TAKAGI-SUGENO MULTIPLE-MODEL CONTROLLER FOR A CONTINUOUS BAKING YEAST FERMENTATION PROCESS

Enrique Herrera[1], Bernardino Castillo
Centro de investigación y de Estudios Avanzados del I.P.N., Unidad Guadalajara, Av. Científica 1145, Colonia el bajío C.P. 45010, Zapopan, México

Jesús Ramírez
Centro de Investigación y Asistencia en Tecnología y Diseño del Estado de Jalisco A.C., Av. Normalistas 800 C.P. 44270, Guadalajara, México

Eugénio C. Ferreira
IBB-Institute for Biotechnology and Bioengineering, Centre of Biological Engineering, Universidade do Minho 4710-057, Braga, Portugal

Keywords: Nonlinear system, Fuzzy model, Fuzzy controller, Fermentation process.

Abstract: The purpose of this work is to design a fuzzy integral controller to force the switching of a bioprocess between two different metabolic states. A continuous baking yeast culture is divided in two sub-models: a respiro-fermentative with ethanol production and a respirative with ethanol consumption. The switching between both different metabolic states is achieved by means of tracking a reference substrate signal. A substrate fuzzy integral controller model using sector nonlinearity was built for both nonlinear models.

1 INTRODUCTION

Control applications in bioprocesses have increased in the last decades due to the fast advances on computer and electronic technology. An adequate control of fermentation processes allows reducing production costs and increases the yield, while at the same time achieving the quality of the desired product (Yamuna and Ramachandra 1990).

In the case where the nonlinear model of the process is known, a fuzzy system may be used. A first approach can be done using the Takagi-Sugeno (TS) fuzzy model, (Takagi and Sugeno, 1985), where the consequent part of the fuzzy rules are replaced by linear systems. This can be attained, for example, using the method of sector nonlinearities which allows constructing an exact fuzzy model from the original nonlinear system by means of linear subsystems (Tanaka and Wang, 2001). From this exact model a controller may be designed based on the linear subsystems.

Along this line of reasoning, in this work a fuzzy integral controller based on sector nonlinearities is proposed and applied to a continuous baker's yeast process. An interesting feature of this model is the splitting in two different partial models: a respiro-fermentative (RF) model with ethanol production and the respirative (R) model with ethanol consumption. The fuzzy integral controller is used to force the switching of a bioprocess between both different metabolic states by means of tracking a reference substrate signal.

[1]On Ph. D. studies leave from the Centro de Investigación y Asistencia en Tecnología y Diseño del Estado de Jalisco.

Herrera E., Castillo B., Ramírez J. and C. Ferreira E. (2007).
TAKAGI-SUGENO MULTIPLE-MODEL CONTROLLER FOR A CONTINUOUS BAKING YEAST FERMENTATION PROCESS.
In *Proceedings of the Fourth International Conference on Informatics in Control, Automation and Robotics*, pages 436-439
DOI: 10.5220/0001622704360439

2 FUZZY MODELS BASICS

2.1 Takagi-Sugeno Fuzzy Model

The Takagi-Sugeno fuzzy models are used to represent nonlinear dynamics by means of a set of IF-THEN rules. The consequent parts of the rules are local linear systems. The ith rule of a continuous fuzzy model has the following form:

$$IF\ z_1(t)\ is\ M_1^i\ and ...and\ z_p(t)\ is\ M_p^i$$
$$THEN \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \qquad i = 1,...,r, \end{cases} \tag{1}$$

where M_j^i is a fuzzy set and r is the number of rules in the fuzzy model; $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^q$ is the output vector, $A_i \in R^{nxn}$, $B_i \in R^{nxm}$, y $C_i \in R^{qxn}$ are suitable matrices and $z(t)=[z_1(t),\ldots,z_p(t)]$ is a known vector of premise variables which may depend partially on the state $x(t)$. Given a pair of $(x(t), u(t))$ and a product inference engine the aggregate TS fuzzy model can be inferred as:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\}, \\ y(t) &= \sum_{i=1}^{r} h_i(z(t)) C_i x(t), \end{aligned} \tag{2}$$

where

$$h_i(z(t)) = \prod_{j=1}^{p} M_j^i(z_j(t)) \Bigg/ \sum_{i=1}^{r}\left(\prod_{j=1}^{p} M_j^i(z_j(t))\right),$$

for all t. The term $M_j^i(z_j(t))$ is the membership value of $z_j(t)$ in M_j^i. We have that $h_i(z(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$ for all t and i=1,…,r.

2.2 Parallel Distributed Compensator

The parallel distributed compensator (PDC) is used to design a fuzzy controller from a TS fuzzy model. Each control rule is designed from the corresponding rule of a TS model.

$$IF\ z_1(t)\ is\ M_1^i\ and ...and\ z_p(t)\ is\ M_p^i$$
$$THEN \quad u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) \qquad i = 1,...,r, \tag{3}$$

where F_i is the controller gain for the ith subsystem, which makes Hurwitz the matrices A_i-B_iF_i.

2.3 Integral Control

Consider the linear system

$$\dot{\xi}(t) = A_\xi \xi(t) + B_\xi u_\xi(t) + y_R(t), \tag{4}$$

where $\dot{\xi}, \xi, A_\xi, B_\xi$, $u_\xi(t)$ and y_R are given by

$$\dot{\xi}(t) = \begin{bmatrix} \dot{x} \\ \dot{\sigma} \end{bmatrix}, \quad \xi(t) = \begin{bmatrix} x \\ \sigma \end{bmatrix}, \quad A_\xi = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix},$$
$$B_\xi = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad y_R(t) = \begin{bmatrix} 0 \\ y_R \end{bmatrix}, \quad u_\xi(t) = -Fx + k\sigma$$

where $\dot{\sigma} = e = y_R - y$ is a tracking error and $y_R(t)$ is a reference signal. It is desired to design a state feedback control such that $y(t) \rightarrow y_R(t)$ as $t \rightarrow \infty$ (Khalil, 1996). If the pair (A,B) is controllable and the following condition is achieved

$$rank \begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} = n + p, \tag{5}$$

then it is possible to find a matrix K such that A_ξ-$B_\xi K$ is Hurwitz, assuring that $y(t)$ - $y_R(t) \rightarrow 0$ as $t \rightarrow \infty$; where $K=[-F, k]$ and k must be nonsingular.

3 THE EXACT FUZZY CONTROLLER

A continuous baking yeast culture may be represented by the following nonlinear system $\dot{x}(t) = f_i(x(t)) + Bu(t) + d(x(t))$ where $f_i(x(t))$ describes a respiro-fermentative baking yeast partial model (RF) with ethanol production and a respirative baking yeast partial model (R) with ethanol consumption (Pormeleau, 1990). The RF partial model is described by

RF model

$$f_{RF} = \begin{bmatrix} \frac{x_4}{Ko + x_4} q_o^{\max}\left(Y_{O2} - Y_{r_1}\frac{Y_{O2}}{Y_O}\right) & Y_r q_s^{\max}\frac{x_1}{Ks + x_2} & 0 & 0 \\ \frac{x_4}{Ko + x_4} q_o^{\max}\left(-k_1 Y_{O2} + k_2 Y_{r_1}\frac{Y_{O2}}{Y_O}\right) & -k_2 Y_r q_s^{\max}\frac{x_1}{Ks + x_2} & 0 & 0 \\ -k_3 Y_{r_1} q_o^{\max}\frac{Y_{O2}}{Y_O}\frac{x_4}{Ko + x_4} & k_3 Y_r q_s^{\max}\frac{x_1}{Ks + x_2} & 0 & 0 \\ -k_5 Y_{O2} q_o^{\max}\frac{x_4}{Ko + x_4} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{6}$$

The R model can be divided in two sub models (Ferreira, 1995):

Rqe1 model

$$f_{Rqe1}=\begin{bmatrix} Y_e q_e^{\max} Ki\dfrac{x_3}{(Ke+x_3)(Ki+x_2)} & Y_O q_s^{\max}\dfrac{x_1}{Ks+x_2} & 0 & 0 \\ 0 & -k_1 Y_O q_s^{\max}\dfrac{x_1}{Ks+x_2} & 0 & 0 \\ -k_4 Y_e q_e^{\max} Ki\dfrac{x_3}{(Ke+x_3)(Ki+x_2)} & 0 & 0 & 0 \\ -k_6 Y_e q_e^{\max} Ki\dfrac{x3}{(Ke+x_3)(Ki+x_2)} & -k_5 Y_O q_s^{\max}\dfrac{x_1}{Ks+x_2} & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (7)$$

Rqe2 model

$$f_{Rqe2}=\begin{bmatrix} Y_{O2} e q_o^{\max}\dfrac{x_4}{Ko+x_4} & \dfrac{x_1}{Ks+x_2} q_s^{\max}\left(Y_O - Y_{O2} e\dfrac{Y_O}{Y_{O2}}\right) & 0 & 0 \\ 0 & -k_1 Y_O q_s^{\max}\dfrac{x_1}{Ks+x_2} & 0 & 0 \\ -k_4 Y_{O2} e q_o^{\max}\dfrac{x_4}{Ko+x_4} & k_4 Y_{O2} e q_s^{\max}\dfrac{x_1}{Ks+x_2}\dfrac{Y_O}{Y_{O2}} & 0 & 0 \\ -k_6 Y_{O2} e q_o^{\max}\dfrac{x_4}{Ko+x_4} & \dfrac{x_1}{Ks+x_2} q_s^{\max}\left(-k_5 Y_O + k_6 Y_{O2} e\dfrac{Y_O}{Y_{O2}}\right) & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (8)$$

The input B matrix for all the models is given by

$$B=[-x_1,\ -x_2+S_{in},\ -x_3,\ -x_4]^T \quad (9)$$

where x_1 is the biomass, x_2 is the substrate, x_3 is the ethanol, x_4 is the dissolved oxygen, S_{in} is the inlet substrate concentration, D is the dilution rate, $u(t)=D$. The yield coefficients k_1 *to* k_6 and the remaining parameters values are described in Ferreira, (1995). The oxygen transfer rate (OTR) is assumed to be a measurable and known perturbation, and thus d=[0 0 0 OTR]^T. Before designing a fuzzy controller an exact fuzzy model must be first built.

When the nonlinear dynamic model for the baking yeast is known, as well as all their parameters, a fuzzy exact model can be derived from the given nonlinear model. This requires a sector nonlinearity approach (Tanaka and Wang, 2001). From the models (6-9) the fuzzy exact model can be constructed. The premise variables for the RF partial model (6) and the input B matrix (9) are chosen as:

$$z_1(t)=\frac{x_4}{Ko+x_4} \qquad z_2(t)=\frac{x_1}{Ks+x_2}$$

$$z_{x_1}(t)=x_1,\ z_{x_2}(t)=x_2,\ z_{x_3}(t)=x_3,\ z_{x_4}(t)=x_4.$$

The membership functions can be obtained from $z(t)=\sum_{i=1}^{2} M_i(z(t))a_i$ where the following property $M_1(z(t))+M_2(z(t))=1$ must be accomplished (Tanaka and Wang , 2001). The linear subsystems A_{ijklmn}^{RF}, B_{ijklmn}^{RF} are derived from

$$A_{ijklmn}^{RF}=\begin{bmatrix} a_i q_o^{\max}\left(Y_{O2}-Y_r\dfrac{Y_{O2}}{Y_O}\right) & Y_r q_s^{\max} b_j & 0 & 0 \\ a_i q_o^{\max}\left(-k_1 Y_{O2}+k_2 Y_r\dfrac{Y_{O2}}{Y_O}\right) & -k_2 Y_r q_s^{\max} b_j & 0 & 0 \\ -k_3 a_i Y_r q_o^{\max}\dfrac{Y_{O2}}{Y_O} & k_3 Y_r q_s^{\max} b_j & 0 & 0 \\ -k_5 a_i Y_{O2} q_o^{\max} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (10)$$

$$B_{ijklmn}^{RF}=\left[-c_k,\ -d_l+S_{in},\ -e_m,\ -f_n\right]^T$$

i,j,k,l,m,n = 1,2.

where a_i, b_j, c_k, d_l, e_m, f_n are the maximum and minimum values of $z_1(t)$, $z_2(t)$, $z_{x1}(t)$, $z_{x2}(t)$, $z_{x3}(t)$ and $z_{x4}(t)$ respectively. The following ranges for $x_1(t)\in[0, 10]$, $x_2(t)\in[0, 1]$, $x_3(t)\in[0, 5]$ and $x_4(t)\in[0, 0.007]$ were assumed.

From the model (10) the substrate integral controller for the RF partial model can be designed using the following model:

$$A_j^{RF}=\begin{bmatrix} -k_2 Yr q_s^{\max} b_j & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x_2 \\ \sigma \end{bmatrix} \quad (11)$$

$$B_l^{RF}=\left[-d_l+S_{in},\right]^T$$

From models (10) and (11) the A_{ijklmn}^{RF}, B_{ijklmn}^{RF} matrices for the RF partial integral PDC can be written as

$$A_{ijklmn}^{RF}=\begin{bmatrix} a_i q_o^{\max}\left(Y_{O2}-Y_r\dfrac{Y_{O2}}{Y_O}\right) & Y_r q_s^{\max} b_j & 0 & 0 & 0 \\ a_i q_o^{\max}\left(-k_1 Y_{O2}+k_2 Y_r\dfrac{Y_{O2}}{Y_O}\right) & -k_2 Y_r q_s^{\max} b_j & 0 & 0 & 0 \\ -k_3 a_i Y_r q_o^{\max}\dfrac{Y_{O2}}{Y_O} & k_3 Y_r q_s^{\max} b_j & 0 & 0 & 0 \\ -k_5 a_i Y_{O2} q_o^{\max} & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \sigma \end{bmatrix} \quad (12)$$

$$B_{ijklmn}^{RF}=\left[-c_k,\ -d_l+S_{in},\ -e_m,\ -f_n,\ 0\right]^T$$

i,j,k,l,m,n = 1,2.

A general fuzzy rule to infer all the fuzzy rules for the RF PDC can be stated as:

IF $z_1(t)$ is *"$M_{1i}(z_1(t))$" and* $z_2(t)$ is *"$M_{2j}(z_2(t))$" and* $zx1(t)$ is *"$M_{3k}(z_{x1}(t))$" and* $z_{x2}(t)$ is *"$M_{4l}(z_{x2}(t))$" and* $z_{x3}(t)$ is *"$M_{5m}(z_{x3}(t))$" and* $z_{x4}(t)$ is *"$M_{6n}(z_{x4}(t))$"*

THEN $u^{RF}(t)=-F_{ijklmn}x(t)$ (13)

From (12) and using the notation given by (4) the aggregated fuzzy controller for the RF partial model turns to be

$$\dot{x}^{RF}(t)=\sum_{1}^{64}h_{\psi}(z(t))\left[\left\{A_{ijklmn}^{RF}-B_{ijklmn}^{RF}F_{ijklmn}\right\}\xi(t)+y_R(t)+d\right], \quad (14)$$

where

$$\begin{aligned}\psi &= n+2(m-1)+4(l-1)+8(k-1)\\ &\quad +16(j-1)+32(i-1),\\ h_{\psi}(z(t)) &= M_{1i}(z_1(t))M_{2j}(z_2(t))M_{3k}(z_{x_1}(t))\\ &\quad \times M_{4l}(z_{x_2}(t))M_{5m}(z_{x_3}(t))M_{6n}(z_{x_4}(t))\end{aligned} \quad (15)$$

It has to be noticed that x_1, x_3 and x_4 are not taken into account in the PDC design; this is because these states are not intended to be stabilized but to switch between the RF and R partial models. The fuzzy controller for the models Rqe1 and Rqe2 were constructed following the same procedure.

4 SIMULATION RESULTS

The application of the proposed controller scheme was simulated using MATLAB™. In order to force the switching between the RF and the R baking yeast partial models, the substrate fuzzy controller was forced to track a square reference signal, varied between 0.01 g/l and 0.07 g/l. S_{in} was set to 5 g/l. The behavior of the substrate fuzzy tracking controller as well as the biomass and ethanol behavior are shown in figure 1.

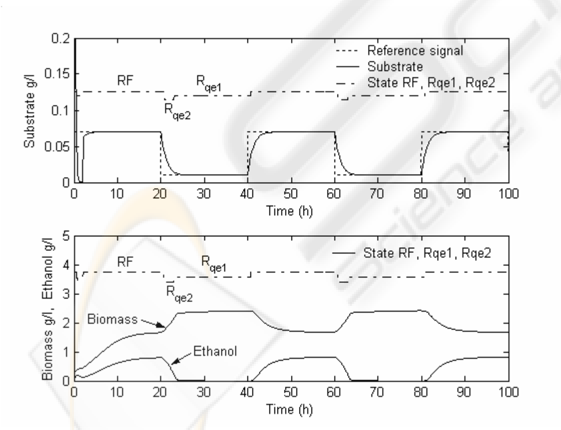

Figure 1: Fuzzy controller performance and biomass and ethanol behavior.

It can be notice that on the RF model, ethanol is produced, limiting the biomass growth; when the Rqe2 partial model is enable the ethanol is consumed promoting biomass growth, and on the Rqe1 state partial model the ethanol is consumed and the biomass is growing just due to the substrate uptake. To test the fuzzy integral controller performance, it was enabled when 2 hours of fermentation elapsed time was accomplished.

5 CONCLUSIONS

Based on the idea of splitting a continuous baking yeast model, a TS fuzzy model was proposed using the sector nonlinearities method, giving an exact representation of the original nonlinear plant. Moreover, a controller for each partial model was constructed. It is worth noting that the controller was capable to force the switching along the partial models. Therefore, the approach presented here may be considered a valid method to design a controller.

ACKNOWLEDGEMENTS

This paper has been supported by the Mexican Consejo Nacional de Ciencia y Tecnología (CONACyT), under grants 46538, 41148 and the Ph.D. Schoolarship 70662.

REFERENCES

Ferreira, E., 1995. *Identificação e controlo adaptivo de processos biotecnológicos*. Ph. D Thesis, Universidade do Porto, Portugal.

Khalil, H. 1996. *Nonlinear Systems*, second edition. Prentice Hall. United States of America.

Pormeleau, Y., 1990. *Modelisation et controle d'un procéde fed-batch de culture des levures á pain (saccharomyces cerevisiae)*. Ph. D. Thesis. Ecole Polytechnique de Montréal, Canada.

Takagi, T., Sugeno, M., 1985. *Fuzzy identification of systems and its applications to modelling and control*. IEEE Trans. Sys, Man Cyber, 15, 116-132.

Tanaka, K., Wang, H., 2001. *Fuzzy control systems design and analysis, a linear matrix inequality approach*. John Wiley & Sons, United States of America.

Yamuna, R., Ramachandra, R., 1999. *Control of fermenters - a review*. Bioprocess Engineering, 21, 77-88.