

# STUDY OF A CONTROLLED COMPLEX MECHANICAL SYSTEM IN ANTI VIBRATORY DOMAIN

## *Application to a Hard Landing of an Aircraft*

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**Abstract:** This paper studies problematic of a mechanical system composed of different parts mechanically coupled and submitted to a high speed shock.

After a shock, different parts of the system oscillate. If one of them is excited at a particular frequency, such as its proper frequency, important oscillations appear and can lead to the deterioration of the system by introducing important stresses. In this paper, we propose an analysis in order to understand this kind of problem and what we can do to avoid it. Firstly we discuss problematic and we expose the studied system. In a second time, we present model which allows us to understand the phenomenon by carrying out numerical simulations. Then we complete a comparative analysis of different methods of control. Prospects and problematic of real controlled device are studied. Finally experimental set up is described.

## 1 INTRODUCTION

The topic of this paper takes place in the problematic of the struggle against vibrations. More particularly in the minimisation of induced vibrations by a high speed shock in a complex mechanical system.

Vibrations and their effects are very problematic phenomenoms for all mechanical systems. Although there are a lot of applications, the overall of anti vibratory devices aim the increase of the service life of machines and structures but also the increase of the comfort of passengers in means of transportation.

In fact several complex systems are submitted to external and internal excitations. There are external excitations, like earthquakes or wind for buildings and structures for example and road disturbances (pothole for example) for vehicles. Internal excitations are issued from mechanical pieces in movement or out of balance for mechanical system.

Here we study vibrations induced by external excitation and more especially these ones induced by shock.

Aeronautics is a domain where it is important to study the behaviour of an excited system. In fact progress in the domain of materials leads frames of aircrafts to be lighter. These ones easily bend under an excitation. During taxiing, the fuselage is submitted to excitations which lead to uncomfortable situation for passengers and stressful vibrations for the frame (Kruger, 2000). Moreover aircrafts are particularly constrained during a landing and especially a hard landing which is equivalent to a high speed shock. In fact because of the mechanical coupling existing between the fuselage and the landing gear, the frame of the aircraft bends and important deformations, resulting of a particular excitation of the frame, can lead to the deterioration of the aircraft. Reinforcement of the fuselage can be made. But this passive solution

makes the aircraft heavier.

So in order to insure comfort of passengers and to evict vibrations in fuselage, Ghiringhelli proposes to control the landing gears (Ghiringhelli, 2000). In this study, he only takes into account the cabin of an aircraft. Here we take into account the tail beam, a particular critic component that can easily bends under a high speed shock and whose oscillations lead to important stress in the area of the joint between the cabin and the tail beam. This phenomenon is particularly enhanced on helicopter.

Thus in order to analyse problematic and to understand the phenomenon, we study the behaviour of a mechanical system composed of different parts mechanically coupled and submitted to a high speed shock.

In order to reproduce a high speed shock, we study the free fall and the impact on the ground of the system. The behaviour of the upper part of the system is particularly studied because it represents for example the tail beam of an aircraft and so we want to understand and to avoid its oscillations.

Thus we firstly present modelization of studied system in order to carry out numerical simulations. Then we complete an analysis of different methods of control. A prospect of real device is introduced. Finally experimental set-up is exposed.

## 2 MODELING

### 2.1 Description

In a first time, in order to simplify the study only the main movement of bounce is taken into account. The studied system is composed of a system which is equivalent to a quarter part of a vehicle with another sprung mass located on the upper mass of the quarter part of a vehicle.

The quarter part of a vehicle is composed of a wheel, an unsprung mass (mns) and a sprung mass (ms) linked by a suspension (cf. Figure 1 and Figure 5). The subsystem located on the sprung mass of the quarter part of the vehicle, is composed of a mass (mq), a spring and a damper. Its damping rate is about 3%, which corresponds to a structural damping.

We have a free fall of the system; so the speed of the shock is proportional to the height of the fall. Here we study a shock with a speed of 3 m/s. The height of the fall is 0.4 m. In all following simulations, initial conditions on positions of different masses making up the system, allow us to adjust the speed of the shock.

Two approaches have been studied. An analytical approach and a multi body approach have been presented in a previous paper. Multi body approach corresponds to a non linear model based on experimental characterizations of some different constitutive parts of the system such as the tire and the hydraulic shock absorber. After study, the non linear model can be linearized. In this paper, we only present and study the linear analytical model. Moreover this one has been cross checked with experimental tests made on the drop test bench, described in the following of this paper.

The studied system is also described by the following figure:

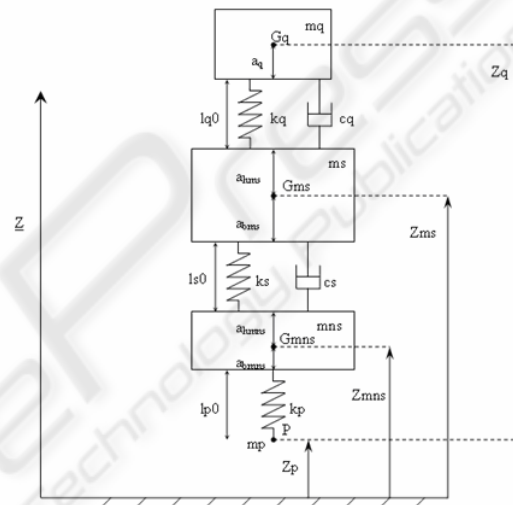


Figure 1: Model and definition of parameters.

We consider four degrees of freedom (d.o.f), which are:

- Zq, absolute displacement of the centre of mass mq.
- Zms, absolute displacement of the centre of mass ms.
- Zmns, absolute displacement of the centre of mass mns.
- Zp, absolute displacement of the point P.

REMARK.— setting conditions on the absolute displacement of the point P, which corresponds to the bottom point of the tire, allow us to differentiate the phase of fall and the phase of contact with the ground during simulations.

In fact we have following conditions:

- Zp > 0, phase of fall.
- Zp ≤ 0, phase of evolution of the system on ground.

Notations:

- mq, mass of the upper system.
- Gq, centre of mass mq.

- ms, sprung mass.
- Gms, centre of mass ms.
- mns, unsprung mass.
- Gmns, centre of mass mns.
- kq, stiffness of the upper system.
- lq0, length of the unloaded spring kq.
- cq, damping coefficient of the upper system.
- cs, damping coefficient of the suspension.
- ks, stiffness of the suspension.
- ls0, length of the unloaded spring ks.
- kp, stiffness of the tire.
- lp0, length of the unloaded tire.
- P, point of contact of the tire.
- ai, distance between a centre of mass and the point of application of a spring. The index i corresponds to the different notations used in Figure 1.

The behaviour of the system is described by the following equations:

$$m_q \cdot \ddot{Z}_q = -m_q \cdot g - k_q \cdot (Z_q - Z_{ms}) - c_q \cdot (\dot{Z}_q - \dot{Z}_{ms}) - k_q \cdot (-a_q - a_{hms} - l_{q0}) \quad (1)$$

$$m_s \cdot \ddot{Z}_{ms} = -m_s \cdot g + k_q \cdot (Z_q - Z_{ms}) + c_q \cdot (\dot{Z}_q - \dot{Z}_{ms}) + k_q \cdot (-a_q - a_{hms} - l_{q0}) - k_s \cdot (Z_{ms} - Z_{mns}) - k_s \cdot (-a_{bms} - a_{hmns} - l_{s0}) - c_s \cdot (\dot{Z}_{ms} - \dot{Z}_{mns}) \quad (2)$$

$$m_{ns} \cdot \ddot{Z}_{mns} = -m_{ns} \cdot g + k_s \cdot (-a_{bms} - a_{hmns} - l_{s0}) + k_s \cdot (Z_{ms} - Z_{mns}) + c_s \cdot (\dot{Z}_{ms} - \dot{Z}_{mns}) - k_p \cdot (Z_{mns} - Z_p - a_{bmns} - l_{p0}) \quad (3)$$

$$m_p \cdot \ddot{Z}_p = -m_p \cdot g + k_p \cdot (Z_{mns} - Z_p) + k_p \cdot (-a_{bmns} - l_{p0}) \quad (4)$$

The mass  $m_p$  is set to zero. When  $Z_p > 0$ , the system is falling, the tire represented by the spring with stiffness  $k_p$  doesn't apply any force on the mass  $m_{ns}$ .

## 2.2 Simulations and Analysis

We study vibrations induced by a high speed shock. In this study, free fall of system is considered. Thus the speed of the shock is determined by the height of the fall ie initial positions of different masses. Here we analyse a shock with a speed of 3 m/s (a 0.4 m high fall). Moreover we set the following condition; no bounce of the system can occur. This is a condition of stability for an aircraft during landing or a condition of safety for a car riding on a chaotic road.

The upper system is composed of the mass  $m_q$ , the spring  $k_q$  and the damper  $c_q$ . It has a low proper frequency about 7 Hz.

The damping coefficient ( $c_s$ ) of the suspension is different between the phase of compression and the phase of extension. This difference makes the suspension softer and guaranties no bounce.

After several simulations, we chose a damping rate of 60% for compression and 90% for extension. The damping rate, noted  $\lambda$ , is calculated as following:

$$\lambda = \frac{c_s}{2 \cdot \sqrt{k_s \cdot m_s}} \quad (5)$$

The stiffness of the spring  $k_p$  modelling the tire is set to 250000 N/m. This is an average value of the used tire on the test bench.

We study the excitation force transmitted to the sprung mass ( $m_s$ ). We obtain the following result of simulation:

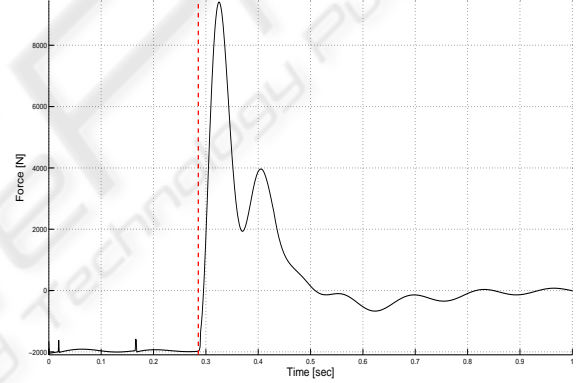


Figure 2: Excitation force on ms.

The impact occurs at the time 0.28 sec (outlined on the graph by the red vertical dashed lined). As soon as the impact occurs, we notice the presence of a double bump. The first peak depends on characteristics of the suspension (stiffness, damping rate). The second peak depends on the stiffness of the tire. The stiffer these elements are, the higher the peaks are. The duration of the double bump is equal to 0.13 sec.

Because the coupling between the mass  $m_s$  and the mass  $m_q$ , the double bump excites the upper system in a frequency band near its proper frequency; leading to important displacements of the mass  $m_q$ .

We can conclude that the duration and the particular shape of the excitation transmitted by the suspension and resulting of the high speed shock are responsible for important displacements of the mass  $m_q$ .

Thus in order to prevent important oscillations of the upper system, we have to control the transmitted excitation. This one is transmitted by the suspension. To control the dynamic behaviour of the suspension allows us to minimize oscillations of the mass  $m_q$  and also to minimize the force on the upper system.

In the following, different methods of control of the dynamic behaviour are designed and a comparative analysis is presented. Then problematic and prospects of real device are exposed.

### 3 CONTROLLED SYSTEM

#### 3.1 Problematic

The previous work shows that the particular excitation transmitted by the suspension to the mass  $m_s$ , leads to important oscillations of the mass  $m_q$ .

Several studies propose different controlled suspensions in order to minimize the acceleration of the mass  $m_s$  (Giua et al., 2004; Guglielmino and Edge, 2004; Kim et al., 2003). The aim of all these studies is to minimize the acceleration of the mass  $m_s$  in order to insure the comfort of passengers (Yagiz, 2004). Our aim is to minimize acceleration of the mass  $m_q$ . In fact, according to the coupling between the sprung mass ( $m_s$ ) and the upper mass ( $m_q$ ), we will control the transmitted force on  $m_s$  in order to minimize acceleration of the upper mass ( $m_q$ ).

In fact we can't add a control force on the upper system; that would mean a collocated actuator on the tail beam on a real aircraft. This is more difficult and less practicable than control the landing gear.

#### 3.2 Comparative Analysis of Different Methods of Control

Here we compare different methods of control. First we study two classical methods of PID with feedback on  $m_s$  measure of acceleration and then on  $m_q$  measure of acceleration in order to respectively minimize acceleration on  $m_s$  and on  $m_q$ .

Then we design sliding mode controller with state feedback on  $m_s$  using the existing coupling between the sprung mass ( $m_s$ ) and the upper mass ( $m_q$ ) in order to minimize the acceleration of  $m_q$ .

We want to control the excitation force transmitted by the suspension to the sprung mass ( $m_s$ ). We introduce a control force, noted  $u$ , in the equations defining the system. This force is added on the sprung mass in parallel with passive force of

damping and stiffness. According to equations (2) and (3) previously exposed we obtain:

$$\begin{aligned} m_s \cdot \ddot{Z}_m s &= -m_s \cdot g + k_q \cdot (Z_q - Z_m s) + c_q \cdot (\dot{Z}_q - \dot{Z}_m s) \\ &+ k_q \cdot (-a_q - a_{hms} - l_q 0) - k_s \cdot (Z_m s - Z_m n s) \\ &- k_s \cdot (-a_{bms} - a_{hmns} - l_s 0) - c_s \cdot (\dot{Z}_m s - \dot{Z}_m n s) \\ &+ u \end{aligned} \quad (6)$$

$$\begin{aligned} m_n s \cdot \ddot{Z}_m n s &= -m_n s \cdot g + k_s \cdot (-a_{bms} - a_{hmns} - l_s 0) \\ &+ k_s \cdot (Z_m s - Z_m n s) + c_s \cdot (\dot{Z}_m s - \dot{Z}_m n s) \\ &- k_p \cdot (Z_m n s - Z_p - a_{bmns} - l_p 0) - u \end{aligned} \quad (7)$$

##### 3.2.1 Design of PID Controller

Considering the Laplace domain, the transfer function used for the PID controller is the following:

$$H(p) = \frac{U(p)}{\varepsilon(p)} = K_p \cdot \left( 1 + \frac{1}{T_i \cdot p} + \frac{T_d \cdot p}{a \cdot T_d \cdot p + 1} \right) \quad (8)$$

Where  $K_p$ ,  $T_d$ ,  $T_i$  and  $a$  are tuning parameters determined from simulations.  $\varepsilon(p)$  is the offset between the set point and the measure of the considered parameter.

We study two approaches. First, we minimize the acceleration of the sprung mass ( $m_s$ ). On a second time, we minimize the acceleration of the upper mass ( $m_q$ ). In fact, we firstly minimize the acceleration of the sprung mass ( $m_s$ ) because according to mechanical coupling between the two masses, we want to analyse the behaviour of the upper mass ( $m_q$ ) using a PID controller in order to minimize the acceleration of the sprung mass ( $m_s$ ). Then we use the same PID controller with minimization of the upper mass ( $m_q$ ), always exerting the control force  $u$  on the sprung mass. Results of the simulations of these two controlled systems are presented and discussed in the following of this paper (cf. part 3.2.3).

##### 3.2.2 Design of Sliding Mode Controller

Always using the mechanical coupling between the sprung mass ( $m_s$ ) and the upper mass ( $m_q$ ), we control the behaviour of the sprung mass ( $m_s$ ) using a sliding mode controller in order to minimize the acceleration of the upper mass ( $m_q$ ).

In this part we develop the design of the sliding mode controller which we will implement in the following. We have the following state vector:

$$\underline{\dot{x}} = \begin{bmatrix} Zq \\ Zms \\ Zmns \\ \dot{Z}q \\ \dot{Z}ms \\ \dot{Z}mns \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \quad (9)$$

In order to design the sliding mode controller, we explain the system model as an affine system of the form:

$$\dot{\underline{x}} = \underline{f}(\underline{x}) + \underline{g} \cdot u \quad (10)$$

Using this form, we can write:

$$\begin{aligned} \dot{x}_1 &= x_4 \\ \dot{x}_4 &= f_4(x_j) \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{x}_2 &= x_5 \\ \dot{x}_5 &= f_5(x_j) + g_5 \cdot u \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{x}_3 &= x_6 \\ \dot{x}_6 &= f_6(x_j) + g_6 \cdot u \end{aligned} \quad (13)$$

Where  $j=1 \dots 6$ . Moreover we have:

$$f_4(x_j) = \frac{1}{mq} \cdot \begin{pmatrix} kq \cdot (x_2 - x_1) + cq \cdot (x_5 - x_4) \\ -mq \cdot g \end{pmatrix} \quad (14)$$

$$f_5(x_j) = \frac{1}{ms} \cdot \begin{pmatrix} kq \cdot x_1 + (-kq - ks) \cdot x_2 + ks \cdot x_3 \\ +cq \cdot x_4 + (-cq - cs) \cdot x_5 + cs \cdot x_6 \\ -ms \cdot g \end{pmatrix} \quad (15)$$

$$f_6(x_j) = \frac{1}{mns} \cdot \begin{pmatrix} ks \cdot x_2 + (-ks - kp) \cdot x_3 \\ +cs \cdot x_5 + (-cs) \cdot x_6 - mns \cdot g \end{pmatrix} \quad (16)$$

We consider the desired state  $x_2^d$ . The error between the actual and the desired state can be written as:

$$e = x_2 - x_2^d \quad (17)$$

Here we consider the switching surface  $s$  defined for second order system by:

$$s = \dot{e} + \lambda \cdot e \quad (18)$$

$\lambda$  sets the dynamic in the sliding phase ( $s=0$ ). The control force  $u$  must be chosen so that trajectory of the state approaches the switching surface and then stay on it for all future time; guarantying stability and convergence to desired state. It is compound of a sum of two terms as following:

$$u = u_{eq} + u^* \quad (19)$$

The first term called equivalent control, is defined according to parameters of the nominal system. It is expressed as:

$$u_{eq} = g_5^{-1} \cdot \left( \ddot{x}_2^d - \lambda \cdot \dot{e} - f_5(x_j) \right) \quad (20)$$

The second term is defined in order to tackle uncertainties and to introduce reaching law. It is defined by:

$$u^* = g_5^{-1} \cdot (-k \cdot s) \quad (21)$$

The parameter  $k$  is chosen by the designer in order to define a reaching rate.

Thus we obtain the following law of control:

$$u = g_5^{-1} \cdot \left( \ddot{x}_2^d - \lambda \cdot \dot{e} - k \cdot s \right) \quad (22)$$

Results of the simulations of this controlled system are discussed in the following part.

### 3.2.3 Analysis of Simulations Results

Simulations of the previous designed controllers lead to following results:

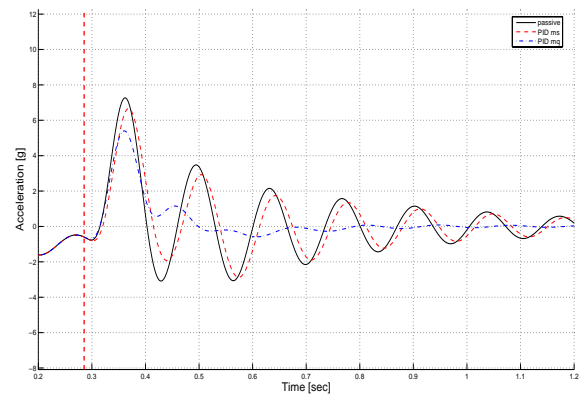


Figure 3: Acceleration of the mass (mq) - comparison between passive and PID controllers.

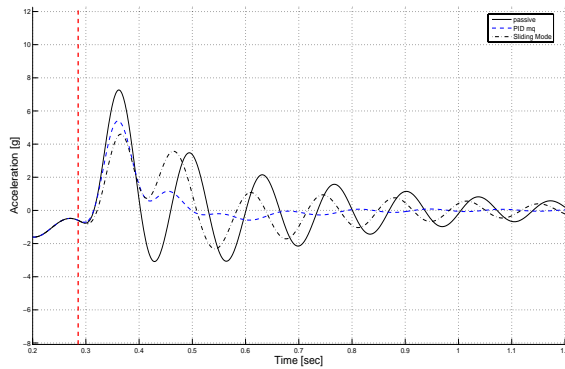


Figure 4: Acceleration of the mass ( $m_q$ ) - comparison between passive, PID and Sliding mode controllers.

The critical point occurs at the landing during the first compression of the landing gear. Thus, we want to minimize the amplitude of the first peak on the acceleration.

On Figure 3, we compare the passive system with PID controllers minimizing the acceleration of  $m_s$  and  $m_q$  (cf. part 3.2.1). The minimization of acceleration of  $m_s$  using PID is not effective on the minimization of the acceleration of  $m_q$ . Nevertheless minimization of acceleration of  $m_q$  using the same PID is effective. In fact we have respectively a gain of 9% and 25% in comparison with the passive system.

On Figure 4, we compare the passive system with PID and sliding mode controllers. Here minimization of  $m_s$  using sliding mode controller in order to minimize the acceleration of  $m_q$  is very effective. We notice a gain of 35% in comparison with the passive system on the first peak of acceleration of the upper mass ( $m_q$ ).

Moreover in order to guaranty the stability of the system and optimize the behaviour in minimization of the acceleration of  $m_q$ , the sliding mode controller is operative only at the impact of the system on the ground and during a defined time corresponding to the proper period of the upper system. This characteristic allows the maximum minimization of the first peak of the acceleration of the upper system ( $m_q$ ).

Thus using mechanical coupling, in order to minimize the acceleration of the upper mass ( $m_q$ ), the sliding mode controller is the most effective.

### 3.3 Prospects of Real Device

On the real device, we can't add an actuator in parallel of the passive landing gear.

In order to guaranty the maximum of stability and to follow the control force  $u$  which will lead to

an optimal transmitted force, we keep a passive hydraulic shock absorber that will dissipate the majority of the shock energy and in parallel of the passive shock absorber we add a controlled throttling device that will dissipate the rest of the energy.

This device is a semi active device where only the damping coefficient of suspension will be modified. Such a device doesn't need a lot of energy and moreover in case of failure of the controller, the stability of the system is insured.

## 4 EXPERIMENTAL SET UP

We build a drop test bench in order to test free falls of the system. The drop test bench is composed of a static part and a mobile part. Two columns and a base make up the static part. The mobile part is composed of the quarter part of a vehicle (wheel, suspension, sprung mass ( $m_s$ ) and unsprung mass ( $m_{ns}$ )) and the upper system (mass  $m_q$ , springs).

The stiffness of the suspension is insured by two parallel linear springs. Damping is insured by a hydraulic shock absorber. Four tuning parameters on it, allow us to modify its characteristic damping curve, in order to differentiate the damping rate in domains of low and high speeds for phases of compression and extension.

The used wheel is a wheel of an industrial vehicle. This one has been selected because its capacity to support heavy loads. A ball-bearing runner insures the guide of the mobile and leads the shock to be purely vertical.

Here frequential similitudes between a real aircraft and each subsystem of the drop test bench have been made. Drop test bench is represented on the following figure:

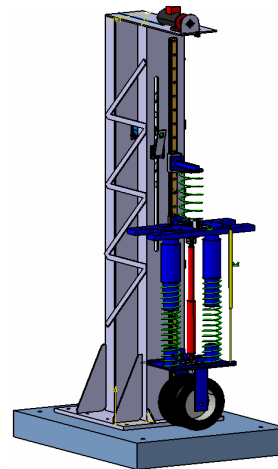


Figure 5: Numerical mock-up of drop test bench.

Accelerometers on each mass insure the knowledge of accelerations. Speeds and displacements are determined by numerical integrations. A force transducer between the shock absorber and the sprung mass (ms) measures the force transmitted by the damper. A linear inductive displacement transducer gives the stroke of the suspension. Thus redundancy of data on stroke of the suspension is insured.

Several tests in different configurations have been realized and have allowed us to cross check the previous exposed model. Cross checking leads us to have an accurate numerical model that allows us to develop the controlled device. In following of the study, this one will be test on the drop test bench.

## 5 CONCLUSION

In this paper, a study of the induced vibrations by a high speed shock on a complex mechanical system has been presented. Different anti vibratory methods of control have been designed from a cross checked numerical model which has been previously exposed. Cross checking results from an experimental study that has been realized on a drop test bench. Using mechanical coupling, a sliding mode controller has proved its efficacy in order to minimize the acceleration of an upper system located on an equivalent quarter part of vehicle system submitted to high speed shock.

Nevertheless this control force must be reachable by a dynamical tuning of the damping coefficient of the hydraulic shock absorber.

Thus in prospect, a semi active device has been designed and will have to be tested.

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