TIME-FREQUENCY REPRESENTATION OF INSTANTANEOUS FREQUENCY USING A KALMAN FILTER

Jindřich Liška and Eduard Janeček
Department of Cybernetics, University of West Bohemia in Pilsen, Univerzitní 8, Plzeň, Czech Republic

Keywords: Instantaneous frequency, Kalman filter, time-frequency analysis, state estimation, Hilbert transform.

Abstract: In this paper, a new method for obtaining a time-frequency representation of instantaneous frequency is introduced. A Kalman filter serves for dissociation of signal into modes with well defined instantaneous frequency. A second order resonator model is used as a model of signal components – 'monocomponent functions'. Simultaneously, the Kalman filter estimates the time-varying signal components in a complex form. The initial parameters for Kalman filter are obtained from the estimation of the spectral density through the Burg's algorithm by fitting an auto-regressive prediction model to the signal. To illustrate the performance of the proposed method, experimental results show the contribution of this method to improve the time-frequency resolution.

1 INTRODUCTION

Data analysis is a necessary part in pure research and in practical applications. The problem of estimating of a signal is of great interest in many areas of engineering, such as energy processing, speech recognition, vibration analysis and time series modeling. To analyze a non-stationary data, previous methods repeatedly apply block data processing such as the short-time Fourier transform, with the assumption, that the frequency characteristics are time-invariant (or that the process is stationary) for the duration of the time block. The resolution of such methods is limited by the Heisenberg-Gabor uncertainty principle.

In this work a different approach is proposed, in which a Kalman filter is used to decompose the time-varying signal into analytic components. As is well known, the Kalman-filter can estimate the state vectors of time-varying systems with knowledge of the stochastic characteristics of the measurement noise. The estimated components are then used for computation of instantaneous amplitude and frequency.

The rest of the paper is organized as follows. In Section 2, a summary of the common non-stationary data processing methods is presented. In Section 3, we mention the instantaneous frequency phenomenon. In Section 4, the use of Kalman filter to obtain complex signal component estimation is described. In Section 5, the results from experiments and from real application are discussed. Conclusions are drawn in Section 6.

2 NON-STATIONARY DATA PROCESSING METHODS

The spectrogram is the most basic method, which is a limited time window-width Fourier spectral analysis. Since it relies on the traditional Fourier transform, one has to assume the data to be piecewise stationary. There are also practical difficulties in applying the method: in order to localize an event in time, the window width must be narrow, but, on the other hand, the frequency resolution requires longer time series (uncertainty principle).

The wavelet approach is essentially a Fourier spectral analysis with an adjustable window. For specific applications, the basic wavelet function can be modified according to special needs, but the form has to be given before the analysis. In most common applications, the Morlet wavelet is defined as Gaussian enveloped sine and cosine wave groups with 5.5 waves. It is very useful in analysing data with gradual frequency changes. Difficulty of the wavelet analysis is among others its non-adaptive nature. Once the basic wavelet is selected then is used to analyse all the data.

Liška J. and Janeček E. (2007).
TIME-FREQUENCY REPRESENTATION OF INSTANTANEOUS FREQUENCY USING A KALMAN FILTER.
In *Proceedings of the Fourth International Conference on Informatics in Control, Automation and Robotics*, pages 40-46
DOI: 10.5220/0001621900400046

The Wigner-Ville distribution is sometimes also referred to as the Heisenberg wavelet. By definition it is the Fourier transform of the central covariance function.

Above mentioned methods were used in Section 5 to compare their results with the output of the method based on Kalman estimation.

3 INSTANTANEOUS FREQUENCY AND THE COMPLEX SIGNAL

Instantaneous frequency, $\omega(t)$, is often defined as derivation of phase

$$\omega(t) = \frac{d\varphi(t)}{dt} = 2\pi f(t) \tag{1}$$

One of the ways how the unknown phase can be obtained is to introduce a complex signal $z(t)$ which corresponds to the real signal. As mentioned in (Hahn, 1996) or in (Huang, 1998), the Hilbert transform can be the elegant solution of this problem.

The Hilbert transform, $v(t)$, of a real signal $u(t)$ of the continuous variable t is

$$v(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{u(\eta)}{\eta - t} d\eta \tag{2}$$

where P indicates the Cauchy Principle Value integral. The complex signal $z(t)$

$$z(t) = u(t) + j \cdot v(t) = a(t) e^{j\varphi(t)} \tag{3}$$

whose imaginary part is the Hilbert transform $v(t)$ of the real part $u(t)$ is then called the analytical signal and its spectrum is composed only of the positive frequencies of the real signal $u(t)$.

From the complex signal, an instantaneous frequency and amplitude can be obtained for every value of t. Following (Hahn, 1996) the instantaneous amplitude is defined as

$$a(t) = \sqrt{u(t)^2 + v(t)^2} \tag{4}$$

and the instantaneous phase can be defined as

$$\varphi(t) = \arctan \frac{v(t)}{u(t)} \tag{5}$$

The instantaneous frequency then simplifies to

$$\omega(t) = \frac{d}{dt}(\arctan \frac{v(t)}{u(t)}) = \frac{u(t)\dot{v}(t) - v(t)\dot{u}(t)}{u(t)^2 + v(t)^2} \tag{6}$$

Even with the Hilbert transform, there is still considerable controversy in defining the instantaneous frequency as in (Boashash, 1992a). Applying the Hilbert transform directly to a multicomponent signal provides values of $a(t)$ and $\omega(t)$ which are unusable for describing the signal. The idea of instantaneous frequency and amplitude does not make sense when a signal consists of multiple components at different frequencies. This leads Cohen in (Cohen, 1995)to introduce term 'monocomponent function' where at any given time, there is only one frequency value. Huang (Huang, 1998) introduced a so called Empirical Mode Decomposition method to decompose the signal into monocomponent functions (Intrinsic Mode Functions).

4 USE OF KALMAN FILTER TO OBTAIN THE SIGNAL COMPONENTS

In this paper, an adaptive Kalman filter based approach is used to decompose the analyzed signal into monocomponent functions. As mentioned above, it is required that the estimated components are complex functions because of efficient computation of the instantaneous frequency. The analyzed signal is modeled as a sum of resonators in this study.

4.1 Complex Signal Component Model

The second-order model ($n = 2$) of auto-regressive (AR), linear time-invariant (LTI) system is considered as a resonator. Its external description in continuous domain is defined by the following differential equation

$$y(t) = a \cdot \sin(\omega \cdot t) \tag{7}$$

$$\dot{y}(t) = a \cdot \omega \cdot \cos(\omega \cdot t) \tag{8}$$

$$\ddot{y}(t) = -a \cdot \omega^2 \sin(\omega \cdot t) = -\omega^2 \cdot y(t) \tag{9}$$

where a is the amplitude and ω is the natural frequency of the resonator. Let the measured system is described by its state equations:

$$\dot{x}(t) = A \cdot x(t) \tag{10}$$

$$y(t) = C \cdot x(t) \tag{11}$$

where $x(t)$ denotes the vector of system internal states ($u(t)$ and $v(t)$) at time *t*, $y(t)$ is the output signal, A is the state matrix and C is the output matrix. Hence it follows that the internal model representation of the resonator with suitable selected state variables ($u(t) = \sin(\omega \cdot t)$ and $v(t) = \cos(\omega \cdot t)$) is then

$$\begin{bmatrix} \dot{u}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \cdot \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \tag{12}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} u(t) \\ v(t) \end{bmatrix}. \tag{13}$$

The state equation (12) shows the state matrix of the continuous model as a 2D rotation matrix whose eigenvalues are pure imaginary numbers. The trajectory in state space of such a system is a circle.

There is need to discretize the continuous state space model for a digital computation needs. This can be done by solving the state differential equation (14) and substitution of the time t with sampling step h (see Fairman, 1998)

$$x(t) = e^{At} x(0). \tag{14}$$

The discretized state model ($\Delta t = h$) with state noise $\xi(k)$ and output noise $\eta(k)$ is then

$$\begin{bmatrix} u(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} \cos(h \cdot \omega) & -\sin(h \cdot \omega) \\ \sin(h \cdot \omega) & \cos(h \cdot \omega) \end{bmatrix} \cdot \begin{bmatrix} u(k) \\ v(k) \end{bmatrix} + \Gamma \cdot \xi(k) \tag{15}$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} u(k) \\ v(k) \end{bmatrix} + \Delta \cdot \eta(k) \tag{16}$$

The variables $\xi(k)$ and $\eta(k)$ are white noise vectors with identity covariance matrices. The specific features of the noises are characterized by the covariance matrices Γ and Δ.

This resonator model forms together with Kalman filtering approach an estimator of complex signal. The estimation of the first model state is a real part (sine function) and the estimation of the second model state is an imaginary part (cosine function) of the complex signal.

4.2 Discrete Kalman Filter

A discrete-time Kalman filter realizes a statistical estimation of the internal states of noisy linear system and it is able to reject uncorrelated measurement noise – a property shared by all Kalman filters. Let's assume a system with more components. Then the state matrix consists of following blocks:

$$A_i = \begin{bmatrix} \cos(h \cdot \omega_i) & \sin(h \cdot \omega_i) \\ -\sin(h \cdot \omega_i) & \cos(h \cdot \omega_i) \end{bmatrix} \tag{17}$$

and the state noise matrix blocks may be defined as a derivative of the state matrix blocks:

$$\Gamma_i = \frac{\partial A_i}{\partial (h \cdot \omega_i)} = \begin{bmatrix} -\sin(h \cdot \omega_i) & \cos(h \cdot \omega_i) \\ -\cos(h \cdot \omega_i) & -\sin(h \cdot \omega_i) \end{bmatrix} \tag{18}$$

The derivation of state matrix blocks as an estimation of the state noise matrix was selected experimentally, because the derivation produces blocks also in the state noise matrix and the components relate to each other in the same manner as in the state matrix.

The state-variable representation of the whole system, which is characterized by the sum of resonators, is given by the following matrices:

$$A = \underbrace{\begin{bmatrix} A_1 & 0 & 0 & \dots & 0 \\ 0 & A_2 & 0 & \dots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \dots & 0 & A_n \end{bmatrix}}_{2n \times 2n};$$

$$C = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \end{bmatrix}}_{1 \times 2n};$$

$$\Gamma = \underbrace{\begin{bmatrix} \Gamma_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \Gamma_2 & 0 & \dots & 0 & 0 \\ 0 & 0 & \ddots & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & 0 & 0 \\ 0 & 0 & \dots & 0 & \Gamma_n & 0 \end{bmatrix}}_{2n \times (2n+1)}; \tag{19}$$

$$\Delta = \underbrace{\begin{bmatrix} 0 & \dots & 0 & \delta \end{bmatrix}}_{1 \times (2n+1)}.$$

Commonly, the Kalman estimation includes two steps – prediction and correction phase. Let's assume that the state estimate μ_0 is known with an error variance P_0. An a priori value of the state at instant $k+1$ can be obtained as

$$\mu_{k+1} = A \cdot \mu_k \tag{20}$$

The measured value $y(k)$ is then used to update the state at instant k. The additive correction of the a priori estimated state at $k+1$ is according to (Vaseghi, 1987) proportional to the difference between the a priori output at instant k defined as $C \cdot \mu_k$ and the measured $y(k)$:

$$\mu_{k+1} = A \cdot \mu_k + K_k \cdot (y_k - (C \cdot \mu_k)) \tag{21}$$

where K_k is the Kalman gain which guarantees the minimal variance of the error $x_k - \mu_k$.

Also, at each step the variance $P(k+1)$ of the error of μ_{k+1} is calculated (see (Vaseghi, 1987)):

$$P_{k+1} = AP_kA^T + \Gamma\Gamma^T - K_k \cdot (CP_kA^T + \Delta\Gamma^T) \tag{22}$$

It is used for calculation of Kalman gain in the next step of the recursive calculation (correction phase):

$$K_k = (AP_kC^T + \Gamma\Delta^T) \cdot (CP_kC^T + \Delta\Delta^T)^{-1} \tag{23}$$

4.3 Estimation of Initial Parameters

The initial parameters for Kalman filter are obtained from the estimation of the spectral density by fitting an AR prediction model to the signal. The used estimation algorithm is known as Burg's method (Marple, 1987), which fits an AR linear prediction filter model of a specified order to the input signal by minimizing the arithmetic mean of the forward and backward prediction errors. The spectral density is then computed from the frequency response of the prediction filter. The AR filter parameters are constrained to satisfy the Levinson-Durbin recursion.

The initial Kalman filter parameters (frequencies of the resonators) are then obtained as local maxima of estimated spectral density which are greater than a predefined level. These values indicate significant frequencies in spectral density and determine the order of the model (see Section 3.2).

5 RESULTS

Within this work three test signals are analyzed. The first test signal s_1 (t) contains three harmonic components. $1kHz$ sampling rate was used and the signal was 1 second long ($N = 1000$ points). Signal was formed by sinus functions with oscillation frequencies f_1=10Hz, f_2=30Hz and f_3=50Hz. The amplitude was for all three components set to 10, but the second component was zero for the first 0.5 seconds. The output noise with mean $m = 0$ and variance $\sigma = 1$ was added to the simulation signal.

The initial parameters for Kalman filter were obtained through Burg's AR linear prediction filter of order 10 and the level for local maxima was determined as $\max > 1$. Under these conditions the initial frequencies (n=3) for Kalman estimator were obtained from Burg's spectral density. The initial conditions of Kalman estimator were set up in the following way: μ_0=[1…1], P_0=10^6.I, $\delta = 1$, where dim(μ_0)=$1 \times n$ and dim(P_0)= $2n \times 2n$.

To take a look at the convergence of the estimate, the comparison of the Hilbert approach and Kalman filter is considered. In figure 1 the complex signal of second component of the simulated signal is displayed. The results were obtained through Hilbert transform and through Kalman estimation. The disadvantage of the Hilbert transform is that it requires the pre-processing of the signal through some signal decomposition method. To decompose the signal into its components, the above introduced algorithm uses the model of sum of resonators and simultaneously the Kalman estimator is used to estimate the time progression of these components.

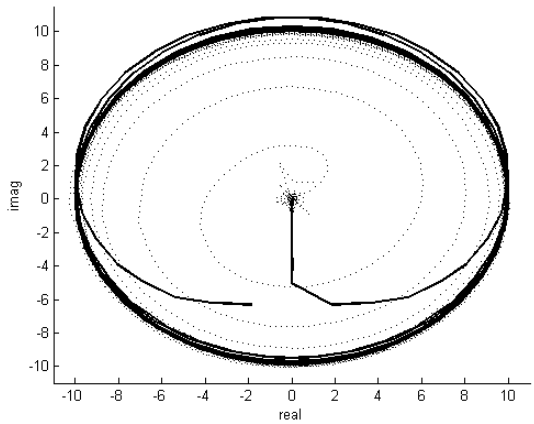

Figure 1: Second component of test signal s_1 (t) - complex signal obtained through Hilbert transform (solid line) and through Kalman estimation (dotted line).

In figure 2, there is shown the instantaneous frequency of all components of test signal s_1 (t).

The additive noise in simulation signal is the cause of the instantaneous frequency oscillating.

The algorithm based on Kalman estimation is also illustrated on another two non-stationary test signals s_2 (t) and s_3 (t). The initial conditions of Kalman estimator were set up as mentioned above and initial frequencies of the model were obtained through Burg's AR linear prediction filter of order 25 as maxima in estimated power spectrum (*n=10*).

The test signal s_2 (t) consists of two components in time-frequency domain - stationary harmonic signal with constant frequency and concave parabolic chirp signal. Both components exist in time between t = 100 and t = 900. The results of the Kalman estimation is compared with the methods mentioned in Section 2 and the results are shown in Figure 4. The output of Kalman estimation in time-frequency domain has relatively better time-frequency resolution in both components than the other methods.

The test signal s_3 (t) consists of four harmonic components and the accuracy of the method to identify the frequency and also the time of the origin and end of the components is tested. The signal begins again in time t = 100 and ends in t = 900. The frequency changes in t = 300 and t = 600. There are two components simultaneously in time between t = 300 and t = 600. The ability of methods to distinguish between these two frequencies is visible in Figure 5. The smoothed pseudo Wigner-Ville distribution and Kalman filter have better time-frequency resolution compared to short-time Fourier transform and to Morlet wavelet.

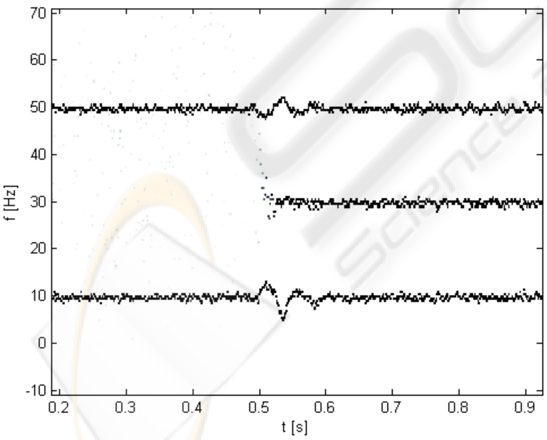

Figure 2: Estimation of instantaneous frequency of test signal components.

The last example is the transform of the acoustic signal from the real equipment where the instantaneous event took place. The signal was measured with 80 kHz sampling rate. For comparison, in figure 3, the time-frequency-amplitude responses of the short-time Fourier transform (STFT) and of the Kalman estimator approach are compared. The black column at first 4 milliseconds in the left spectrogram is the adaptation phase of Kalman filter. This example was obtained with following initial conditions: Burg's filter of order 400 was used to identify the power spectral density and all local maxima (*n=148*), which satisfy the inequality $\max > 10^{-7}$ were appointed as monitored frequencies. All resonance frequencies (5, 6, 14 and 27kHz) in the STFT spectrogram are also presented in the left one (Kalman). An event which occurs at time 0.022 seconds is displayed also in both spectrograms (see the frequency band 2 - 15 kHz). It is visible that the Kalman version of spectrogram offers a better resolution in time and frequency than the spectrogram obtained through STFT.

6 CONCLUSIONS

The new method for obtaining the time-frequency representation of instantaneous frequency has been introduced in this work. The procedure is based on the Kalman estimation and shares its advantages regarding the suppression of measurement noise. In this method the Kalman filter serves for dissociation of signal into modes with well defined instantaneous frequency. Simultaneously the time progression of signal components is estimated. This procedure utilizes the adaptive feature of the Kalman filter.

In cases where the short–time Fourier transform cannot offer sufficient resolution in frequency-time domain, there can be taken advantage of this method despite of higher computational severity. In vibrodiagnostic methods, where frequency-time information is used for localizing of non-stationary events, the sharpness of the introduced method can be helpful for the improvement of the event localization.

ACKNOWLEDGEMENTS

This work was supported by AREVA NP GmbH, Department SD-G in Erlangen (Germany) and from the specific research of Department of Cybernetics at the University of West Bohemia in Pilsen.

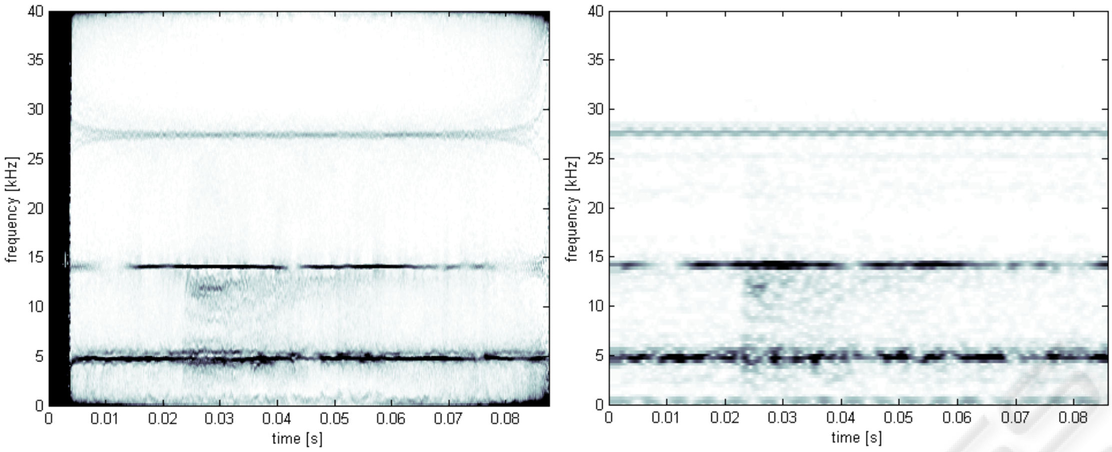

Figure 3: Spectrogram using Kalman estimation (left) and using short-time Fourier transform (right).

REFERENCES

Boashash, B., 1992a. Estimating and Interpreting the Instantaneous Frequency of a Signal – Part 1: Fundamentals. In *Proceedings of the IEEE*, vol. 80, no. 4, pp. 520-538.

Boashash, B., 1992b. Estimating and Interpreting the Instantaneous Frequency of a Signal – Part 2:Algorithms and Applications. In *Proceedings of the IEEE*, vol. 80, no. 4, pp. 540-568.

Cohen, L., 1995. *Time-Frequency Analysis*, Prentice Hall PTR. New Jersey.

Hahn, S.L., 1996. *Hilbert Transforms in Signal Processing*, Artech House. Boston.

Huang, N.E., et al., 1998. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. In *Proc.R.Soc.Lond. A,* vol. 454, no. 1971, pp. 903-995.

Huang, N. E., 2003. A confidence limit for the empirical mode decomposition and the Hilbert spectral analysis. In *Proc. Roy. Soc. Lond.*, 459, pp. 2317-2345, 2003

Rilling, G., Flandrin, P., Goncalves, P., 2003. On Empirical Mode Decomposition and its Algorithms. In *IEEE-EURASHIP workshop on nonlinear signal an image processing NSIP-03*

Maragos, P., Kaiser, J.F., Quantieri, T.F., 1993. On Amplitude and Frequency Demodulation Using Energy Operators. In *IEEE Trans. on Signal Processing,* vol. 41, no. 4, pp. 1532-1550.

Marple, S.L., 1987. *Digital Spectral Analysis*, Prentice Hall. New Jersey.

Vaseghi, S.V., 2000. *Advanced Digital Signal Processing and Noise Reduction*, John Wiley & Sons. New Jersey.

Fairman, F.W., 1998. *Linear Control Theory: The State Space Approach*, John Wiley& Sons. Toronto

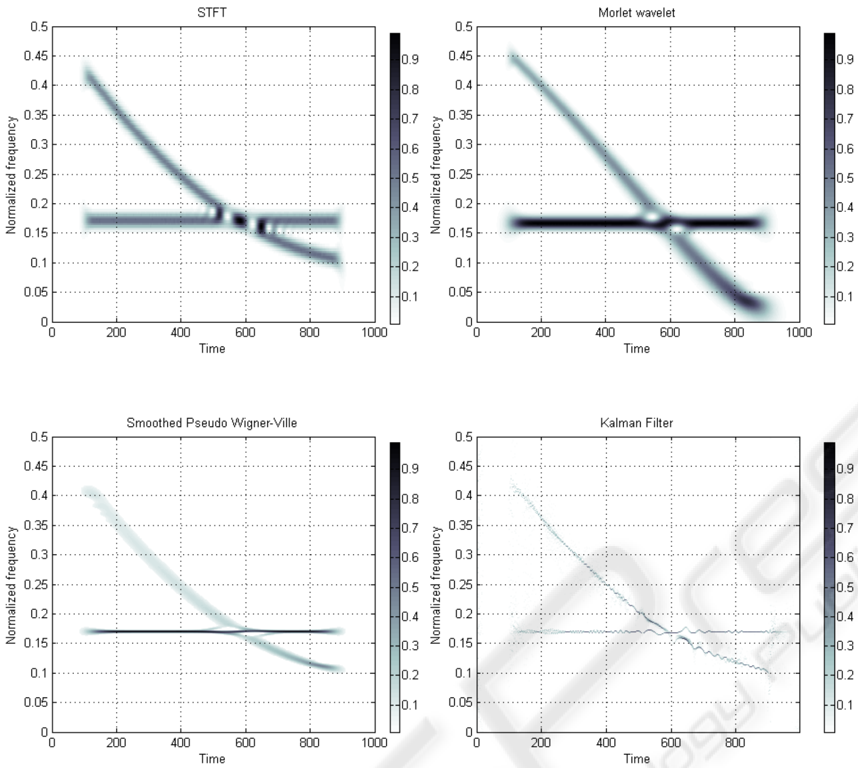

Figure 4: Time-frequency analysis results of the test signal $s_2(t)$.

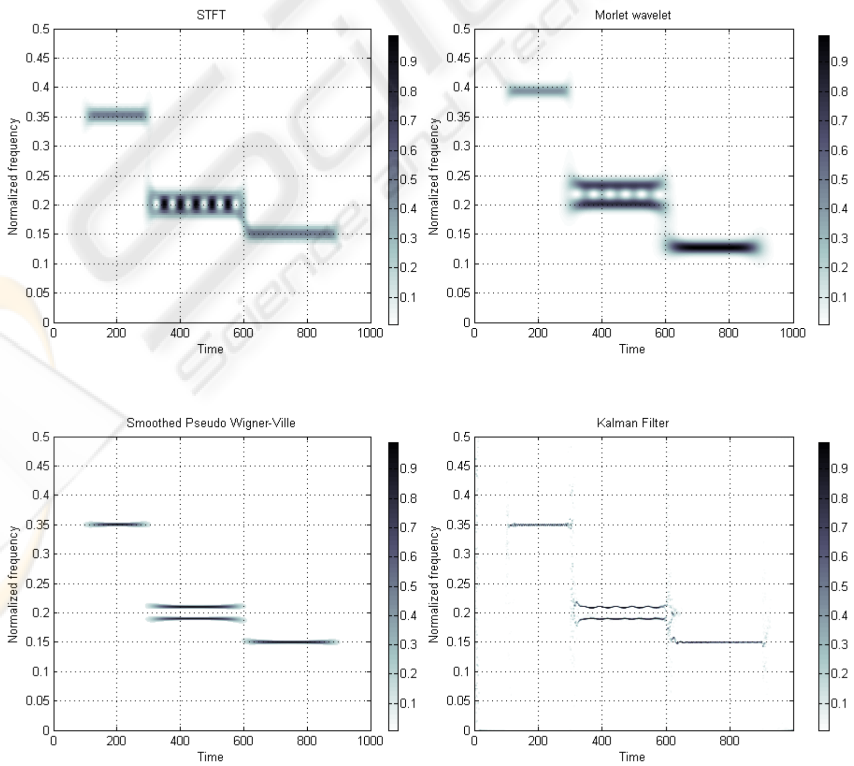

Figure 5: Time-frequency analysis results of the test signal $s_3(t)$.