BELL SHAPED IMPEDANCE CONTROL TO MINIMIZE JERK WHILE CAPTURING DELICATE MOVING OBJECTS

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Abstract: Catching requires the ability to predict the position and intercept a moving object at relatively high speeds. Because catching is a contact task, it requires an understanding of the interaction between the forces applied and position of the object being captured. The application of force to a mass results in a change in acceleration. The rate of change of acceleration is called jerk. Jerk causes wear on the manipulator over time and can also damage the object being captured. This paper uses a curve that asymptotes to zero gradient at +/- infinity to develop an impedance controller, to decelerate an object to a halt after it has been coupled with the end effector. It is found that this impedance control method minimizes the jerk that occurs during capture, and eliminates the jerk spikes that are existent when using spring dampers, springs or constant force to decelerate an object.

1 INTRODUCTION

A catch can be defined as the entire process of intercepting a moving object (by a manipulator), wherein the object becomes attached to the manipulator, and decelerating the object to bring it to a halt. Catching in robotics is an important task since it is an extension to being able to pick up stationary objects. Catching has a wide variety of application areas including manufacturing industries, sports and space robotics. The ability to consistently catch objects can be useful in certain sports like baseball for repeated pitching practice. Catching a ball using a baseball glove (Riley and Atkeson, 2002) and juggling and catching balls (Sakaguchi et. al, 1991, Beuhler et. al 1994) have been studied previously. Burridge et. al (1995), provide an insight into dynamical pick and place robots. This can be useful in picking moving objects randomly from conveyor belts. Most of the literature on catching describes trajectory planning and interception of the object before the catch. The catch itself is generally thought to be an inelastic collision. Minimizing impact during capture and regulating the forces thereafter is important to limit damage to the object.

The task of capturing a moving object by robotic manipulators presents significant difficulties. The process involves being able to accurately predict the moving object's position in time and move the manipulator to the position where it can intercept the object (Sakaguchi et. al, 1991). Once the object has been intercepted, it becomes a part of the manipulator (Kovecses et. al, 1999) and hence, the dynamics of the manipulator change. These need to be taken into consideration during the post-capture phase. It is required to decelerate the object within the allowable workspace of the manipulator (Lin et. al, 1989) to prevent mechanical damage to the system. At the same time, care must be taken to decelerate the object within its permissible limits.

During the capture phase, a certain amount of impact occurs depending on the mismatch in velocities of the manipulator and the moving object. Yoshikawa et. al (1994) present a relationship between the relative velocities between moving objects and the resulting impulse forces and go on to calculate the optimum attitude of arms to minimize mechanical shock. Once the object has been captured, the kinetic energy of the object must be dissipated as work done. This is achieved by decelerating the object over a certain distance. There

are several methods of decelerating an object after capture. A well known method is the use of damped springs. Constant force or springs can also be used in order to perform the same task. The force profile used (models of spring dampers, springs or constant force) is crucial in determining the deceleration and jerk experienced by the object.

During the process of catching, position control of the manipulator is an important task. Although position control can be used to move a manipulator to intercept the object, this alone is insufficient to successfully capture the object. While decelerating the object, it is important to take into account, both the position of the manipulator with respect to its workspace and also the force being applied to decelerate the object. Hogan N (1985) in his three-part paper presents an approach to control the dynamic interaction between the manipulator and its environment. The author states that control of force or position alone is insufficient and that dynamic interaction between the two is required. This is referred to as Impedance Control. Applying force depending on time is inappropriate since it does not ensure that the object is stopped over a certain distance. By applying a force, depending on the position of the object, the method ensures that the moving body is brought to a halt by removing it's kinetic energy over a certain distance.

The first derivative of acceleration is called jerk. Jerk is undesirable as it increases the rate of wear on the manipulator and can also cause damage to the object being captured. It is known to cause vibration and is a measure of impact levels that can excite unmodelled dynamics. This effect is more evident in delicate or flexible structures (Muenchhof and Singh, 2002, Barre et. al, 2005). It has been stated (Kyriakopoulos and Saridis, 1991) that jerk adversely affects the efficiency of the control algorithms and joint position errors increase with jerk. P Huang et. al (2006) in their work state that jerk affects the overall stability of the system and also causes vibrations of the manipulator and hence must be minimized. Macfarlane and Croft (2001) state that jerk limitation results in improved path tracking, reduced wear on the robot and also results in smoothed actuator loads.

In this paper, we assume that the process of tracking and intercepting an object has been completed. We then analyze the use of springs, spring dampers and constant force in decelerating the object during post-capture (once capture has occurred). It is found that these methods result in a high jerk. Hence a method to decelerate an object over a certain distance keeping the jerk to a minimum is proposed. The method establishes a bell shaped impedance relationship between force and position. The results of this method are then compared to the other methods.

2 CAPTURE METHODS

A moving object has a certain amount of kinetic energy associated with it. This is dependant on the mass of the object and its velocity. For a body of mass 'm' kg, travelling with a velocity 'v' m/s, the kinetic energy is given by:

$$Kinetic\ Energy = \tfrac{1}{2}\ m\ v^2 \qquad (1)$$

In order to bring the object to rest, a certain amount of force must be applied in a direction, opposite to that of the motion of the object. For the object to completely come to rest, it is required that the amount of work done be equal to the kinetic energy of the object. The work done is given by:

$$Work\ Done = Force * Displacement \qquad (2)$$

Equating (1) and (2),

$$Force * Displacement = \tfrac{1}{2}\ m\ v^2 \qquad (3)$$

Using equation (3), the force required to decelerate an object over a certain distance can be worked out. This however is a constant force. As the distance over which the object must be decelerated to a halt becomes small, the amount of force to be applied becomes large and vice versa. Since force is directly proportional to acceleration (from Newton's equation $F = m * a$), it follows that the deceleration experienced by an object is greater when the object is brought to a halt over a shorter distance than over a longer distance. Hence, if the maximum deceleration tolerable by a body is known, the distance over which it can be brought to a halt by applying a certain amount of force can be worked out using equation (3). To decelerate the body, force can be applied in different ways. Although force control alone is sufficient to decelerate the object, it is important to take into account, both the position of the object and the force being applied to it (Hogan, 1985). An impedance controller can be used wherein the output force is dependant on the position of the object. This ensures that the amount of deceleration experienced by the object at any position can be kept within predefined limits. Impedance control requires measuring the position of the object, and applying a force depending on the desired impedance. The desired impedance determines the amount of force to be applied depending on the object's position. The

amount of force applied controls the position of the object, thus establishing a dynamic relationship between force and position. Although the term impedance control is usually associated with spring damper response, in a broader sense, the desired impedance can be a constant force, a spring or a spring damper.

3 SIMULATION

The dimensional parameters used in the simulation are mass, velocity and distance. We define the following dimensionless variables in order to perform non dimensional analysis of the results:

$$\hat{x} = \frac{x}{s}; \quad \hat{F} = \frac{F}{\frac{mv^2}{s}}; \quad \hat{a} = \frac{a}{\frac{v^2}{s}}; \quad \hat{j} = \frac{j}{\frac{v^3}{s^2}} \qquad (4)$$

where x is displacement, s is total distance over which body decelerates, m is mass, v is velocity, F is force, a is acceleration and j is jerk.

To compare the above impedance control methods a simulation model was built using Visual Nastran 4D software. This was interfaced to a simulink model of the impedance controller. It involves an object of mass 5 kg, moving with a velocity of 5m/s. It is assumed that the object has been successfully intercepted and coupled to the end effector. A linear actuator is used to decelerate the object. The impedance controller varies the amount of force exerted by the linear actuator depending on the position of the moving object (and the force model – spring. etc). In order to make a fair comparison of the different impedance controllers, it was decided to decelerate the object to a halt over a fixed distance of 2m. The results for each of the methods are discussed below.

3.1 Jerk Analysis - Constant Force

The first model of the impedance controller was designed to exert a constant force to decelerate the object. Because the desired impedance is a constant force irrespective of the position, the requirement for a feedback loop is eliminated. The constant force required was worked out using equation (3). For the chosen values of mass (5kg) and velocity (5m/s), the kinetic energy of the object is 62.5Nm. The distance over which the object must decelerate is given to be 2m. Hence using (3), the force required is 31.25N. This constant force was applied to the moving object

in the simulation. When constant force is used to decelerate the vehicle, the sudden application of force at the point of contact and also the sudden removal of force at the end, result in a jerk. A graph of $\hat{x}$ against $\hat{j}$ is shown in Figure 1. The spikes at the beginning and the end indicate a high jerk at the points of application and removal of the force, and in theory are infinite.
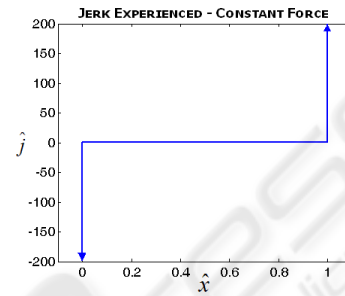

Figure 1: Jerk experienced when constant force is used.

3.2 Jerk Analysis - Spring

In order to minimize the jerk that occurs at the beginning of the capture, it is important that the force being applied gradually increases from zero to a maximum value, with time. This kind of behaviour is characteristic of a spring, since the amount of force applied by the spring is proportional to the displacement of the object. As the spring is compressed, the force being applied increases. This behaviour was simulated using the impedance controller shown in Figure 2. The relation between the force and position (or desired impedance) is given as *Force = Spring Constant * displacement*. The distance over which the body comes to rest is kept the same as before (2m). The spring constant 'k' was chosen to achieve this behaviour by equating the energy of the object to the energy of a spring:

$$\tfrac{1}{2}\ m\ v^2 = \tfrac{1}{2}\ k\ x^2 \qquad (5)$$

where 'k' is the spring constant, and 'x' is the displacement. The kinetic energy of the object is 62.5 Nm. The displacement 'x' is 2m, which is the distance over which the body must decelerate. Using these values in the equation (5), 'k' is found to be 31.25 N/m. The free body diagram equivalent to the resulting system is shown in Figure 3. It must be noted, that using a spring to stop the object over the same distance as before (2m) requires the maximum value of deceleration to be twice as much as when using constant force.
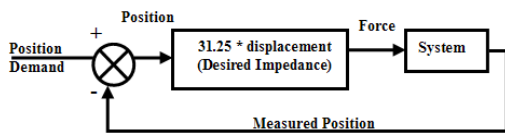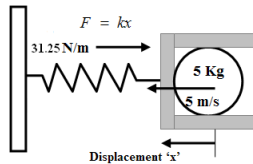
Figure 2: Impedance controller as spring.

Figure 3: Free Body Diagram: Spring system.

The jerk profile when using a spring to decelerate the object is shown in Figure 4. It can be seen that the jerk is zero initially when a spring is used as compared to when applying a constant force. However, at the end, when the body comes to rest, the spring continues to apply a force proportional to the displacement, and stopping the body at that position results in a jerk spike as indicated.
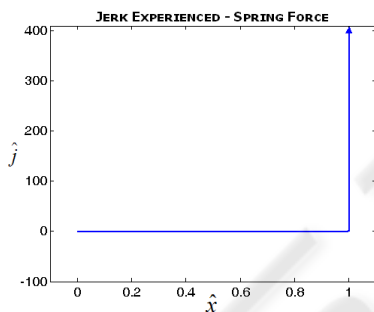

Figure 4: Jerk when spring behaviour impedance is used.

3.3 Jerk Analysis – Spring Damper

In order to eliminate the jerk that occurs towards the end of a spring system, the use of a critically damped spring damper system is considered. The impedance controller for this system is shown in Figure 5. The desired impedance for this system is given by $Force = kx + c\dot{x}$, where 'k' is the spring constant, 'c' is the damping constant, 'x' is the displacement and '$\dot{x}$' is the velocity of the object. The spring constant and damping constant are chosen so that the body decelerates over 2m.
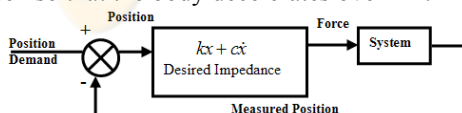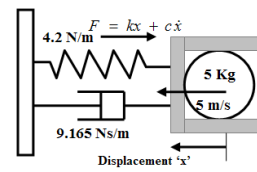

Figure 5: Spring Damper impedance control.

Figure 6: Free Body Diagram: Spring Damper System.

The values of 'c' and 'k' to achieve this are found to be 9.165 Ns/m and 4.2 N/m respectively. The resulting system would then behave as a spring and a damper, the free body diagram of which is shown in Figure 6. The force exerted to stop the object is high initially and gradually decreases when a spring damper is used.
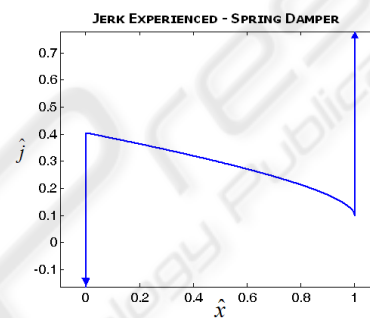

Figure 7: Jerk when spring damper impedance is used.

Because the force is less towards the end, the jerk towards the end is lower (for the chosen sampling interval) than in the case of the spring. However, the large amount of force applied at the beginning results in a high jerk as shown in Figure 7.

4 BELL SHAPED IMPEDANCE CONTROL

From the above analysis of using constant force, spring and a spring damper to decelerate a body, it is immediately clear that jerk is an issue with all the methods. In theory, all these methods cause an infinite amount of jerk on the body, and for the chosen sample interval, a finite but large amount of jerk as shown in the graphs. This jerk can be responsible for an unsuccessful catch as the object may bounce off on impact, or sustain damage. In order to keep the jerk to a minimum, we propose a new method of impedance control, where the relationship between force and position is in the form of a bell curve. The method uses knowledge of statistics and probability distributions to establish the required relationship. The graph of the probability density of a raised cosine distribution is

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in the shape of a bell curve. This knowledge can be used to establish a relationship between the force and position. The probability density function of this distribution is given as:

$$f(x; u, s) = \frac{1}{2s}\left[1 + \cos\left(\frac{x-u}{s}\pi\right)\right] \qquad (6)$$

and is supported in the interval $u - s$ to $u + s$. The amplitude of this distribution is $1/s$ and occurs at u (Figure 8).
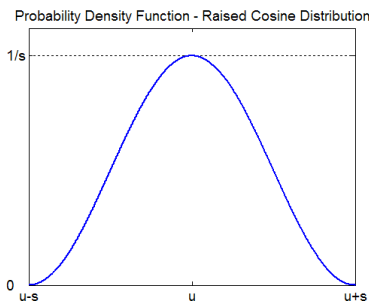

Figure 8: Raised Cosine Distribution - Bell Shaped.

It will be advantageous to establish a relationship between force and position such that the body being captured decelerates over a known distance and experiences a certain maximum deceleration. From the above equation (6), the distance over which the object must decelerate is between u – s and u + s. Hence, u and s are chosen as half the maximum distance. Because the maximum amplitude is dependant on s, a scaling factor is required to achieve the required maximum deceleration for a given distance. Hence, equation (6) is modified to include a scaling factor A chosen such that A/s is the maximum force tolerable. If the maximum deceleration is known, the maximum force tolerable by the object, using Newton's equation is *Force = mass * deceleration*. In order to establish an impedance relationship, a force must be applied depending on the position of the object and hence, equation (6) can be written in terms of force and position as

$$Force = \frac{A}{2s}\left[1 + \cos\left(\frac{Position - u}{s}\pi\right)\right] \qquad (7)$$

Equation (7) results in a force being output depending on the position of the object and ensures that the deceleration of the object is kept within the tolerable limit. It is important to note that the area under this bell curve determines the total work done,

and in order to decelerate the body to a complete halt, this must be equal to the total kinetic energy of the object. The area under this bell curve is 50% of the total area under the rectangle with sides equal to the maximum deceleration and maximum distance over which the body decelerates. This is illustrated in the example that follows. We compare this method to the example used with the spring-damper, spring and constant force methods. The distance over which the body decelerates is 2m. Hence, *u* and *s* are chosen to be 1 and the relative position of the object is from 0m to 2m during which the force is applied to decelerate the object. The maximum amplitude of this curve is however *1/s* which is equal to 1, for the chosen *s*. The area under the curve must be equal to the kinetic energy of the object. For the 5kg mass travelling at 5m/s, the kinetic energy is $62.5 \text{kgm}^2/\text{s}^2$ (or Nm), as established previously. The area under the bell curve is given as *Area = ½ * Force * displacement* where *Force* is worked out using Newton's equation and displacement is the distance over which the body decelerates (50% area as mentioned earlier). Equating this to the kinetic energy of the object, the force required is found to be 62.5N. Hence, *A* must be chosen such that $A/s = 62.5$. Since $s = 1$, $A = 62.5$. Using the calculated values of *A, u* and *s*, the final equation for force, in terms of position or the desired impedance to minimize jerk is implemented.

The force applied to decelerate the object was determined by the impedance relationship established in equation (7). The maximum deceleration experienced by the object is the same as when a spring is used. A graph of force applied using the impedance relationship to decelerate the object against time is shown in Figure 9. Because the position of the object changes faster initially due to its approach velocity, the force required rises steeply at the beginning. The force applied based on the object's position, slows the object down and gradually eases off so as to stop the object over the desired distance of 2m. The jerk profile for this method is shown in Figure 10.
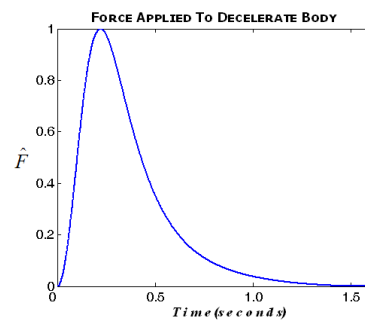

Figure 9: Force applied using Bell Shaped Impedance.

It is a smooth curve, with no spikes and the amount of maximum jerk is very low as compared to any of the other methods. In reality, actuators themselves have inherent dynamics that prevent them from generating instantaneous changes in force. The greater the required instantaneous change in force, the more pronounced the actuator dynamics will become. Therefore, minimum jerk profiles, that limit the required rate of change of force, can be implemented with a greater degree of accuracy.

5 DISCUSSIONS & CONCLUSION

The jerk graphs reveal that the amount of jerk is greatly reduced if a bell shaped curve of force against position is used to capture the object (Figure 10). However, in comparison with the constant force method, the amount of deceleration experienced by the object is high. A trade off between the amount of tolerable jerk and tolerable acceleration is required to be able to generate the required response. An important assumption in this method is that the velocity and mass of the object at the point when capture occurs is known. This ensures that the body decelerates within a certain maximum distance and allows for the force to be specified at every position along its path. Any error in this estimation can result in incorrect calculation of kinetic energy and the object will not stop within the required distance.
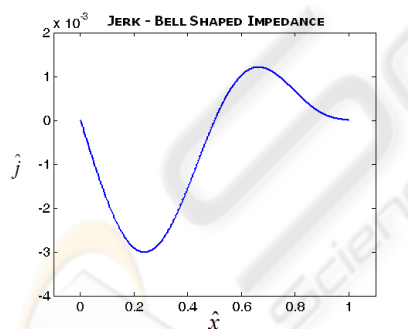

Figure 10: Jerk for Bell Shaped Impedance control.

For accurate calculation, the velocity and mass of the object must be estimated in real time, after which self tuning can be used to generate the required bell shaped impedance control. Additionally, capturing an object requires a high speed of operation and it is much more difficult to apply quick changing forces from actuators at high speeds. The smooth bell shaped acceleration profile also means that forces can be applied with much more ease, due to the gradually changing curve.

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