DYNAMIC SENSOR NETWORKS: AN APPROACH TO OPTIMAL DYNAMIC FIELD COVERAGE

Simone Gabriele and Paolo Di Giamberardino
Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
Università degli Studi di Roma "La Sapienza"
Via Eudossiana 18, Roma, Italia

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Abstract: In the paper a solution to the sensor network coverage problem is proposed, based on the usage of moving sensors that allow a larger fields coverage using a smaller number of devices. The problem than moves from the optimal allocation of fixed or almost fixed sensors to the determination of optimal trajectories for moving sensors. In the paper a suboptimal solution obtained from the sampled optimal problem is given. First, in order to put in evidence the formulation and the solution approach to the optimization problem, a single moving sensor has been addressed. Then, the results for multisensor systems are shown. Some simulation results are also reported to show the behavior of the sensors network.

1 INTRODUCTION

Distributed sensors systems and networks are growing relevance in the scientific and engineering community. Their introduction into several applications for monitoring or surveillance, like for example temperature, ground humidity and solar radiation in farms or parks, presence and distribution of people in critical structures, temperature for fire prevention (buildings as well as woods), and so on, together with the growth of decentralized control in large and complex structures (factories, refineries, energy production and distribution, etc) makes the interest of many researchers for these kind of problems growing and growing, as proved for example by (Akyildiz et al., 2002; Lewis, 2004).

The use of several sensors, suitably deployed, makes the range of measurements as wide as required. Then, one common features required by sensor networks is the full coverage of a given (large) area with the union of each single field of measurement. This problem has been usually faced studying optimal, suboptimal or heuristic solutions to the coverage problem in terms of *good* allocation of sensors in the area under measurement. In other terms, the problem usually has been posed answering the question ""which are the *best* places to put the N sensors?", where *best* is often considered with respect to energetic costs (for the deployment as well as for the communications) or number of sensors.

Such a problem has been well studied in a lot of works, such as (V. Isler and Daniilidis, 2004; Yung-Tsung Hou, 2006; Zou and Chakrabarty, 2004; Lin, 2005; Huang and Tseng, 2005; Meguerdichian et al., 2001).

In (Tan et al., 2004; Howard, 2002) the problem of self-deploying mobile sensors, able to configure according to the environment, is addressed and some solutions are proposed. In these kind of approaches a common fact is the use of a lot of quasi static sensor units to cover a given area.

An alternative idea is to use a reduced number of sensor units moving continuously; such an approach is the one followed by the authors in the present work. A result based on the solution of a suitable coordinated optimal control problem is presented in the sequel. The only limit of this approach is the impossibility of getting a continuous measure for a given point of the area under monitoring, allowing the user only to fix the maximum acceptable time between two consecutive measures of the same point. The problem is than to plan trajectories optimally in sense of area coverage. An optimal control formulation for this problem is proposed in (Wang, 2003). In (Tsai, 2004;

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Cecil and Marthler, 2006) the same problem has been studied in the level set framework and some suboptimal solutions are proposed. An approach based on space decomposition and Voronoy graphs is proposed in (Acar et al., 2006).

In the present work we suggest a customizable optimal control framework that allow the study of a set of different case of the described problem.

The motion problem both for a single sensor and for a set of sensors, under kinematic and dynamic constraints on the motion, with the objective to maximize the area covered during the movement is formulated as an optimal control problem Since this problem, in the general case, cannot be solved analytically, a discretization of space and time is performed, so obtaining a discrete time optimal control problem tractable as a Non Linear Programming (NLP) one. Similar approach to optimal control problems was proposed for industrial manipulators control in (Bicchi and Tonietti, 2004) and for path planning in (Ma and Miller, 2006).

The paper is organized as follows. In Section 2 the mathematical model of the sensor(s) is given, together with the constraints to be satisfied. Model and constraints are then used to propose a formulation for the optimal control problem. In Section 3 the discrete problem, obtained by spatial and temporal discretization, is formulated in terms of a solvable NLP problem. Section 4 is devoted to the particularization of the problem for some cases, showing the respective simulation results. Some final comments in Section 5 end the paper.

2 PROBLEM FORMULATION

2.1 The Mathematical Model

A mobile sensor is modeled, from the dynamic point of view, as a material point of unitary mass, moving on a space $W \subset \mathbb{R}^2$, called the *workspace*, under the action of two independent control input forces named $u_1(t)$ and $u_2(t)$. Then, the position of the sensor in W at time t is described by its Cartesian coordinates $(x_1(t), x_2(t))$. The motion satisfies the well known equations:

$$\begin{aligned} \ddot{x}_1(t) &= u_1(t) \\ \ddot{x}_2(t) &= u_2(t) \end{aligned} \tag{1}$$

The linearity of 1 allows one to write the dynamics in the form

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu(t) \\ y(t) &= Cz(t) \end{aligned} \tag{2}$$

where

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

once the state vector $z(t) = \begin{pmatrix} \dot{x}_1(t), x_1(t), \dot{x}_2(t), x_2(t) \end{pmatrix}^T$ and the output $y(t) = \begin{pmatrix} x_1(t), x_2(t) \end{pmatrix}^T$ are defined. Clearly, $y(t)$ denotes the trajectory followed by the mobile sensor.

If the workspace M is supposed to be a rectangular subset of $\mathbb{R}^2$, the trajectory must satisfy the constraints

$$\begin{aligned} x_{1,min} \le x_1(t) \le x_{1,max} \\ x_{2,min} \le x_2(t) \le x_{2,max} \end{aligned}$$

Moreover, physical limits on the actuators (for the motion) and/or on the sensors (in terms of velocity in the measure acquisition) suggest the introduction of the following additional constraints

$$\begin{aligned} |\dot{x}_1(t)| &\le v_{max} \\ |\dot{x}_2(t)| &\le v_{max} \\ |u_1(t)| &\le u_{max} \\ |u_2(t)| &\le u_{max} \end{aligned}$$

In this work the hypothesis that the mobile sensor at time t can take measures within a circular area of radius ρ around its current position $y(t)$ is considered. Such an area under sensor *visibility* will be denoted as

$$M(t) = \sigma(y(t), \rho)$$

In other words, $M(t)$ denotes the area over which the sensor can take measures at time t.

2.2 The Mathematical Formulation of the Coverage Problem

According to what stated in subsection 2.1, given a time interval $\Theta = [0, t_f]$, the geometric expression of the area covered by the measures during Θ, say M_Θ, can be easily given by

$$M_\Theta = \bigcup_{t \in \Theta} M(t) = \bigcup_{t \in \Theta} \sigma(y(t), \rho) \tag{3}$$

However, such a formulation is not easy to be used in an analytical optimal control problem formulation. Then, alternative expressions that gives in an analytic form how a given trajectory reflects on the space coverage for the sensor measure must be found.

The one used in this work is based on the distance $d(y(t),P)$ between the points $\{P|P \in W\}$ of the workspace and the trajectory.

Once the distance between a point P of the workspace and a given trajectory $y(t)$ is defined as

$$d(y(t),P) = \min_{t\in\Theta} ||y(t)-P|| \tag{4}$$

and making use of the function

$$\text{pos}(\xi) = \begin{cases} \xi & \text{if} \quad \xi > 0 \\ 0 & \text{if} \quad \xi \leq 0 \end{cases} \tag{5}$$

that fixes to zero any nonpositive value, the function

$$\hat{d}(y(t),P,\rho) = \text{pos}\,(d(y(t),P)-\rho) \geq 0$$

can be defined. Then, a measure of how the trajectory $y(t)$ produces a good coverage of the workspace can be given by

$$J(y(t)) = \int_{P\in W} \hat{d}(y(t),P,\rho) \tag{6}$$

Smaller is $J(y(t))$, better is the coverage. If $J(y(t)) = 0$ than $y(t)$ covers completely the workspace.

2.3 The Optimal Control Problem Formulation

Making use of the element introduced in previous subsections, the Optimal Control Problem can be formulated in order to find the best trajectory $y^*(t)$ that maximizes the area covered by sensor measurement during the time interval Θ, as defined in previous subsection, and satisfies the constraints. Then a constrained optimal control problem is obtained, whose form is

$$\begin{aligned} &\min J(\Lambda(u(t))) \\ &f(u(t)) = 0 \\ &g(u(t)) \leq 0 \end{aligned} \tag{7}$$

In (7), the cost functional $J(\cdot)$ is given by (from (6))

$$J(\Lambda(u(t))) = \int_{p\in W} \hat{d}(\Lambda(u(t)),p,\rho) \tag{8}$$

The optimal solution $u(t) = u^*(t)$ $(t \in \Theta)$is the control that produces the optimal trajectory $y^*(t) = \Lambda(u^*(t))$ $(t \in \Theta)$.

In general is not possible to solve analytically the optimal control problem defined in the precedent section, due the functional form of $J(\cdot)$ in (7). In next section a solvable discrete problem is defined.

3 DISCRETE TIME FORMULATION

In order to overcome the difficulty of solving a problem as (7) due to the complexity of the cost function $J(\cdot)$, a discretization is performed, both with respect to space W, and with respect to time in all the time dependent expressions.

The workspace is divided into square cells $c_{i,j}$ with resolution (size) l_{res}, and the trajectory is discretized with sample time T_s. The equations of the discrete time dynamics are:

$$\begin{aligned} &z((k+1)T_s) = A_d z(kT_s) + B_d u(kT_s) \\ &y(kT_s) = Cz(kT_s) \end{aligned} \tag{9}$$

where $A_d = e^{AT_s}$ and $B_d = \int_0^{T_s} e^{A\tau} B d\tau$

The state vector $z(t)$ at the generic time instant $t = NT_s$ depends on the initial state z_0 and on the controls from time $t = 0$ to time $t = (N-1)T_s$

$$z(NT_s) = A_d^N z_0 + \sum_{i=0}^{N-1} A_d^i B_d u((N-1)T_s - iT_s) \tag{10}$$

The following matrices are now defined:

$$Z_N = \begin{bmatrix} z(T_s) \\ \vdots \\ z(NT_s) \end{bmatrix}$$

$$Y_N = \begin{bmatrix} y(0) \\ \vdots \\ y(NT_s) \end{bmatrix} \qquad A_N = \begin{bmatrix} A_d \\ A_d^2 \\ \vdots \\ A_d^N \end{bmatrix}$$

$$B_N = \begin{bmatrix} B_d & 0 & \dots & 0 & 0 \\ A_d B_d & B_d & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_d^{N-1} B_d & A_d^{N-2} B_d & \dots & A_d B_d & B_d \end{bmatrix}$$

$$U_N = \begin{bmatrix} u(0) \\ \vdots \\ u((N-1)T_s) \end{bmatrix} \qquad U_{max} = \begin{bmatrix} u_{max} \\ \vdots \\ u_{max} \end{bmatrix}$$

$$Z_{max} = \begin{bmatrix} v_{max} \\ x_{1,max} \\ v_{max} \\ x_{2,max} \\ -- \\ \vdots \\ -- \\ v_{max} \\ x_{1,max} \\ v_{max} \\ x_{2,max} \end{bmatrix} \qquad Z_{min} = \begin{bmatrix} -v_{max} \\ x_{1,min} \\ -v_{max} \\ x_{2,min} \\ -- \\ \vdots \\ -- \\ -v_{max} \\ x_{1,min} \\ -v_{max} \\ x_{2,min} \end{bmatrix}$$

Making use of such matrices, the sequence of values for the sampled state vector $z(kT_s)$, with $0 \leq K \leq (N+1)$, can be expressed in the simple compact form

$$Z_N = A_N z_0 + B_N U_N \tag{11}$$

The cost function can then be written as:

$$J(Y_N) = \sum_{i=1}^{\nu_x} \sum_{j=1}^{\nu_y} \hat{d}(\Lambda(U_N), c_{i,j}, \rho) \tag{12}$$

where $\nu_x = \frac{(x_{max} - x_{min})}{l_{res}}$, $\nu_y = \frac{(y_{max} - y_{min})}{l_{res}}$ and $\Lambda(U_N) = Y_N$.

3.1 The Nonlinear Programming Problem Formulation

The problem of finding the maximum area coverage trajectory can now be written as a tractable discrete optimization problem with linear inequality and box constraints

$$\begin{aligned} &\min_{U_N} \sum_{i=1}^{\nu_x} \sum_{j=1}^{\nu_y} \hat{d}(\Lambda(U_N), c_{i,j}, \rho) \\ &A_{model} U_N \leq B_{model} \\ &-U_{max} \leq U_N \leq U_{max} \end{aligned} \tag{13}$$

where $A_{model} = \begin{bmatrix} B_N \\ -B_N \end{bmatrix}$

and $B_{model} = \begin{bmatrix} Z_{max} - A_N z_0 \\ -Z_{min} + A_N z_0 \end{bmatrix}$

Suboptimal solutions can be computed using numerical methods. In the simulations performed, the SQP (Sequential Quadratic Programming) method has been applied. The obtained model can be customized according to the specific task, as shown in the following section.

4 MODEL CUSTOMIZATION AND CASE SOLUTIONS

In this section some cases are faced in order to put in evidence the capabilities and he effectiveness of the proposed solution. The values of parameters used in all the simulations are:

$u_{max} = 0.5N$, $v_{max} = 1.5\frac{m}{sec}$, $x_{max} = y_{max} = 4m$, $x_{min} = y_{min} = -4m$, $T_s = 0.5sec$

4.1 The Case of a Single Sensor

Fixed initial state

The formulation adopted allows to find covering trajectories for a single sensor who start from a given initial state. In figure 1 the corresponding simulation results, for $z_0 = \begin{bmatrix} 0 & 0 & 0 & 0. \end{bmatrix}^T$ are depicted. With $t_f = 20sec$ the sensor covers the 70.9% of total area.

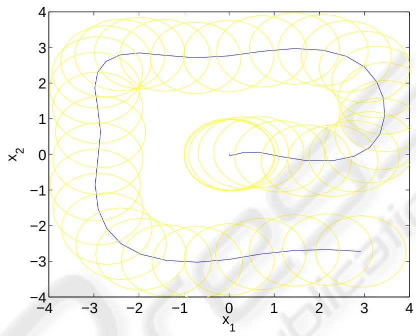

Figure 1: Suboptimal trajectory for one moving sensor with arbitrary starting point ($z = 0$).

Optimal initial state

The initial state z_0 can be included among the set of variables of the optimization problem. In fact, defining

$$V_N = \begin{bmatrix} z(0) \\ u(0) \\ \vdots \\ u((N-1)T_s)) \end{bmatrix}$$

$$H_N = \begin{bmatrix} A_d & B_d & \dots & 0 & 0 \\ A_d^2 & A_d B_d & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_d^N & A_d^{N-1} B_d & \dots & A_d B_d & B_d \end{bmatrix}$$

it is possible to write

$$Z_N = H_N V_N \tag{14}$$

The new optimization problem is then obtained setting

$$A_{model} = \begin{bmatrix} H_N \\ -H_N \end{bmatrix} \quad \text{and} \quad B_{model} = \begin{bmatrix} Z_{max} \\ -Z_{min} \end{bmatrix}$$

Leaving the initial condition free, better results are obtained since the initial state is also optimal, as it is shown in figure 2.

Here in the same time of the precedent simulation ($t_f = 20sec$), the sensor covers 73.49% of total area, versus the 70.9% of the fixed starting state case.

Periodic trajectory

Cyclic trajectories can be very useful in area monitoring or surveillance tasks because this choice, once

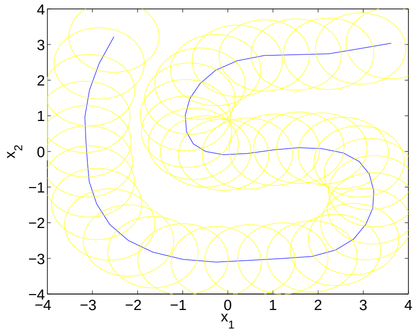

Figure 2: Suboptimal trajectory for one moving sensor with suboptimal starting point.

the maximum time NT_S between measures on the same point is fixed, allows to repeat the measure in the same point periodically.

According to the present formulation, the sampled dynamics over N sampled instants has a periodic behavior if and only if

$$z((N+1)T_s) = z(0) \tag{15}$$

Observing that the computation of the $(N+1)-th$ sampled values for the state gives

$$z((N+1)T_s) = [\ A_d^{N+1} \quad \ldots \quad A_d B_d \quad B_d\ ] V_N$$

while

$$z(0) = [\ I \quad 0 \quad \ldots \quad 0 \quad 0\ ] V_N$$

condition (15) can be rewritten as

$$[\ A_d^{N+1} - I \quad \ldots \quad A_d B_d \quad B_d\ ] V_N = 0 \tag{16}$$

Equation (16) must be added as a new constraint in the optimization problem in order to get periodic solutions.

The figure 3 shows the results obtained by simulations for this case.

With $t_f = 40sec$ the $98,17\%$ of the workspace area is covered.

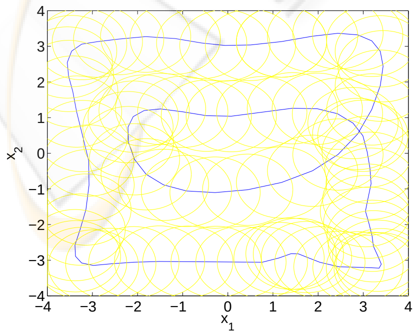

Figure 3: Sub-optimal trajectory for one moving sensor.

4.2 The Case of Multiple Sensors

The models shown above are very easily extended to the case under interest of area coverage with multiple moving sensors. The use of multiple sensors instead of one allows to reduce the time t_f within the same coverage or, equivalently, increase the coverage for the same t_f.

If n is the number of the moving sensors, the optimization problem can be formulated in the same way once the following matrices are defined and used instead of the corresponding ones:

$$U_N^n = \begin{bmatrix} U_{N,1} \\ \vdots \\ U_{N,n} \end{bmatrix}$$

where $U_{N,i}$ stands for the control set (U_N) of the $i-th$ sensor.

$$A_{model}^n = \begin{pmatrix} A_{model,1} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & A_{model,n} \end{pmatrix}$$

$$B_{model}^n = \begin{pmatrix} B_{model,1} \\ \vdots \\ B_{model,n} \end{pmatrix}$$

where $A_{model,i}$ and $B_{model,i}$ are the A_{model} and the B_{model} matrices of the $i-th$ single sensor model.

In figure 4 the result for the multi-sensor case with $n = 2$ is depicted. In time $t_f = 25sec$ the 99.86% of the workspace area is covered. The gain of time respect to the single sensor case is evident.

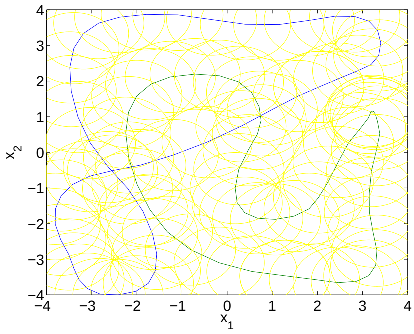

Figure 4: Sub-optimal cyclic trajectories for moving sensor 1 (blue) and moving sensor 2 (green), the yellow circles show the measures area.

5 CONCLUSIONS AND FUTURE WORKS

In the present paper a measurement system composed by several sensors moving within the area under measure has been considered. This system has been called *dynamic sensor network*. For this kind of system the formulation for an optimal solution to the area coverage problem has been provided. The complexity of the cost function makes very hard (actually impossible) the computation of the optimal solution. Then, in order to get a solution, a sampled model has been considered, bringing to a nonlinear programming problem that has been solved numerically. The results for a single sensor with different choices for initial conditions (freely given or optimal) and for behavior of the trajectory (non periodic or periodic) show the effectiveness of the proposed procedure. The case of a n-sensors systems has also been considered and, for $n = 2$ has been simulated in order to show the results when more sensors are present. The problem at present under investigation concerns the inclusion of non collision constraints, where non collisions are to be considered both between moving sensors and with fixed obstacles that can be present in the measurement area.

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