault diagnostic scheme driven by plant measurements is used to detect, locate, and estimate faults; while a fault accommodation scheme driven by fault information from the diagnostic scheme is used to modify the nominal control law in order to reduce the fault effect on the system. Based on the above idea, the total task of the proposed FTC system is divided into three parts:

- Plant control: attempts to stabilize the closed-loop system and provide the desired tracking properties in the absence of faults. The controller is designed using LPV technique (Apkarian et al., 1995; Apkarian and Adams, 1998; Gahinet et al., 1996; Kose et al., 1998; Tuan and Apkarian, 2002).
- Fault diagnosis: deals with the problem of saturation fault detection, location, and level estimation. To achieve that, a suitable LPV model is derived to describe the faulty system. Then the results in (Polycarpou and Helmicki, 1995) are used to construct the diagnostic scheme.
- Fault accommodation: attempts to reduce the fault effect on the system by modifying the nominal control law through the reference reshaping filter and feed-forward gain. The accommodation scheme is designed with the help of the bounded real lemma for LPV system presented in (Gahinet et al., 1996).

The notation $\mathcal{H}(\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F})$ is used throughout the paper to denote the symmetric matrix $\begin{pmatrix} \mathcal{A} & \mathcal{B} & \mathcal{C} \\ \mathcal{B}^T & \mathcal{D} & \mathcal{E} \\ \mathcal{C}^T & \mathcal{E}^T & \mathcal{F} \end{pmatrix}$.

Abdullah A. (2007).
SATURATION FAULT-TOLERANT CONTROL FOR LINEAR PARAMETER VARYING SYSTEMS.
In *Proceedings of the Fourth International Conference on Informatics in Control, Automation and Robotics*, pages 19-24

2 PROBLEM STATEMENT

Consider a class of LPV systems of the form:

$$\dot{x}_s(t) = A_s(\rho(t))x_s(t) + B_s u(t) \quad (1)$$

$$y(t) = x_s(t) \quad (2)$$

where $x_s(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control signal, and $y(t) \in \Re^n$ is the measured output signal. $A_s(\rho(t)) = A_{s_o} + \sum_{i=1}^{N} \rho_i(t) A_{s_i}$, and A_{s_j} and B_s are known constant matrices with appropriate dimensions. Furthermore, $\rho(t) = (\rho_1(t), \ldots, \rho_N(t))^T$ is the vector of real time varying parameters ranging inside the hyper-rectangle region defined by $\rho_i(t) \in [\underline{\rho}_i, \overline{\rho}_i]$. Also, its rate $\dot{\rho}(t) = (\dot{\rho}_1(t), \ldots, \dot{\rho}_N(t))^T$ is ranging inside another hyper-rectangle region defined by $\dot{\rho}_i(t) \in [\underline{\nu}_i, \overline{\nu}_i]$.

The actuator saturation fault considered in this study is given by Definition 1.

Definition 1 (Actuator Saturation Fault) The actuator saturation fault is defined mathematically as:

$$\sigma_j(\delta_j u_j) = \begin{cases} u_j & |u_j| < \delta_j \overline{u}_j \\ \delta_j \overline{u}_j \, sign(u_j) & |u_j| \geq \delta_j \overline{u}_j \end{cases}$$

where u_j is the input to the jth actuator, σ_j is the output of the jth actuator, and $0 \leq \delta_j \leq 1$ is the reduced level of the jth saturation limit $\overline{u}_j$.

Remark 1 The value of δ_j represents the reduced level of the actuator saturation limit where $\delta_j = 0$ means a complete failure, $\delta_j = 1$ means no failure exists, and $0 < \delta_j < 1$ means the saturation limit has been reduced to the value of $\pm\delta_j \overline{u}_j$.

Now the main problem is presented.

Problem 1 Design an FTC system for the LPV system (1)-(2) such that:

- In the absence of actuator saturation fault, the nominal control objectives are achieved.
- In the presence of actuator saturation fault, the control objectives are achieved as close as possible to the nominal one.

3 FAULT-TOLERANT CONTROL SYSTEM

The main schemes of the FTC system are shown in Figure 1. These schemes are controller, fault diagnosis, and fault accommodation. Furthermore, the fault accommodation scheme consists of reconfiguration mechanism, feed-forward gain, and reference reshaping filter. The controller is designed to achieve

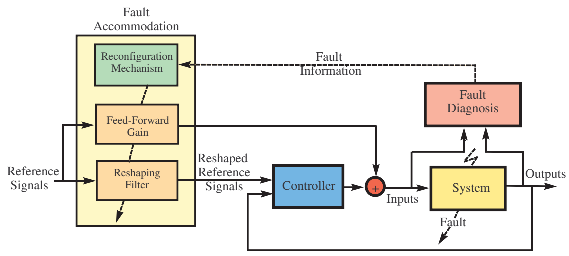

Figure 1: Structure of fault-tolerant control system.

the desired system performances assuming that the system is under normal operation. The fault diagnostic scheme is designed to detect, locate, and estimate a fault. The fault diagnostic scheme is driven by the available system input and output signals. The fault information (no fault, fault, location and magnitude of fault) is supplied to the reconfiguration mechanism to trigger an appropriate reconfiguration of the feed-forward gain and reference reshaping filter. The feed-forward gain and reference reshaping filter are designed to fulfill the new physical constraints imposed by the fault.

3.1 Control Design

The controller is designed based on the concept of affine quadratic stability (Gahinet et al., 1996) defined below.

Definition 2 (Affine Quadratic Stability) The LPV system $\dot{x} = A_c(\rho)x$ is affinely quadratically stable (AQS) if there exists N+1 symmetric matrices P_i such that the following inequalities:

$$P(\rho) = P_o + \rho_1 P_1 + \ldots + \rho_i P_i + \ldots + \rho_N P_N > 0 \quad (3)$$

$$F(\rho, \dot{\rho}) = A_c(\rho)^T P(\rho) + P(\rho) A_c(\rho) + \frac{dP(\rho)}{dt} < 0, \quad (4)$$

where $\frac{dP(\rho)}{dt} = \dot{\rho}_1 P_1 + \ldots + \dot{\rho}_i P_i + \ldots + \dot{\rho}_N P_N$, hold for all admissible trajectories of the parameter vector ρ. In this case, the function $V(x,\rho) = x^T P(\rho) x$ is a quadratic Lyapunov function for the LPV system $\dot{x} = A_c(\rho)x$.

The difficulty associated with the control design using Definition 2 is that the matrix inequality (4) is not linear in terms of $P(\rho)$ and $A_c(\rho)$. However, Lemma 1 (Bara et al., 2001) can be used instead of Definition 2 to simplify a controller design.

Lemma 1 The LPV system $\dot{x}(t) = A_c(\rho)x(t)$ is AQS if there exist a constant matrix W and a symmetric matrix $P(\rho)$ such that the following LMI:

$$\mathcal{H}(-W - W^T, W^T A_c(\rho)^T + P(\rho), W^T,$$

$$-P(\rho)+\frac{dP(\rho)}{dt},0,-P(\rho))<0 \tag{5}$$

holds for all admissible trajectories of the parameter vector ρ.

Since the parameter vector ρ ranges over a polytope, the LMI (5) involves infinite number of constraints. Theorem 1 is used to reduce the infinite number of constraints into a finite one, hence simplifying the controller design.

Theorem 1 If there exist matrices W, R_i, and symmetric matrices P_i such that the following LMIs:

$$\mathcal{H}(-W-W^T,(A_s(Ve_i)W-B_sR_i+P(Ve_i))^T,W^T,$$

$$-P(Ve_i)-P_o+P(\tilde{Ve}_i),0,-P(Ve_i))<0 \tag{6}$$

are feasible for all $(Ve_i,\tilde{Ve}_i)$ where $\rho=\sum_{i=1}^{2^N}\alpha_i(t)Ve_i$, and $\dot{\rho}=\sum_{i=1}^{2^N}\beta_i(t)\tilde{Ve}_i$ with $\sum_{i=1}^{2^N}\alpha_i(t)=1$, $\sum_{i=1}^{2^N}\beta_i(t)=1$, $\alpha_i(t)\geq 0$, and $\beta_i(t)\geq 0$. Then, the control law $u=-(\sum_{i=1}^{2^N}\alpha_i(t)K_i)x(t)$ where $K_i=R_iW^{-1}$ stabilizes the system (1)-(2).

Proof: is omitted.

To implement the control law $u=-(\sum_{i=1}^{2^N}\alpha_i(t)K_i)x(t)$, $\alpha_i(t)$ must be available online. $\alpha_i(t)$ can be computed from the relation $\rho=\sum_{i=1}^{2^N}\alpha_i(t)Ve_i$ using the known Ve_i and $\rho(t)$.

3.2 Fault Diagnosis

To diagnosis a fault, consider the case where the saturation limit of the jth actuator is reduced due to a fault. Then, equation (1) is written as:

$$\dot{x}_s=A_s(\rho)x_s+B_{s_1}u_1\ldots+B_{s_j}\sigma_j(\delta_ju_j)\ldots+B_{s_m}u_m \tag{7}$$

For fault diagnosis, equation (7) is expressed in terms of control outputs by defining:

$$\lambda_j=\begin{cases}\frac{\sigma_j(\delta_ju_j)}{u_j}-1 & u_j\neq 0\\ 0 & u_j=0\end{cases}$$

and then writing (7) as:

$$\dot{x}_s=\begin{cases}A_s(\rho)x_s(t)+B_su+\lambda_jB_{s_j}u_j & u_j\neq 0\\ A_s(\rho)x_s+B_su & u_j=0\end{cases}$$

Theorem 2 is used to estimate the value of λ_j which will be used to detect the fault and to estimate the saturation level δ_j.

Theorem 2 Consider the estimated model:

$$\dot{\hat{x}}_s=A_s(\rho)x_s+B_su+\hat{\lambda}_jB_{s_j}u_j+G(\hat{x}_s-x_s)$$

where $\hat{x}_s\in\mathbb{R}^n$ is the estimated state vector, G is the constant matrix with negative eigenvalues, and $\hat{\lambda}_j\in\mathbb{R}$ is the estimated parameter of λ_j adjusted as:

$$\dot{\hat{\lambda}}_j=\Gamma(B_{s_j}u_j)^Te-\chi\Gamma\frac{\hat{\lambda}_j\hat{\lambda}_j^T}{|\hat{\lambda}_j|^2}\Gamma(B_{s_j}u_j)^Te;\ \hat{\lambda}_j(0)=0 \tag{8}$$

where $e=x-\hat{x}$ is the estimated error, Γ is the positive definite matrix, and χ is the indicator function for the projection algorithm (to prevent parameter drift) defined as:

$$\chi=\begin{cases}0 & (|\hat{\lambda}_j|<M)\ or\ (|\hat{\lambda}_j|=M\ and\ \hat{\lambda}_j^T\Gamma(B_{s_j}u_j)^Te\leq 0)\\ 1 & (|\hat{\lambda}_j|=M\ and\ \hat{\lambda}_j^T\Gamma(B_{s_j}u_j)^Te>0)\end{cases}$$

Then $\hat{\lambda}_j$ is uniformly bounded, and $lim_{t\longrightarrow\infty}e(t)=0$.

Proof: is omitted.

For fault detection, $\hat{\lambda}_j$ is tested for the likelihood of saturation fault. A decision about the existence of saturation fault is made as follows: if $\nu_j\leq\varepsilon_j$, the saturation fault doest not exist; if $\nu_j>\varepsilon_j$, the saturation fault exist. $\nu_j=[\frac{1}{\alpha}\int_t^{t+\alpha}(\hat{\lambda}_j(\tau))^2d\tau]^{1/2}$ is the average energy of $\hat{\lambda}_j$ over the time interval $[t,t+\alpha]$, α is the detection window, and ε_j is the threshold.
For saturation level estimation, when the saturation fault exists (i.e., $\nu_j>\varepsilon_j$), $\hat{\lambda}_j$ has the value: $\hat{\lambda}_j=\frac{\sigma_j(\hat{\delta}_ju_j)}{u_j}-1=\frac{\hat{\delta}_j\bar{u}_j\,sign(u_j)}{u_j}-1=\frac{\hat{\delta}_j\bar{u}_j}{|u_j|}-1$. Then the saturation level δ_j can be estimated as: $\hat{\delta}_j=\frac{|u_j|}{\bar{u}_j}(\hat{\lambda}_j+1)$.

3.3 Fault Accommodation

To accommodate a saturation fault, a reference reshaping filter and feed-forward gain are used to fulfil the new input constraint $|u_j(t)|<\delta_j\bar{u}_j$. To enforce this constraint the jth system input $u_j(t)$ is generated from the jth control output $u_{c_j}(t)$, which is a function of a modified reference signal $\bar{r}$ generated by the reference reshaping filter, and the jth feed-forward signal u_{f_j} in order to get $|u_j(t)=u_{c_j}(t)+u_{f_j}(t)|<\delta_j\bar{u}_j$. Furthermore, the modified reference signal $\bar{r}$ should be designed to fulfil the input constraint while deviation from the reference signal r is minimized. Based on these suggestions, Problem 2 is addressed as follows.

Problem 2 Given the:

- System: $\begin{cases}\dot{x}_s(t)=A_s(\rho)x_s(t)+B_su(t)\\ y(t)=x_s(t)\end{cases}$
- Controller: $\begin{cases}\dot{x}_c(t)=A_c(\rho)x_c(t)+B_c(\rho)\bar{r}(t)+C_c(\rho)x_s(t)\\ u_c(t)=D_c(\rho)x_c(t)+E_c(\rho)\bar{r}(t)+F_c(\rho)x_s(t)\end{cases}$

- Saturation fault level: δ_j

Design a:

- Reference reshaping filter: $\begin{cases} \dot{x}_r(t) = A_r(\delta_j)x_r(t) + B_r(\delta_j)r(t) \\ \bar{r}(t) = C_r(\delta_j)x_r(t) + D_r(\delta_j)r(t) \end{cases}$
- Feed-forward: $u_f(t) = F(\delta_j)r(t)$

Such that the whole system is stable, $|u_j(t) = u_{c_j}(t) + u_{f_j}(t)| < \delta_j \bar{u}_j$, and $\|\bar{r}(t) - r(t)\|_2$ is minimized.

The reference reshaping filter and feed-forward gain are designed based on the concept of affine quadratic H_∞ performance (Gahinet et al., 1996) defined below.

Definition 3 (Affine Quadratic H_∞ Performance) The LPV system:

$$\dot{x}(t) = A(\rho)x(t) + B(\rho)r(t) \quad (9)$$
$$u_j(t) = C_j(\rho)x(t) + D_j(\rho)r(t) \quad (10)$$

has affine quadratic H_∞ performance γ_j if there exist $N+1$ symmetric matrices P_i such that:

$$P(\rho) = P_o + \rho_1 P_1 + \ldots + \rho_i P_i + \ldots + \rho_N P_N > 0 \quad (11)$$
$$\mathcal{H}(A(\rho)P(\rho) + P(\rho)A^T(\rho) + P(\dot{\rho}) - P_o, P(\rho)C_j^T(\rho), B(\rho), -\gamma_j I, D_j(\rho), -\gamma_j I) < 0 \quad (12)$$

holds for all admissible parameter vector ρ. In such a case, the Lyapunov function $V(x,\rho) = x^T P(\rho)x$ establishes that the system (9) is asymptotically stable and its L_2 gain does not exceed γ_j. That is, $|u_j(t)| < \gamma_j \|r(t)\|_2$ for all L_2-bounded $r(t)$ (provided that $x(0) = 0$).

The difficulty of using Definition 3 to design the reference reshaping filter and feed-forward gain resides in the fact that matrix inequality (12) is not linear in terms of $P(\rho)$ and $A(\rho)$. Therefore, the matrix inequality (12) is not convex and thus difficult to solve. To convert the problem into an LMI problem and make it tractable, the following relaxations and selections are proposed:

- To get a LMI, $A_r(\delta_j)$ and $C_r(\delta_j)$ are predesigned and denoted as: $A_r(\delta_j) = A_{r_j}$ and $C_r(\delta_j) = C_{r_j}$, where A_{r_j} has negative eigenvalues
- To satisfy $|u_j(t)| < \delta_j \bar{u}_j$, γ_j should satisfy: $\gamma_j \le \frac{\delta_j \bar{u}_j}{max(\|r(t)\|_2)}$
- Minimizing $\|\bar{r}(t) - r(t)\|_2$ is relaxed and making $\bar{r}(t) = \Lambda_j r(t)$ is considered at the steady state, that is $D_r(\delta_j) = \Lambda_j + C_{r_j} A_{r_j}^{-1} B_r(\delta_j)$, where Λ_j is a constant diagonal matrix with its elements $0 < \mu_{i,j} \le 1$
- The structures of the remaining design matrices are selected as: $B_r(\delta_j) = \delta_j B_{r_j}$ and $F(\delta_j) = \delta_j F_j$, where B_{r_j} and F_j are constant design matrices.

The state space representation of the plant with controller, reference reshaping filter, and feed-forward gain is given by:

$$\dot{x}(t) = A(\rho)x(t) + B(\rho)H_j r(t) \quad (13)$$
$$u_j(t) = C_j(\rho)x(t) + D_j(\rho)H_j r(t) \quad (14)$$

where

$x(t) = (x_s^T(t), x_c^T(t), x_r^T(t))^T$, $H_j = (\Lambda_j^T, B_{r_j}^T, F_j^T)^T$,

$$A(\rho) = \begin{pmatrix} A_s(\rho) + B_s F_c(\rho) & B_s D_c(\rho) & B_s E_c(\rho) C_{r_j} \\ C_c(\rho) & A_c(\rho) & B_c(\rho) C_{r_j} \\ 0 & 0 & A_{r_j} \end{pmatrix},$$

$B(\rho) = (((B_s E_c(\rho))^T, B_c^T(\rho), 0)^T \vdots ((\delta_j B_s E_c(\rho) C_{r_j} A_{r_j}^{-1})^T, (\delta_j B_c(\rho) C_{r_j} A_{r_j}^{-1})^T, \delta_j I)^T \vdots (\delta_j B_s^T, 0, 0)^T)$,

$C_j(\rho)$=jth row of $\{(F_c(\rho), D_c(\rho), E_c(\rho) C_{r_j})\}$, and

$D_j(\rho)$=jth row of $\{(E_c(\rho) \vdots (\delta_j E_c(\rho) C_{r_j} A_{r_j}^{-1}) \vdots \delta_j I)\}$.

Corollary 1 is used to convert Problem 2 into an LMI problem.

Corollary 1 Consider the LPV system (13)-(14) with known δ_j, A_{r_j}, and C_{r_j}. If there exists a solution (P_i, H_j, γ_j) that maximize $\mu_{i,j}$ subject to:

$$P(\rho) = P_o + \rho_1 P_1 + \ldots + \rho_i P_i + \ldots + \rho_N P_N > 0 \quad (15)$$
$$\mathcal{H}(A(\rho)P(\rho) + P(\rho)A^T(\rho) + P(\dot{\rho}) - P_o, P(\rho)C_j^T(\rho), B(\rho)H_j, -\gamma_j I, D_j(\rho)H_j, -\gamma_j I) < 0 \quad (16)$$
$$\gamma_j \le \frac{\delta_j \bar{u}_j}{max(\|r(t)\|_2)} \quad (17)$$
$$0 < \mu_{i,j} \le 1 \quad (18)$$

for all admissible parameter vectors ρ. Then, the system (13) is asymptotically stable, $|u_j(t)| < \delta_j \bar{u}_j$, and $\bar{r}(t) = \Lambda_j r(t)$ as $t \longrightarrow \infty$ (provided that $r(t)$ is a bounded constant reference).

Proof: is omitted.

The LMIs (15)-(16) need to be solved for all admissible parameter vectors ρ which imply infinite number of LMIs. However, the infinite number of LMIs can be reduced to a finite number of LMIs using the following procedures:

- Write the matrices in (13)-(14) as: $A(\rho) = A_o + \sum_{i=1}^N \rho_i A_i$, $B(\rho) = B_o + \sum_{i=1}^N \rho_i B_i$, $C_j(\rho) = C_{j_o} + \sum_{i=1}^N \rho_i C_{j_i}$, and $D_j(\rho) = D_{j_o} + \sum_{i=1}^N \rho_i D_{j_i}$
- Use the matrix expressions in the previous step to write the LMI (16) as:

$$M_o + \sum_{i=1}^{N} \dot{\rho}_i M_{dot} + \sum_{i=1}^{N} \rho_i M_i + \sum_{i,l=1, i \neq l}^{N} \rho_i \rho_l M_{il}$$

$$+\sum_{i=1}^{N}\rho_i^2 M_{ii} < 0 \quad (19)$$

where $M_o = \mathcal{H}(A_oP_o + P_oA_o^T, P_oC_{j_o}^T, B_oH_j, -\gamma_j I, D_{j_o}H_j, -\gamma_j I)$, $M_{dot} = \mathcal{H}(P_i, 0, 0, 0, 0, 0)$, $M_i = \mathcal{H}(A_oP_i + A_iP_o + P_oA_i^T + P_iA_o^T, P_oC_{j_i}^T + P_iC_{j_o}^T, B_iH_j, 0, D_{j_i}H_j, 0)$, $M_{il} = \mathcal{H}(A_iP_l + A_lP_i + P_iA_l^T + P_lA_i^T, P_iC_{j_l}^T + P_lC_{j_i}^T, 0, 0, 0, 0)$, and $M_{ii} = \mathcal{H}(A_iP_i + P_iA_i^T, P_iC_{j_i}^T, 0, 0, 0, 0)$.

- To reduce the number of parameters ($\dot{\rho}_i$, ρ_i, $\rho_i\rho_l$, and ρ_i^2) and hence to reduce the design complexity, define fewer parameters σ_i, $i = 1, 2, \ldots, K$, to bound the parameters $\dot{\rho}_i$, ρ_i, $\rho_i\rho_l$, and ρ_i^2. Then, the LMIs (15) and (19) will be a function of σ_i.
- Solve the LMIs (15), (17), (18), and (19) for P_i, H_j, and γ_j at all vertices of the σ parameter space. In this case, the existing solutions guarantee the feasibility of the LMIs for all admissible parameter vector σ, see for example (Gahinet et al., 1996).

The following procedures are implemented inside the reconfiguration mechanism scheme in order to adapt the right reference reshaping filter and feed-forward gain after estimating the level of saturation fault.

- If there is no saturation fault, i.e. $v_j(k) \leq \varepsilon_j$, set $F_j = 0$ and $\bar{r}(t) = r(t)$, then stop. Otherwise, go to the next step.
- Given the estimated level $\hat{\delta}_j \in S_j$, where $S_j = \{\underline{\kappa}_j \leq \hat{\delta}_j \leq \overline{\kappa}_j : 0 \leq \underline{\kappa}_j, \overline{\kappa}_j \leq 1\}$, adapt the reference reshaping filter and feed-forward gain designed using the above procedures $\forall \hat{\delta}_j \in S_j$, then stop.

4 ILLUSTRATION EXAMPLE

In this section a second-order LPV model is used to illustrate the proposed design. The LPV model is given by:

$$\underbrace{\begin{pmatrix} \dot{x}_{s1} \\ \dot{x}_{s2} \end{pmatrix}}_{\dot{x}_s} = \underbrace{\begin{pmatrix} \rho(t) & 1 \\ 0 & -1 \end{pmatrix}}_{A_s(\rho)} \begin{pmatrix} x_{s1} \\ x_{s2} \end{pmatrix} + \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{B_s} u \quad (20)$$

$$y = x_{s1} \quad (21)$$

where -0.5= $\underline{\rho} \leq \rho(t) \leq \overline{\rho}$ =0.1.

4.1 Control System

To design a controller using the result of Theorem 1, the LPV model (20) is written in the polytopic form $\dot{x}_s = (\alpha_1 A_s(\overline{\rho}) + \alpha_2 A_s(\underline{\rho}))x_s + B_s u$, where $\alpha_1 = (\rho(t) - \underline{\rho})/(\overline{\rho} - \underline{\rho})$, and $\alpha_2 = (-\rho(t) + \overline{\rho})/(\overline{\rho} - \underline{\rho})$. Then the results of Theorem 1 are used to design the controller to stabilize the closed loop system, and to track the desired constant reference signal. Figure 2 shows that the controlled system output reaches the desired set point with "almost" perfect tracking.

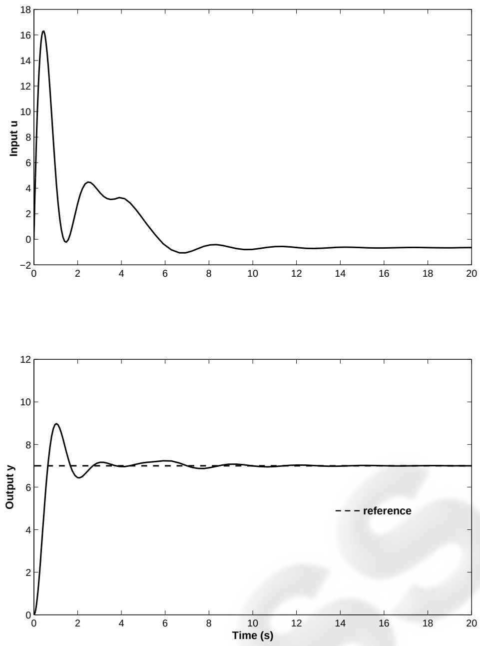

Figure 2: Simulation results of nominal control system.

4.2 Fault Diagnosis

The proposed design of saturation fault diagnosis is simulated for the LPV model (20). Figure 3 shows the results of fault diagnostic system with the saturation level of $\delta = 0.5$. The fault is occurred at 5.00 sec and detected at 6.12 sec using the threshold value of 0.05. The results of Theorem 2 are used to estimate the saturation level. The learning rate Γ is chosen as $\Gamma = 10$, and the filter pole is set to $p = -60$. Figure 3 shows that after the occurrence of the fault, the level of the saturation fault is converged to the true value within about 3 sec. The result indicates that the estimation scheme provides an accurate estimation of the saturation level within a reasonable time.

4.3 Fault Accommodation

Once the saturation level is estimated and sent to the reconfiguration mechanism scheme. An appropriate reconfiguration of the feed-forward gain and refer-

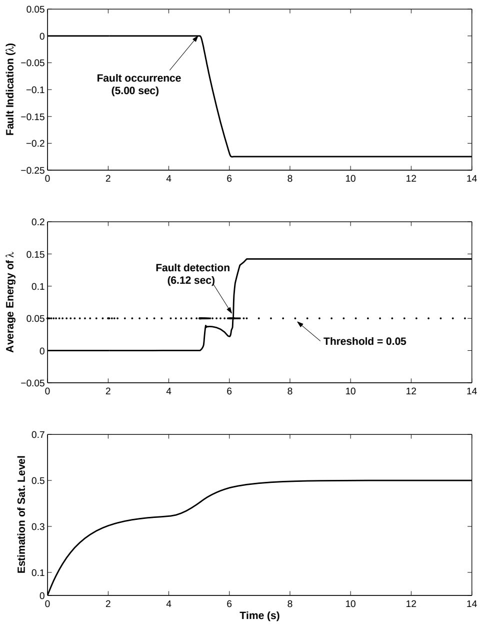

Figure 3: Simulation results of fault diagnostic scheme.

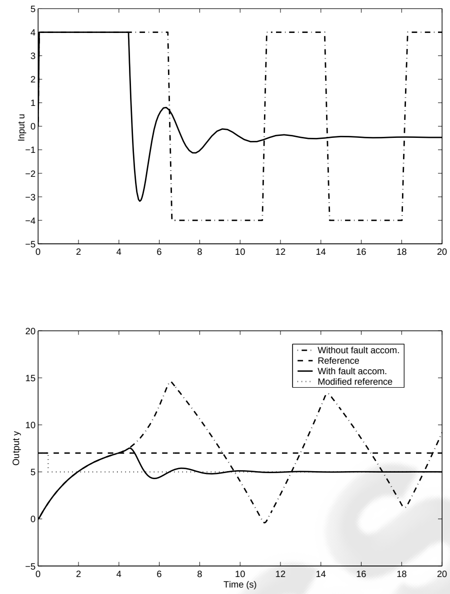

Figure 4: Simulation results with and without fault accommodation.

ence reshaping filter is triggered to accommodate the fault. Figure 4 shows the simulation results of the control system with and without fault accommodation for the case of 0.2 reduction in the saturation limit. Without fault accommodation, the input signal reaches its limit and the output diverges from the desired set point. With fault accommodation, on the other hand, the input signal is reduced within its new limit and the output tracks the desired set point.

5 CONCLUSION

Designing a FTC system for LPV systems subject to actuator saturation fault is considered. The FTC system consists of a nominal control, fault diagnostic, and fault accommodation schemes in order to achieve control objectives in the absence and presence of actuator saturation fault. Simulation results demonstrate the effectiveness of the proposed FTC system.

REFERENCES

Apkarian, P. and Adams, R. J. (1998). Advanced gain-scheduling techniques for uncertain systems. *IEEE Transaction on Control Systems Technology*, 6:21–32.

Apkarian, P., Gahinet, P., and Becker, G. (1995). Self-scheduled h_∞ control of linear parameter-varying systems: a design example. *Automatica*, 31:1251–1261.

Bara, G. I., Daafouz, J., Kratz, F., and Ragot, J. (2001). Parameter-dependent state observer design for affine lpv systems. *International Journal of Control*, 74:1601–1611.

Blanke, M., Staroswiecki, M., and Wu, N. E. (2001). Concepts and methods in fault-tolerant control. In *proc. American Control Conference*.

Bodson, M. (1995). Emerging technologies in control engineering. *IEEE Control Systems Magazine*, 15.

Gahinet, P., Apkarian, P., and Chilali, M. (1996). Affine parameter-dependent lyapunov functions and real parametric uncertainty. *IEEE Transaction on Automatic Control*, 41:436–442.

Isermann, R., Schwarz, R., and Stolzl, S. (2002). Fault-tolerant drive-by-wire systems. *IEEE Control Systems Magazine*, 22.

Kose, I. E., Jabbari, F., and Schmitendorf, W. E. (1998). A direct characterization of l2-gain controllers for lpv systems. *IEEE Transaction on Automatic Control*, 43:1302–1307.

Patton, R. J. (1997). Fault-tolerant control systems: the 1997 situation. In *SAFEROCESS'97, IFAC Symp. Fault Detection, Supervision and Safety for Technical Processes*.

Polycarpou, M. M. and Helmicki, A. J. (1995). Automated fault detection and accommodation: a learning systems approach. *IEEE Transactions on Systems, Man and Cybernetics*, 25.

Rauch, H. E. (1994). Intelligent fault diagnosis and control reconfiguration. *IEEE Control Systems Magazine*, 14.

Stengel, R. F. (1991). Intelligent failure-tolerant control systems. *IEEE Control Systems Magazine*, 11.

Tuan, H. D. and Apkarian, P. (2002). Monotonic relaxations for robust control: new characterizations.