MODELING AND OPTIMAL TRAJECTORY PLANNING OF A BIPED ROBOT USING NEWTON-EULER FORMULATION

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Abstract: The development of an algorithm to achieve optimal cyclic gaits in space for a thirteen-link biped and twelve actuated joints is proposed. The cyclic walking gait is composed of successive single support phases and impulsive impacts with full contact between the sole of the feet and the ground. The evolution of the joints are chosen as spline functions. The parameters to define the spline functions are determined using an optimization under constraints on the dynamic balance, on the ground reactions, on the validity of impact, on the torques and on the joints velocities. The criterion considered is represented by the integral of the torque norm. The algorithm is tested for a biped robot whose numerical walking results are presented.

1 INTRODUCTION

The design of walking gaits for legged robots and particularly the bipeds has attracted the interest of many researchers for several decades. Due to the unilateral constraints of the biped with the ground and the great number of degrees of freedom, this problem is not trivial. Intuitive methods can be used to obtain walking gaits as in (Grishin et al., 1994). Using experimental data and physical considerations, the authors defined polynomial functions in time for a prototype planar biped. This method is efficient. However to build a prototype and to choose the appropriate actuators or to improve the autonomy of a biped, an optimization algorithm can lead to very interesting results. In (Rostami and Besonnet, 1998) the Pontryagin's principle is used to design impactless nominal trajectories for a planar biped with feet. However the calculations are complex and difficult to extend to the 3D case. As a consequence a parametric optimisation is a useful tool to find optimal motion.

The choice of optimisation parameters is not unique. The torques, the Cartesian coordinates or joint coordinates can be used. Discrete values for the torques defined at sampling time are used as optimization parameters in (Roussel et al., 2003). However it is necessary, when the torque is an optimised variable, to solve the inverse dynamic problem to find the joint accelerations and integrations are used to obtain the evolution of the reference trajectory in velocity and in position. Thus this approach require many calculation : the direct dynamic model is complex and many evaluations of this model is used in the integration process. In (Beletskii and Chudinov, 1977), (Bessonnet et al., 2002), (Channon et al., 1992), (Zonfrilli et al., 2002), (Chevallereau. and Aoustin, 2001) or (Miossec and Aoustin, 2006) to overcome this difficulty, the parametric optimization defines directly the reference trajectories of Cartesian coordinates or joint coordinates for 2D bipeds with feet or without feet. An extension of this strategy is given in this paper for a 3D biped with with twelve motorized joints. The dynamic model is more complex than for a 2D biped, so its computation cost is important in the optimisation process and the use of Newton-Euler method to calculate the torque is more appropriate than the Lagrange method usually used. Since the inverse dynamic model is used only to evaluate the torque for the constraints and criterion calculation, the number of evaluation of the torque can be limited. The desired motion is based on the solution of an optimal problem whose constraints depend on the nonlinear multibody system dynamics of the 12 DoF biped and physical contact constraints with the environment.

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A half step of the cyclic walking gait is composed uniquely of a single support and an instantaneous double support that is modelled by passive impulsive equations. This walking gait is simpler that the human gait, but with this simple model the coupling effect between the motion in frontal plan and sagittal plane can be studied. A finite time double support phase in not considered in this work currently because for rigid modelling of robot, a double support phase can usually be obtained only when the velocity of the swing leg tip before impact is null. This constraint has two effects. In the control process it will be difficult to touch the ground with a null velocity, as a consequence the real motion of the robot will be far from the ideal cycle. Furthermore, large torques are required to slow down the swing leg before the impact and to accelerate the swing leg at the beginning of the single support. The energy cost of such a motion is higher than a motion with impact in the case of a planar robot without feet (Chevallereau. and Aoustin, 2001), (Miossec and Aoustin, 2006).

Therefore a dynamic model is calculated for the single phase. An impulsive model for the impact on the ground with complete surface of the foot sole of the swing leg is deduced from the dynamic model for the biped in double support phase. It takes into account the wrench reaction from the ground. This model is founded on the Newton Euler algorithm, considering that the reference frame is connected to a stance foot. The evolution of joint variables are chosen as a spline function of time instead of usual polynomial functions to prevent oscillatory phenomenon during the optimization process (see (Chevallereau. and Aoustin, 2001), (Saidouni and Bessonnet, 2003) or (L. Hu and Sun, 2006)). The coefficients of the spline functions are calculated as function of initial, intermediate and final configurations and initial and final velocities of the robot which are optimization variables. Taking into account the impact and the fact that the desired walking gait is cyclic, the number of optimization variables is reduced. The criterion considered is the integral of the torque norm. During the optimization process, the constraints on the dynamic balance, on the ground reactions, on the validity of impact, on the limits of the torques, on the joints velocities and on the motion velocity of the biped robot are taken into account. The paper is organized as follows. The 3D biped and its dynamic model are presented in Section II. The cyclic walking gait and the constraints are defined in Section III. The optimization parameters, optimization process and the criterion are discussed in Section IV. Simulation results are presented in Section V. Section VI contains our conclusion and perspectives.

2 MODELS OF THE STUDIED BIPED ROBOT

2.1 Biped Model

We considered an anthropomorphic biped robot with thirteen rigid links connected by twelve motorized joints to form a tree structure. It is composed of a torso, which is not directly actuated, and two identical open chains called legs that are connected at the hips. Each leg is composed of two massive links connected by a joint called knee. The link at the extremity of each leg is called foot which is connected at the leg by a joint called ankle. Each revolute joint is assumed to be independently actuated and ideal (frictionless). The ankles of the biped robot consist of the pitch and the roll axes, the knees consist of the pitch axis and the hips consist of the roll, pitch and yaw axes to constitute a biped walking system of two 2-DoF ankles, two 1-DoF knees and two 3-DoF hips as shown in figure 1. The action to walk associates single support phases separated by impacts with full contact between the sole of the feet and the ground, so that a model in single support, a model in double support and an impact model are derived.

2.2 Geometric Description of the Biped

To define the geometric structure of the biped walking system we assume that the link 0 (stance foot) is the base of the biped robot while link 12 (swing foot) is the terminal link. Therefore we have a simple open loop robot which geometric structure can be described using the notation of Khalil and Kleinfinger (Khalil and Dombre, 2002). The definition of the link frames is given in figure 1 and the corresponding geometric parameters are given in Table I. The frame R_0 coordinates, which is fixed to the tip of the right foot (determined by the width l_p and the length L_p), is defined such that the axis z_0 is along the axis of frontal joint ankle. The frame R_{13} is fixed to the tip of the left foot in the same way that R_0.

2.3 Dynamic Model in Single Support Phase

During the single support phase the stance foot is assumed to remain in flat contact on the ground, *i.e.*, no sliding motion, no take-off, no rotation. Therefore the dynamics of the biped is equivalent to an 12 DoF manipulator robot. Let $q \in \mathbb{R}^{12}$ be the generalized coordinates, where $q_1, \ldots, q_{12}$ denote the relative angles of the joints, $\dot{q} \in \mathbb{R}^{12}$ and $\ddot{q} \in \mathbb{R}^{12}$ are the velocity

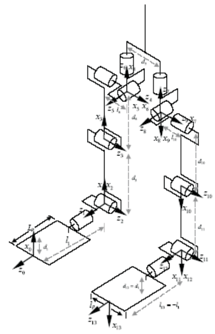


Figure 1: The multi-body model and link frames of the biped robot.

Table 1: Geometric parameters of the biped.

j	$a(j)$	α_j	θ_j	r_j	d_j
1	0	0	q_1	l_1	d_1
2	1	$\frac{\pi}{2}$	q_2	0	0
3	2	0	q_3	0	d_3
4	3	0	q_4	l_4	d_4
5	4	$-\frac{\pi}{2}$	$q_5-\frac{\pi}{2}$	0	0
6	5	$-\frac{\pi}{2}$	q_6	0	0
7	6	0	q_7	0	d_7
8	7	$\frac{\pi}{2}$	$q_8-\frac{\pi}{2}$	0	0
9	8	$-\frac{\pi}{2}$	q_9	0	0
10	9	0	q_{10}	$l_{10}=l_4$	$d_{10}=d_4$
11	10	0	q_{11}	0	$d_{11}=d_3$
12	11	$\frac{\pi}{2}$	q_{12}	0	0
13	12	0	q_{13}	$l_{13}=-l_1$	$d_{13}=d_1$

and acceleration vectors respectively. The dynamic model is computed using the Newton-Euler method (see (Khalil and Dombre, 2002)) represented by the following relation

$$\Gamma = \mathrm{f}(q,\dot{q},\ddot{q},F_t) \quad (1)$$

where $\Gamma \in \mathbb{R}^{12}$ is the joint torques vector and F_t is the external wrench (forces and torques), exerted by the swing foot on the ground. In single support phase $F_t = 0$ and in double support phase $F_t \neq 0$.

In order to denote the dynamic model under the Lagrange form

$$\Gamma = D_s(q)\ddot{q} + H_s \quad (2)$$

with

$$H_s = (C_s(q,\dot{q}) + G_s(q)) \quad (3)$$

the equation (1) is used. In such calculation the matrix D_s and the vector H_s are needed. $C_s \in \mathbb{R}^{12}$ represents the Coriolis and centrifugal forces and $G_s \in \mathbb{R}^{12}$ is the vector of gravity.

The matrix D_s is calculated by the algorithm of Newton-Euler, by noting from the relation (1), (M.W.Walker and D.E.Orin, 1982), that the i^{th} column is equal to Γ if

$$\dot{q} = 0, g = 0, \ddot{q} = e_i, F_t = 0$$

$e_i \in \mathbb{R}^{12\times 1}$ is the unit vector, whose elements are zero except the i^{th} element which is equal to 1.

The calculation of the vector H_s is obtained in the same way that D_s considering that $H_s = \Gamma$ if $\ddot{q} = 0$. Therefore, the dynamic model under the Lagrange form is denoted by the following matrix equations

$$\Gamma = D_s(q)\ddot{q} + H_s(q,\dot{q})$$

where $D_s \in \mathbb{R}^{12\times 12}$ is the symmetric definite positive inertia matrix.

To take easily into account the effect of the reaction force on the stance foot, it is interesting to add 6 coordinates to describe the situation of the stance foot. Newton variables are used for this link, thus its velocity is described by the linear velocity of frame $R_0 : V_0$ and angular velocity ω_0. Since the stance foot is assumed to remain in flat contact, the resultant ground reaction force/moment F_R and M_R are computed by using the Newton-Euler algorithm. $\omega_0 = \mathbf{0}$, $\dot{\omega}_0 = \mathbf{0}$ and $\dot{V}_0 = -g$ are the initial conditions of the Newton-Euler algorithm to take into account the effect of gravity. So, the equation (2) becomes

$$D(X)\dot{V} + C(V,q) + G(X) = D_\Gamma \Gamma + D_R R_{F_R} \quad (4)$$

where $X = [X_0, \alpha_0, q]^T \in \mathbb{R}^{18}$, X_0 and α_0 is the position and the orientation variables of frame R_0, $V = [^0V_0, ^0\omega_0, \dot{q}]^T \in \mathbb{R}^{18}$ and $\dot{V} = [^0\dot{V}_0, ^0\dot{\omega}_0, \ddot{q}]^T \in \mathbb{R}^{18}$. $D \in \mathbb{R}^{18\times 18}$ is the symmetric definite positive inertia matrix, $C \in \mathbb{R}^{18}$ represents the Coriolis and centrifugal forces, $G \in \mathbb{R}^{18}$ is the vector of gravity. $R_{F_R} = [F_R, M_R]^T \in \mathbb{R}^6$ is the ground reaction forces on the stance foot, calculated by the Newton-Euler algorithm, $D_\Gamma = [0_{6\times 12} \mid I_{12\times 12}]^T \in \mathbb{R}^{18\times 12}$ and $D_R = [I_{6\times 6} \mid 0_{12\times 6}]^T \in \mathbb{R}^{6\times 18}$ are constant matrices composed of 1 and 0.

In the optimization process, the torques and force are calculated with the Newton-Euler algorithm and not with the equation (4) . The Newton-Euler is much more efficient from the computation point of view, (Khalil and Dombre, 2002).

2.4 Dynamic Model in Double Support Phase

In double support phase, only the forces and moments of interaction of the left foot with the ground have to be added. Then, the model (4) becomes

$$D(X)\dot{V} + C(V,q) + G(X) + D_f R_f = D_\Gamma \Gamma + D_R R_{F_R} \quad (5)$$

where $R_f \in \mathbb{R}^6$ represents the vector of forces F_{12} and moments M_{12} exerted by the left foot on the ground. This wrench is naturally expressed in frame R_{12} :$^{12}F_{12}$, $^{12}M_{12}$. The virtual work δW_{12} of this wrench is :

$$\delta W_{12} = {}^{12}F_{12}^{T\,12}d_{12} + {}^{12}M_{12}^{12}\delta_{12} \quad (6)$$

where $^{12}d_{12}$ represents an infinitesimal virtual displacement of the link 12 and $^{12}\delta_{12}$ represents an infinitesimal virtual angular displacement. The relation between these virtual displacements, $^{12}d_{12}$ and $^{12}\delta_{12}$, and the virtual joints displacement δq_{12} are the same that between the velocities $^{12}V_{12}, {}^{12}\omega_{12}$ and $\dot{q}_{12}$.

Usually the velocities of link 12 can be expressed as

$$\begin{bmatrix} V_{12} \\ w_{12} \end{bmatrix} = \begin{bmatrix} V_0 + w_0 \times {}^0P_{12} \\ w_{12} \end{bmatrix} + J_{12}\dot{q} \quad (7)$$

where $^0P_{12}$ is the vector linking the origin of frame R_0 and the origin of frame R_{12} expressed in frame R_0, $J_{12} \in \mathbb{R}^{6\times 12}$ is the Jacobian matrix of the robot, $J_{12}\dot{q}$ represents the effect of the joint velocities on the Cartesian velocity of link 12. The velocities V_{12} and w_{12} must be expressed in frame R_{12}, thus we write (7):

$$\begin{bmatrix} {}^{12}V_{12} \\ {}^{12}w_{12} \end{bmatrix} = \begin{bmatrix} {}^{12}A_0 & -{}^{12}A_0^0\hat{P}_{12} \\ 0_{3\times 3} & {}^{12}A_0 \end{bmatrix} \begin{bmatrix} {}^0V_0 \\ {}^0w_0 \end{bmatrix} + {}^{12}J_{12}\dot{q} \quad (8)$$

where $^{12}A_0 \in \mathbb{R}^{3\times 3}$ is the rotation matrix, which defines the orientation of frame R_0 with respect to frame R_{12}. Term $^0\hat{P}_{12}$ is the skew-symmetric matrix of the vector product associated with vector $^0P_{12}$.

$$^0\hat{P}_{12} = \begin{bmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix}$$

Defining matrix $D_f \in \mathbb{R}^{18\times 6}$ as the concatenation of two matrices such that $D_f = [T \;|^{12}J_{12}]^T$, where $^{12}J_{12} \in \mathbb{R}^{6\times 12}$ is the Jacobian matrix of the robot and $T \in \mathbb{R}^{6\times 6}$ equals

$$T = \begin{bmatrix} {}^{12}A_0 & -{}^{12}A_0^0\hat{P}_{12} \\ 0_{3\times 3} & {}^{12}A_0 \end{bmatrix} \quad (9)$$

Then, the linear and angular velocities of the swing foot in frame R_{12} is :

$$\begin{bmatrix} {}^{12}V_{12} \\ {}^{12}w_{12} \end{bmatrix} = D_f^T V \quad (10)$$

Then D_f can be defined by applying the virtual principle on the second leg. However in order to compute the matrix D_f, it is necessary, either to calculate the matrix $^{12}J_{12}$ jacobian by a traditional method, by taking into account the equation (9), or to calculate this matrix by the algorithm of Newton-Euler, by noting from relation (5) that the i^{th} column is equal to $D_\Gamma \Gamma + D_R R_{F_R}$ if

$$\dot{V} = \mathbf{0}, V = \mathbf{0}, g = 0 \text{ and } R_f = e_i$$

$e_i \in \mathbb{R}^{6\times 1}$ is the unit vector, whose elements are zero except the i^{th} element which is equal to 1.

2.5 Impact Equations for Instantaneous Double Support

When the swing foot touches the ground, an impact exists. In reality many possibilities can appear for an impact (partial contact with the sole on the ground, elastic deformations of the bodies and the ground). To simplify our study this impact is assumed to be instantaneous and inelastic with complete surface of the foot sol touching the ground. This means that the velocity of the swing foot touching the ground is zero after its impact. We assume that the ground reaction at the instant of impact is described by a Dirac delta-function with intensity I_{R_f}. Assuming that the previous stance foot is motionless before the impact and does not remains on the ground after the impact the dynamic model during the impact is (see (Formal'sky, 1982) and (M. Sakaguchi and Koizumi, 1995))

$$D(X)\Delta V = -D_f I_{R_f} \quad (11)$$

$$D_f^T V^+ = 0 \quad (12)$$

$$\begin{bmatrix} {}^0V_0^- \\ {}^0w_0^- \end{bmatrix} = \begin{bmatrix} 0_{3\times 1} \\ 0_{3\times 1} \end{bmatrix} \quad (13)$$

where $\Delta V = (V^+ - V^-)$ is the change of velocity caused by the impact and V^+ (respectively V^-) denote the linear and angular velocity of the stance foot and also the joint velocities of the biped after (respectively before) the impact. These equations form a system of linear equations which solution allows to know the impulse forces and the velocity after the impact, thus they can be applied to the biped walking system.

3 DEFINITION OF THE WALKING CYCLE

Because biped walking is a periodical phenomenon our objective is to design a cyclic biped gait. A complete walking cycle is composed of two phases: a single support phase and a double support phase which is modeled through passive impact equations. The single support phase begins with one foot which stays on the ground while the other foot swings from the rear to the front. We shall assume that the double support phase is instantaneous, this means that when the swing leg touches the ground the stance leg takes off. There are two facets to be considered for this problem. The definition of reference trajectories and the method to determine a particular solution of it. This section is devoted to the definition of reference trajectories. The optimal process to choose the best solution of parameters, allowing a symmetric half step, from the point of view of a given criterion will be described in the next section.

3.1 Cyclic Walking Trajectory

Since the initial configuration is a double support configuration, the both feet are on the ground, the twelve joint coordinates are not independent. Because the absolute frame is attached to the right foot we define the situation of the left foot by $(y_{lf}, z_{lf}, \phi_{lf})$ and the situation of the middle of the hips $(x_h, y_h, z_h, \theta_h)$, both expressed in R_0 frame. (y_{lf}, z_{lf}) is the coordinate, in the horizontal plane, of the left foot position, ϕ_{lf} denotes the left foot yawing motion, (x_h, y_h, z_h) is the hip position and θ_h defines the hip pitching motion. The values of the joint variables are solution of the inverse kinematics problem for a leg, which may also be considered as a 6-link manipulator. The problem is solved with a symbolic software, (SYMORO+, see (Khalil and Dombre, 2002)).

In order to deduce the final configuration, we impose a symmetric role of the two legs, therefore from the initial configuration, the final configuration is deduced as:

$$q_{f_{DS}} = E q_{i_{DS}} \tag{14}$$

where $E \in \mathbb{R}^{12\times 12}$ is an inverted diagonal matrix which describes the legs' exchange.

Taking into account the impulsive impact (11)-(13), we can compute the velocity after the impact. Therefore, the velocity after the impact, $\dot{q}^+$, can be calculated when the velocity before the impact, $\dot{q}^-$, is known. The use of the defined matrix E allows us to calculate the initial velocity for the current half step as:

$$\dot{q}_i = E\dot{q}^+. \tag{15}$$

By this way the conditions of cyclic motion are satisfied.

3.2 Constraints

In order to insure that the trajectory is possible, many constraints have to be considered.

3.2.1 Magnitude Constraints on Position and Torque

- Each actuator has physical limits such that

$$|\Gamma_i| - \Gamma_{i,\max} \leq 0, \quad \text{for } i = 1, \ldots, 12 \tag{16}$$

where $\Gamma_{i,\max}$ denotes the maximum value for each actuator.

$$|\dot{q}_i| - \dot{q}_{i,\max} \leq 0, \quad \text{for } i = 1, \ldots, 12 \tag{17}$$

where $\dot{q}_{i,\max}$ denotes the maximum velocity for each actuator.

- The upper and lower bounds of joints for the configurations during the motion are:

$$q_{i,\min} \leq q_i \leq q_{i,\max}, \quad \text{for } i = 1, \ldots, 12 \tag{18}$$

$q_{i,\min}$ and $q_{i,\max}$ stands respectively for the minimum and maximum joint limits.

3.2.2 Geometrical Constraints in Double Support Phase

- The distance $d(hip, foot)$ between the foot in contact with the ground and the hip must remain within a maximal value, *i.e.*,

$$d(hip, foot) \leq l_{hip}. \tag{19}$$

This condition must hold for initial and final configurations of the double support.

- In order to avoid the internal collision of both feet through the lateral axis the heel and the toe of the left foot must satisfy

$$y_{heel} \leq -a \text{ and } y_{toe} \leq -a \tag{20}$$

with $a > \frac{l_p}{2}$ and and l_p is the width of right foot.

3.2.3 Walking Constraints

- During the single support phase to avoid collisions of the swing leg with the stance leg or with the ground, constraints on the positions of the four corners of the wing foot are defined.
- We must take into account the constraints on the ground reaction $R_{F_R} = [R_{F_{Rx}}, R_{F_{Ry}}, R_{F_{Rz}}]^T$ for the stance foot in single support phase as well as impulsive forces $I_{R_f} = [I_{R_{fx}}, I_{R_{fy}}, I_{R_{fz}}]^T$ on the foot touching the ground in instantaneous double support phase. The ground reaction and impulsive forces must be inside a friction cone defined by the friction coefficient μ. This is equivalent to write

$$\sqrt{R_{F_{Ry}}^2 + R_{F_{Rz}}^2} \leq \mu R_{F_{Rx}} \tag{21}$$

$$\sqrt{I_{R_{fy}}^2 + I_{R_{fz}}^2} \leq \mu I_{R_{fx}} \tag{22}$$

The ground reaction forces and the impulsive forces at the contact can only push the ground but may not pull from ground, then the condition of no take off is deduced:

$$R_{f_x} \geq 0 \tag{23}$$

$$I_{R_{fx}} \geq 0. \tag{24}$$

- In order to maintain the balance in dynamic walking, the $ZMP \equiv CoP$, (Zero Moment Point equivalent to the Center of Pressure, see (Vukobratovic and Stepanenko, 1972), point must be within the support polygon, *i.e.*, the distance from CoP to support polygon is negative

$$d(CoP, SP) \leq 0, \tag{25}$$

where SP denotes the support polygon determined by the width l_p and the length L_p of the feet.

4 PARAMETRIC OPTIMIZATION

4.1 The Cubic Spline

To describe the joint motion by a finite set of parameters we choose to use for each joint a piecewise function of the form

$$\begin{aligned} q_i &= \varphi_i(t) = \begin{cases} \varphi_{i1}(t) & if \quad t_0 \leq t \leq t_1 \\ \varphi_{i2}(t) & if \quad t_1 \leq t \leq t_2 \\ \vdots & \\ \varphi_{in}(t) & if \quad t_{n-1} \leq t \leq t_n \end{cases} \\ i &= 1, \ldots 12 \end{aligned}$$

where $\varphi_k(t)$ are polynomials of third-order such that

$$\varphi_{ik}(\mathbf{a}_{ik}, t) = \sum_{j=0}^{3} a_{ikj}(t - t_{k-1})^j, \; k = 1, \ldots, n \; \forall t \in [t_0, t_n] \tag{26}$$

where a_{ikj} are calculated such that the position, velocity and acceleration are always continuous in $t_0, t_1, \ldots t_n$. We used $n = 3$, thus the motion is defined by a specified initial configuration, a final configuration in double support and two intermediate configurations in single support taking into account the initial and final velocity as boundary conditions.

4.2 Optimization Parameters

A parametric optimization problem has to be solved to design a cyclic bipedal gait with successive single and double support phases. This problem depends on parameters to prescribe the two intermediate configurations, q_{int1} and q_{int2}, and the final velocity $\dot{q}_f$ in the single support phase. Taking into account the conditions (14) and (15) the minimal number of parameters necessary to define the joint motion are:

1. Twenty-four parameters are needed to define the two intermediate configurations in single support phase, twelve parameters for the first intermediate configuration $q_{i,int1}$ and twelve parameters for the second intermediate configuration, $q_{i,int2}$ for $i = 1, \ldots, 12$.
2. The velocity before the impact is also prescribed by twelve parameters, $\dot{q}_i^-$ $(i = 1, \ldots 12)$.
3. The left foot yawing motion denoted by ϕ_{lf} and its position (y_{lf}, z_{lf}) in the horizontal plane as well as the situation of the middle of the hips defined by $(x_h, y_h, z_h, \theta_h)$ in double support phase are chosen as parameters.

Let us remark that to define the initial and final configurations in double support nine parameters are required however we define these configurations with only seven parameters. The two others parameters, orientation of the middle of the hips in frontal and transverse plane, are fixed to zero. The duration of a half step, T_s, is fixed arbitrarily.

4.3 Criterion

In the optimization process we consider, as criterion J_Γ, the integral of the norm of the torque divided by the half step length. In other words we are minimizing a quantity proportional to the energy required for a motion

$$J_\Gamma = \frac{1}{d} \int_0^{T_s} \Gamma^T \Gamma dt \tag{27}$$

where T_s is the time of the half step. This general form of minimal energy performance represents the losses by Joule effects to cover distance d.

4.4 Optimization Algorithm

Generally, many values of parameters can give a periodic bipedal gait satisfying constraints (17)-(24). A parametric optimization process, which objective is to minimize J_Γ under nonlinear constraints, is used to find a particular nominal motion. This optimization process can be formally stated as

$$\left.\begin{array}{ll} \text{minimize} & J_\Gamma(p) \\ \text{subject to} & g_i(p) \leq 0 \quad i = 1, 2, \ldots, l \end{array}\right\} \tag{28}$$

where p is the vector of parameters, $J_\Gamma(p)$ is the criterion to minimize with l constraints $g_i(p) \leq 0$ to satisfy. This constraints are given in section 3.2. The nonlinear constrained problem is solved using the Matlab function *fmincon*. This optimization function provide an optimization algorithm based on the Sequential Quadratic Programming (SQP). Therefore, this nonlinear optimization problem with forty-three variables: twenty-four for the two intermediate configurations in single support, twelve for the velocity before the impact and seven to solve the inverse kinematics problem, subject to the constraints given by (17)-(24), is solved numerically.

Table 2: Parameters of SPEJBL.

Physical Parameters	Mass (kg)	Length (m)
Torso	0.3967	$d_7 = 0.140$
Right Leg		
Hip	0.2604	linked to torso
Thigh	0.1224	$d_4 = 0.120$
Shin	0.0558	$d_3 = 0.120$
Ankle	0.1278	$d_1 = 0.042$
Foot	0.3045	$L_p = 0.178$

5 SIMULATION RESULTS

To validate our proposed method, we present the results of an optimal motion for the biped shown in figure 2. The desired trajectory was obtained by the optimization process presented in Section IV, with the minimization of the criterion (27) satisfying the constraints given by (17)-(24). The figure 4 shows the evolution of the optimal motion for a half step with duration, of a single support, which is equal to 0.58 s. For the simulation, we use the physical parameters of the SPEJBL[1]. The physical parameters of SPEJBL are collected in Table 2. Figure 2 shows the photo of SPEJBL and also the dimensional design drawn by VariCAD software.

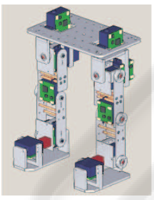


Figure 2: Dimensional drawing of SPEJBL.

The results shown have been obtained with $T_s = 0.58\ s$. The optimal motion is such that the step length is 0.366 m and the optimal velocity is 0.6323 m/s. These values are results of the optimization process.

The normal components of the ground reactions, in function of time, of the stance foot during one half step in single support are presented in figure 3. The average vertical reaction force is 20 N, which is coherent with the weight of the robot which the mass equals 2.1385 Kg. The chosen friction coefficient is 0.7.

The figure 4 shows the evolutions of joint variables $q_i(t)$ $i = 1, \ldots, 12$, defined by the third-order spline function presented in Section III, in the single support phase during one half step. Let us remark that the evolution of each joint variable depends on the boundary conditions ($\dot{q}_{i,ini}$, $\dot{q}_{i,fin}$ for $i = 1, \ldots, 12$) and also on the intermediate configurations ($q_{i,int1}$, $q_{i,int2}$ for $i = 1, \ldots, 12$) whose values are computed in the optimal process.

[1]SPEJBL is a biped robot designed in the Department of Control Engineering of the Technical University in Prague.

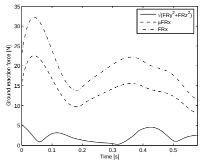
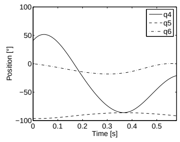
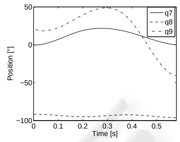


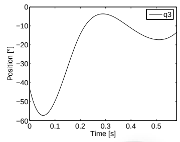
Figure 3: Normal components in the stance foot.



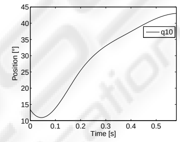
Right hip joint positions



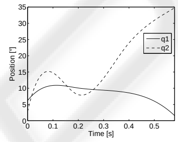
Left hip joint positions



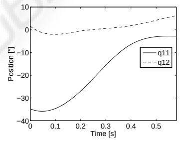
Right knee joint position



Left knee joint position



Right ankle joint positions



Right ankle joint positions

Figure 4: Evolution of joint positions.

The figure 5 shows the CoP trajectory which is always inside the support polygon determined by $l_p = 0.11\ m$ and $L_p = 0.17\ m$., that is, the robot maintains the balance during the motion. Because the minimal distance between of CoP and the boundary of the foot is large, smaller foot is acceptable for this cyclic motion.

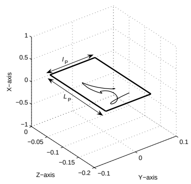


Figure 5: The evolution of CoP trajectory.

For a set of motion velocities, the evolution of J_Γ criterion is presented in figure 6. With respect to the evolution of J_Γ we can conclude that the biped robot

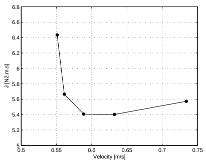


Figure 6: J_Γ in function of several motion velocities for the biped.

consumes more energy for low velocities to generate one half step. Due to the limitations of the joint velocities we could not obtain superior values to 0.73 m/s. The energy consumption increases probably for higher velocity (see (Chevallereau. and Aoustin, 2001)). The robot has been designed to be able to walk slowly, this walk require large torque and small joint velocities. Its design is also based on large feet in order to be able to use static walking, as a consequence the feet are heavy and bulky, thus the resulting optimal motion is close to the motion of a human with snowshoes.

6 CONCLUSION

Optimal joint reference trajectories for the walking of a 3D biped are found. A methodology to design such optimal trajectories is developed. This tool is useful to test a robot design or for the control of the robot. In order to use classical optimization technique, the optimal trajectory is described by a set of parameters: we choose to define the evolution of the actuated relative angle as spline functions. A cyclic solution is desired. Thus the number of the optimization variables is reduced by taking into account explicitly of the cyclic condition. Some inequality constraints such as the limits on torque and velocity, the condition of no sliding during motion and impact, some limits on the motion of the free leg are taken into account. Optimal motion for a given duration of the step have been obtained, the step length and the advance velocity are the result of the optimization process. The result obtained are realistic with respect to the size of the robot under study. Optimal motion for a given motion velocity can also be studied, in this case the motion velocity is consider as a constraint. The proposed method to define optimal motion will be tested on other prototype with dimension closer to human.

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