MULTICRITERIA DECISION MAKING IN BALANCED MODEL OF FUZZY SETS

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Abstract: In the paper aspects of negative information and of information symmetry in context of uncertain information processing is considered. Both aspects are presented in frames of fuzzy sets theory involved in data aggregation and decision making process. Asymmetry of classical fuzziness and its orientation to positive information are pointed out. The direct dependence of symmetry of uncertain information on negative information maintenance is indicated. The symmetrical, so called balanced, extension of classical fuzzy sets integrating positive and negative information an paralleling positiveness/negativeness with symmetry of fuzziness is presented. Balanced counterparts of classical fuzzy connectives are introduced.

1 INTRODUCTION

In the paper a discussion on aspects of negative information and information symmetry is presented. The discussion is based on an observation of asymmetry of operators in classical theories of uncertain information and in theories with focus turned on fuzzy sets. Preliminary discussion on asymmetry of classical fuzzy sets is presented in Section 2. It shows an inclination of classical fuzzy connectives to positive information and incompatibility with negative information. Issues related to negative information are outlined in Section 3. Assumed symmetry of negative and positive information integrates both types of information. The integrated approach to parallelism of negativeness/positiveness and symmetry of information is introduced in Section 4. The integration of negativeness and symmetry is inherently drawn in an idea of so called balanced extension of fuzzy sets. This idea was introduced in (Homenda, 2001) and then discussed in several papers (Homenda, 2004; Homenda, 2003; Homenda and Pedrycz, 2002). Finally, an application of the classical approach to uncertainty versus its balanced model is compared in the example 4.1. The example outlines importance of negative information in decision making process in real environment.

2 ASYMMETRY OF FUZZY SETS

Fuzzy set theory is often used to partition a universe into two subsets if partition criteria are not crisp. If partition criterion is uncertain, definition of subsets as fuzzy sets over the universe is a natural way to model uncertainty. However, neither crisp, nor fuzzy modelling avoids problems with law of excluded middle.

Partitioning the universe into two complementary sets suggests comparable significance of both sets unless additional principle is given. In such partitioning elements of the universe can be classified as *true* and *false*, *like* and *dislike*, *good* and *bad*, etc. without any emotional evaluation of these terms. We will simply talk about *positive* and *negative* information, again - without emotional evaluation of both terms. Using several criteria in partitioning we may classify elements of the universe with regard to every criterion separately. Having a number of pairs of complementary sets it is necessary to aggregate these results in order to get final two sets separation of the universe. Using classical aggregators we can choose between *all good criteria* or *one good criterion*. The first one, where the elements classified as *good* one must have all criteria *good*, is implemented by conjunction. The second one, where the element classified as *good* can have only one *good* criterion, is implemented by dis-

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junction. Both aggregation connectives, conjunction and disjunction, raise clear asymmetry under complement. If elements of one set are classified as having all criteria *good*, elements of the complementary set must have at least one criterion *bad* instead of expected the same condition of all criteria *bad*. Keeping the same condition (either *all criteria*, or *at least one criterion*) in definition of both sets raises troubles with law of excluded middle mentioned above. Following this way of thinking we need other connectives that will balance aggregation of decisions based on singular criterion. The above discussion leads to the conclusion that classical fuzzy set theory is asymmetrical with regard to processing opposite values of given attributes.

2.1 Symmetrization of the Scale

A classical fuzzy set A in the universe X can be defined in terms of its membership function $\mu : X \rightarrow [0,1]$, where the value 0 means exclusion of the element from the set while the values greater than 0 express the grade of inclusion of the element into the set. However, membership function does not define a grade of exclusion, the grade of negative information. Therefore, fuzzy sets theory distinguishes grades of inclusion and reserves only one value - 0 - for exclusion. This raises asymmetry of this interpretation.

Membership function defines fuzzy connectives: union, intersection and complement. The definitions are expressed by max, min and complement to 1, i.e. $d(x,y) = max\{x,y\}$, $c(x,y) = min\{x,y\}$ and $n(x) = 1-x$. Classical connectives are asymmetrical. Union gets its value from the greater argument, despite of the values of both arguments. Similarly, intersection gets its value from the smallers argument only.

We can split values of a given criterion in the spirit of *good* and *bad* allocating the values of the interval $[0,0.5)$ as pieces of *negative* information relevant to *bad* values and the values of the interval $(0.5.1]$ as pieces of *positive information* relevant to *good* values. The value 0.5, the center of the unit interval $[0,1]$, is a numerical representation of the state of no negative/positive information. Being compatible with common meaning of membership function let us assume that the greater the value of positive information, the stronger the *good* value of the criterion. By symmetry, the smaller the value of negative information, the stronger the *bad* value of the criterion.

This interpretation is well-matched with the common sense of ordering of the negative/positive values. The ordering could be seen as *monotonicity* of negative/positive information mapping: it starts from the left end of the unit interval representing strong negative information, then goes toward middle of the unit interval diminishing strength of negative information, then crosses the middle point of the unit interval and then goes towards the right end of the unit interval increasing strength of positive information.

This interpretation is also well-matched with the common sense of symmetry of the negative/positive values with the symmetry center in the value 0.5. The linear transformation $f(x) = 2x-1$ of the unit interval $[0,1]$ into the symmetrical interval $[-1,1]$ points out the symmetry. In this transformation negative information is mapped to the interval $[-1,0)$, positive information - to the interval $(0,1]$ and the state of no information - to the value 0.

2.2 Connectives Asymmetry

The classical fuzzy connectives stay asymmetrical even with the symmetrical bipolar scale of the interval $[-1,1]$ applied. Both classical fuzzy connectives get their values from the maximal argument (union, maximum) and the minimal argument (intersection, minimum). Classical fuzzy connectives were generalized to triangular norms: maximum is an example of t-conorms, minimum is an example of t-norm, c.f. (Schweizer and Sklar, 1983). Strong t-norms and t-conorms, the special cases of triangular norms, have an interesting property: if both arguments are greater than 0 and smaller than 1, the result of strong t-conorm exceeds the greater argument while the result of strong t-norm is less than smaller argument, c.f. (Klement et al., 2000). This property might be interpreted that union tends to positive information despite of the values of its arguments while intersection tends to negative information despite of the values of its arguments. In other words, symmetrical interpretation of the unipolar scale makes that strong t-norm increases certainty of negative information and decreases certainty of positive information. And vice versa, strong t-conorm decreases certainty of negative information and increases certainty of positive information. This observation emphasizes the asymmetry of fuzzy connectives, c.f. Figure 1.

The problem of asymmetry of fuzzy connectives was discussed in number of papers, e.g. (Detyniecki and Bouchon-Meunier, 2000b; Homenda and Pedrycz, 1991; Homenda and Pedrycz, 2002; Silvert, 1979; Yager, 1988; Yager, 1993; Zhang W. R., 1989). In these papers discussion on asymmetry of fuzzy sets and uncertain information processing was undertaken for different reasons, though common conclusions led to importance of the symmetry problem in fuzziness and uncertainty.

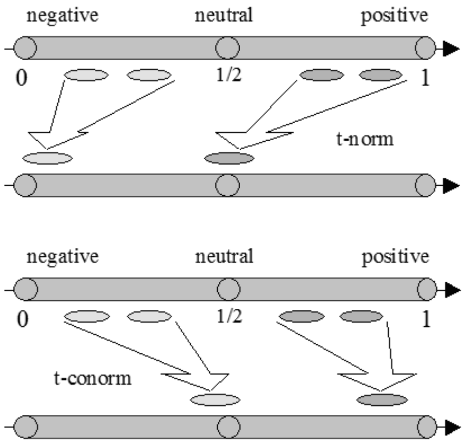

Figure 1: Asymmetry of classical triangular norms.

3 NEGATIVE INFORMATION

The mapping of negative and positive information in the scale of unit interval $[0,1]$ as well as in the symmetrical interval $[-1,1]$ bring incompatibility with connectives, so the question is raised if negative information can be considered as a subject of uncertainty. The question seems justified since negative information is hardly interpretable in classical set theory and classical fuzzy sets theory. However, negative information, as explained in the introductory remarks to this section, play important role in different fields. From psychological studies it is known that human beings convey symmetry in their behavior, c.f. (Grabisch M., 2002). One can be faced with positive (gain, satisfaction, etc.) or negative (loss, dissatisfaction, etc.) quantities, but also with a kind of disinterest (does not matter, not interested in, etc.). For instance, one either likes to listen to the music while reading an interesting novel or does not like to listen to the music then or even music is only a background not affecting him at all. These quantities could be interpreted in context of positive/negative/neutral information. On the other hand, in economy psychological attempt to decision making process with uncertain premises overheads traditional models of customers behavior. The pseudocertainty effect is a concept from prospect theory. It refers to people's tendency to make risk-averse choices if the expected outcome is positive, but risk-seeking choices to avoid negative outcomes. Their choices can be affected by simply reframing the descriptions of the outcomes without changing the actual utility, c.f. (Kahneman and Tversky, 2004). Aggregation of positive and negative premises leads to implementation of a crisp decision. Modelling of such an attempt requires processing of positive/neutral/negative information.

An interesting contribution to positive/negative information maintaining could be found in the theory of intuitionistic fuzzy sets (Atanassov, 1986) and in very similar theory of vague sets (Gau and Buehrer, 1993). Another approach to positive/negative information is discussed in twofold fuzzy sets, c.f. (Dubois and Prade, 1983). In these theories, uncertain information is represented as a pair of positive/negative components numerically described by membership values from the unit interval $[0,1]$. Both components are tied with degree of indeterminacy which stays that sum of membership values of both components cannot exceed the value 1. However, no tool to combine both components is provided in these theories. Since information aggregation leading to non ambiguous result is a clue issue in decision making process, these theories must be supported by information aggregators in such a process.

The very early medical expert system MYCIN, c.f. (Buchanan and Shortliffe, 1984), combine positive and negative information by somewhat ad hock invented aggregation operator. In (Detyniecki, 2000) it was shown that MYCIN aggregation operator is a particular case in a formal study on aggregation of truth and falsity values, c.f. (Detyniecki and Bouchon-Meunier, 2000a) for further discussion on aggregation of positive and negative information.

Having many premisses, usually uncertain, we need to produce nonambiguous information that yields a unique decision. Therefore aggregation of information is crucial in decision making process. The topic of information aggregating has been studied in number of papers. An interesting considerations on information aggregation could be found in - for instance - (Calvo T., 2001; Detyniecki and Bouchon-Meunier, 2000a; Silvert, 1979; Ovchinnikov, 1998; Yager and Rybalov, 1998; Yager and Rybalov, 1996; Zhang W. R., 1989).

4 SYMMETRIZING FUZZINESS

Fuzzy connectives stay asymmetrical with symmetrized scale. The incompatibility of symmetrical interpretation of the scale and asymmetrical behavior of fuzzy connectives suggest incorrectness of scale symmetrization. This discussion leads to the hypothesis that Zadeh's extension of crisp sets to fuzzy sets, c.f. (Zadeh, 1965), relied on dispersion of positive information of the crisp point $\{1\}$ into the interval $(0,1]$. However, negative information of the point $\{0\}$ was still bunched in this point, c.f. Figure 2. This hypothesis can be supported by similarity of balanced

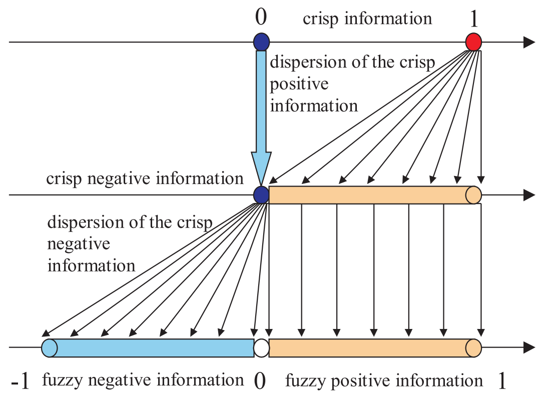

Figure 2: Extension of fuzzy sets to balanced fuzzy sets.

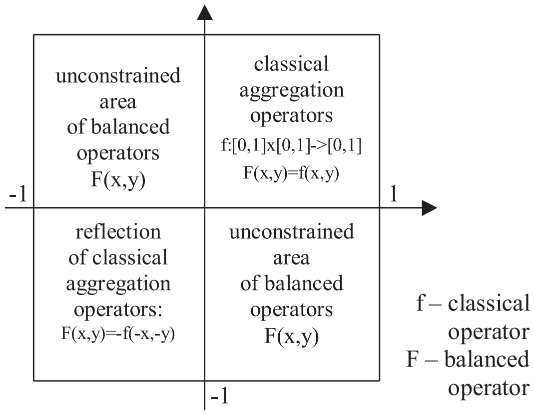

Figure 3: Balanced extension of fuzzy operators.

triangular norms and uninorms and nullnorms - the products of different approaches to fuzzy connectives extension, c.f. (Homenda, 2003)

4.1 Balanced Symmetrization of the Scale

Now, the process of information dispersion is applied again to information concentrated in the point $\{0\}$. This operation extends classical fuzzy sets to balanced fuzzy sets, c.f. (Homenda, 2004; Homenda, 2003). The extension is being done by dispersion of crisp negative information bunched in the point $\{0\}$ into the interval $[-1.0)$ without affecting classical fuzzy sets based on the unit interval $[0,1]$ c.f. Figure 2. Thus, classical fuzzy sets will be immersed in a new space of balanced fuzzy sets. Since both types of information - positive and negative - are assumed to be equally important, it would be reasonable to expect that such an extension will provide a kind of symmetry of positive/negative information.

Concluding, the following symmetry principle can be formulated: *the extension of fuzzy sets to balanced fuzzy sets relies on spreading negative information (information about exclusion) that fit the crisp point $\{0\}$ of fuzzy set into the interval $[-1,0)$.* The extension will preserve properties of classical operators for positive information. It will provide the symmetry of positive/negative information with the center of symmetry placed in the point 0, c.f. Figure 2. It is worth to underline that this operation is entirely different than simple linear rescaling of the unit interval $[0,1]$ into the interval $[-1,1]$. The linear function $f(x)=2x-1$ is replaced by the transformation that is not a function: it allocates the whole interval $[-1,0)$ as a ”value” in the point 0.

4.2 Symmetry of Balanced Connectives

Triangular norms generalize the concept of set operations: union and intersection, c.f. (Schweizer and Sklar, 1983). Triangular norms, t-norms and t-conorms, together with negation, the basic fuzzy connectives are the subject of the discussion of connectives symmetrization.

Definition 4.1 *Triangular norms: t-norm t and t-conorm s, are mappings $t:[0,1]\times[0,1]\rightarrow[0,1]$ and $s:[0,1]\times[0,1]\rightarrow[0,1]$ satisfying the following axioms:*

1. $t(a,t(b,c))=t(t(a,b),c)$
   $s(a,s(b,c))=s(s(a,b),c)$ *associativity*
2. $t(a,b)=t(b,a)$
   $s(a,b)=s(b,a)$ *commutativity*
3. $t(a,b)\leq t(c,d)$ *if* $a\leq c$ & $b\leq d$
   $s(a,b)\leq s(c,d)$ *if* $a\leq c$ & $b\leq d$ *monotonicity*
4. $t(1,a)=a$ *for* $a\in[0,1]$ *boundary*
   $s(0,a)=a$ *for* $a\in[0,1]$ *conditions*

t-norms and t-conorms are dual operations in the sense that for any given t-norm t and given negation operator assumed here to be complement to one, we have the dual t-conorm s defined by the De Morgan formula $s(a,b)=1-t(1-a,1-b)$. And vice-versa, for any given t-conorm s, we have the dual t-norm t defined by the De Morgan formula $t(a,b)=1-s(1-a,1-b)$. Duality of triangular norms causes duality of their properties. Note that the max/min is a pair of dual t-norm and t-conorm.

The idea of balanced extension of classical fuzzy connectives must be compatible with the concept of balanced extension of the unipolar scale and with the symmetry principle formulated in Section 4.1. This requirements and symmetry of the balanced fuzzy scale of the interval $[-1,1]$ determines the domain of symmetrized balanced connectives to be the square $[-1,1]\times[-1,1]$. Preservation of classical fuzzy sets properties requires preservation of properties of classical fuzzy connectives on the unit square $[0,1]\times[0,1]$. Conversely, expected symmetry of positive and negative information puts strict restrictions on balanced extension on the square $[-1,0]\times[-1,0]$. The same factors determine the co-domain of symmetrical fuzzy connectives to the interval $[-1,1]$. This idea of the balanced extension of classical fuzzy connectives is outlined in Figure 3. It is clear that balanced connectives are simple reflection of respective classical connectives on the square $[-1,0]\times[-1,0]$. The remaining parts of the domain of balanced connectives are not explicitly constrained. However, some constrains will be put when other properties of connectives are considered. These properties come from natural extension of the axioms of the triangular norms definition onto the whole domain of balanced operators.

This discussion leads to the definition of balanced negation, balanced t-norms and balanced t-conorms.

Definition 4.2 *of balanced connectives.*
The mapping $N:[-1,1]\to[-1,1], N(x)=-x$ *is the balanced negation.*
The mappings $T:[-1,1]\times[-1,1]\to[-1,1]$ *and* $S:[-1,1]\times[-1,1]\to[-1,1]$ *are balanced t-norm and balanced t-conorm, respectively, assuming that they satisfy the following axioms in the whole domain* $[-1,1]\times[-1,1]$ *unless defined explicitly:*

1., 2., 3.
associativity, commutativity and monotonicity

4. $T(1,a)=a \quad for \quad a\in[0,1]$ *boundary*
   $S(0,a)=a \quad for \quad a\in[0,1]$ *conditions*
5. $T(x,y)=N(T(N(x),N(y)))$
   $S(x,y)=N(S(N(x),N(y)))$ *symmetry*

Conclusion 4.1 *The definitions of balanced t-norm and balanced t-conorm restricted to the unit square* $[0,1]\times[0,1]$ *are equivalent to the classical t-norm and classical t-conorm, respectively.*

Conclusion 4.2 *Balanced t-norm and balanced t-conorm restricted to the square* $[-1,0]\times[-1,0]$ *are isomorphic with the classical t-conorm and classical t-norm, respectively.*

Conclusion 4.3 *Balanced t-norm vanishes on the squares* $[-1,0]\times[0,1]$ *and* $[0,1]\times[-1,0]$.

Example 4.1 *Let us consider the strong t-norm generated by the additive generator* $f(x)=(1-x)/x$ *and the strong t-conorm generated by the additive generator* $f(x)=x/(1-|x|)$, *c.f. (Klement et al., 2000; Schweizer and Sklar, 1983). The formula* $p(x,y)=f^{-1}(f(x)+f(y))$ *defines the respective strong triangular norms. The extension of this t-norm to balanced t-norm comes from the monotonicity and symmetry axioms directly. Alternatively, the additive generator* $f(x)=(1-x)/x$ *of this classical t-norm could be extended to the interval* $[-1,1]$ *with the formula* $f(x)=(1-|x|)/x$. *Of course, this function is undefined in the point* 0. *The formula* $p(x,y)=f^{-1}(f(x)+f(y))$ *defines the balanced t-norm for both arguments being nonnegative or nonpositive.*

The balanced counterpart of the strong t-conorm generated by the additive generator $f(x)=x/(1-|x|)$ *is determined in the squares* $[0,1]\times[0,1]$ *and* $[-1,0]\times[-1,0]$. *The values in remaining parts of the domain are unconstrained besides that they must satisfy axioms of the definition. In this case we can extend the additive generator to the whole interval* $[-1,1]$ *assuming that in points* -1 *and* 1 *it gets the values* $-\infty$ *and* ∞, *respectively. The balanced t-conorm could be defined by the formula* $S(x,y)=f^{-1}(f(x)+f(y))$ *in its whole domain* $[-1,1]\times[-1,1]$ *except the points* $(-1,1)$ *and* $(1,-1)$, *where this balanced t-conorm is undefined, c.f. (Homenda, 2003; Klement et al., 2000; Schweizer and Sklar, 1983) for details. Contour graphs of these two balanced norms are presented in Figure 4.*

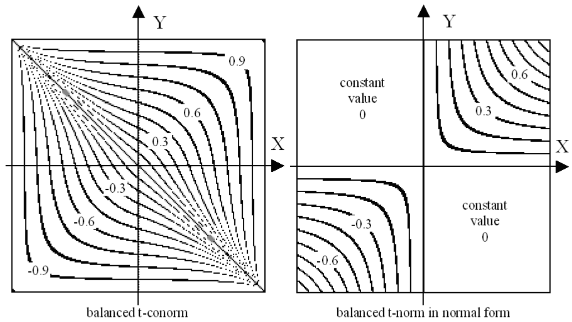

Figure 4: Balanced t-conorm and balanced t-norm.

5 BALANCED FUZZY SETS IN DECISION MAKING

Let us consider simple decision making process in real economical environment, i.e. with uncertain premises. Assume that in the first set of six premises five have the numerical value 0.6 and the last one has the value 0.4. In the second set of six premises the one has numerical value 0.6 and five other have numerical values 0.4. Classical fuzzy connectives employed as aggregators of premises do not distinguish between these two sets. Max and min operators produce the same values for both sets: 0.6 and 0.4, respectively. Employing the strong t-conorm based on the additive operator $f(x) = x/(1-x)$ we get the values 0.89 and 0.83 for both sets, respectively. In the case of dual t-norm we get the respective values equal to 0.17 and 0.11. The linear mapping $f(x) = 2x-1$ to the interval $[-1,1]$ gives the transformed values of premises equal to 0.2 and -0.2 instead of 0.6 and 0.4. Classical triangular norms produce the following the respective values: 0.78, 0.66 and -0.66, -0.78. Therefore, besides small quantitative differences between aggregated numerical results no qualitative indication is given with regard to the decision. Employing balanced modelling based on the additive operator $f(x) = x/(1-|x|)$ balanced t-conorm produces the values 0.50 and -0.50 for both sets of premises, respectively. Dual balanced t-norm produces the values 0.06 and -0.06, respectively. In the case of balanced modelling clear indication is given with regard to the decision.

6 CONCLUSIONS

The balanced extension of fuzzy sets discussed in this paper is a contribution to the discussion on subjects of negative information and symmetry of negative/positive types of information. These aspects of information processing, though controversial in classical and traditional fields of information processing, become useful and necessary in some important areas of research and practice, as indicated in Section 3 and have been studied in number of papers.

Negative and positive types information play important role in information aggregation. Multicriteria decision making process could be seen as a kind of information aggregation leading to a synthetical result applicable in unique choice between given options. The synthesis must consider pros and contras a decision, must consider positive and negative premisses of the decision. The concept of balanced fuzzy sets deals with positiveness and negativeness assuming symmetry of both types of information.

In (Homenda and Pedrycz, 2005) the concept of negativeness and symmetry was applied in construction of the balanced computing unit, a variation of fuzzy neuron. The concept of balanced computing unit involves a generalization of balanced t-norms, so called t-norms in weak form. The balanced computing unit based on weak form of fuzzy connectives may exemplify decision making process with positive and negative premises.

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