HOMOGRAPHY-BASED MOBILE ROBOT MODELING FOR DIGITAL CONTROL IMPLEMENTATION

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Abstract: The paper addresses the development of a kinematic model for a system composed by a nonholonomic mobile robot and a camera used as feedback sensor to close the control loop. It is shown that the proposed homography-based model takes a particular form that brings to a finite sampled equivalent model. The design of a multirate digital control is then descibed and discussed, showing that exact solutions are obtained. Simulation results are also reported to put in evidence the effectiveness of the proposed approach.

1 INTRODUCTION

In several applications a digital implementation of a controller is required even if the plant dynamics is a continuous time one.

The classical approach, especially for nonlinear systems, makes use of a preliminary design of a continuous time control law that is then implemented in a digital way by means of a digital device (computer or microcontroller, for example). In this approach the digital control law is computed as an approximation of the continuous time one. This kind of approach is usually denoted as *indirect digital control*.

Such an approach can lead to poor controller performance due to the system approximation performed: the choice of the sampling time affects the capacity of the digital controller, that generate a piecewise constant signal, to have a behavior close enough to the nominal continuous time controller that assures the desired performances with a continuous time output signal.

All the works done in the field of visual servoing, deal with continuous time systems (see, for example, in the case of mobile robots: (Chen et al., 2006; Lopez-Nicolas et al., 2006; Mariottini et al., 2006)). Image acquisition, elaboration and the nonlinear control law computation are very time consuming tasks. So it is not uncommon that the system is controlled, in a indirect digital control context, with a sampling rate of $0.5Hz$. Clearly this kind of choices strongly affects the system performances, since the use of a slow rate in the controller implies that only a slow evolution of the control system can be required to assure satisfactory behaviors.

To face the drawbacks of poor system approximation, the trivial solution is slowing down the system dynamics slowing down the controls variation rate. This is not always possible, but when the kinematic model is considered, the system dynamics depends only on the velocity imposed by the controller (there is no *drift* in the model) and so it is quite easy to limit the effect of a poor system approximation.

In general, taking into account, also in the design phase, the discrete time nature of the control system can lead to better closed loop system performances. This means that the control law designed *directly* in the discrete time domain can better address the discrete time evolution of the controlled system. This is well understood in the linear case and it is also true for a quite large class of nonlinear systems, the ones that admits a finite or an exact sampled representation (Monaco and Normand-Cyrot, 2001). In fact, a necessary step in the design of a digital controller is to deal with a discrete time plant; if the plant is described by means of a continuus time dynamics, a discretization of the plant is required. The possibility of an exact discretization, better if finite, of the continuous time model clearly affects the overall performances of the

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control system since in this case no approximations are performed in the conversion from continuous time to discrete time.

2 THE CAMERA-CYCLE MODEL

In this section the kinematic model of the system composed of a mobile robot (an unicycle) and a camera, used as a feedback sensor to close the control loop, is presented.

To derive the mobile robot state, the relationship involving the image space projections of points that lie on the floor plane, taken from two different camera poses, are used. Such a relationship is called *homography*. A complete presentation of such projective relations and their properties is shown in (R. Hartley, 2003).

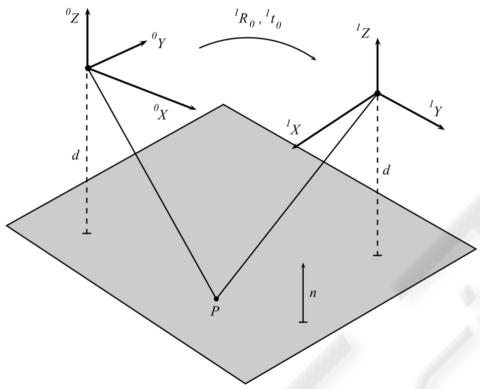

Figure 1: 2-view geometry induced by the mobile robot.

2.1 The Geometric Model

With reference to Figure 1, the relationship between the coordinates of the point P in the two frames is

$$\begin{bmatrix} ^1P \\ 1 \end{bmatrix} = \begin{bmatrix} ^1R_0 & ^1t_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} ^0P \\ 1 \end{bmatrix} \tag{1}$$

It is an affine relation that becomes a linear one in the homogeneous coordinate system. If the point P belongs to a plane in the 3D space with normal versor n and distance from the origin d, it holds that

$$\begin{bmatrix} n^T & d \end{bmatrix} \begin{bmatrix} ^0P \\ 1 \end{bmatrix} = 0 \quad \Rightarrow \quad -\frac{n^{T\,0}P}{d} = 1 \tag{2}$$

note that $d > 0$, since the interest is in the planes observed by a camera and so they don't pass trough the optical center (that is the camera coordinate system origin).

Combining the Equation 1 and the right term of 2, the following relation holds

$$^1P = \left(^1R_0 - \frac{1}{d} {}^1t_0 n^T \right) {}^0P = H^0P \tag{3}$$

The two frame systems in Figure 1 represent the robot frame after a certain movement on a planar floor. Choosing the unicycle like model for the mobile robot, the matrix H become

$$H = \begin{bmatrix} \cos\theta & \sin\theta & \frac{1}{d}(X\cos\theta + Y\sin\theta) \\ -\sin\theta & \cos\theta & \frac{1}{d}(-X\sin\theta + Y\cos\theta) \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

is the homography induced by the floor plane during a robot movement between two allowable poses. Note that $[X, Y, \theta]^T$ is the state vector of the mobile robot with reference to the first coordinate system.

2.2 The Kinematic Model

Taking four entries of matrix H such that

$$\begin{aligned} h_1 &= \cos\theta \\ h_2 &= \sin\theta \\ h_3 &= x\cos\theta + y\sin\theta \\ h_4 &= -x\sin\theta + y\cos\theta \end{aligned} \tag{5}$$

and noting that, for the sake of simplicity, the distance d has been chosen equal to one, since it is just a scale factor that can be taken into account in the sequel, the kinematic unicycle model is

$$\begin{aligned} \dot{X} &= v\cos\theta \\ \dot{Y} &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{6}$$

where v and ω are, respectively, the linear and angular velocity control of the unicycle.

Differentiating the system in Equation 5 with respect to the time and combining it with the system in Equation 6, one obtains

$$\begin{aligned} \dot{h}_1 &= -h_2\omega \\ \dot{h}_2 &= h_1\omega \\ \dot{h}_3 &= h_4\omega + v/d = h_4\omega + {}_d v \\ \dot{h}_4 &= -h_3\omega \end{aligned} \tag{7}$$

that is the kinematic model of the homography induced by the mobile robot movement.

3 FROM CONTINUOUS TO DISCRETE TIME MODEL

In the first part of this section, it will be presented how to derive a the discrete time system model from the continuous time one. Afterwards, a control (and planning) strategy for the discrete time system is introduced and applied. Simulations will be presented to prove the presented strategy effectiveness.

3.1 The General Case

Suppose a system such that

$$\dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) \tag{8}$$

with $f, g_1, \ldots, g_m : M \to \mathbb{R}^n$, analytical vector fields.

To derive a discrete time system from the previous one, suppose to keep constant the controls $u_1, \ldots, u_m$, by means of a zero order holder, for $t \in [kT, (k+1)T)$ and $k \in \mathbb{N}$. Suppose that the system output is sampled (and acquired) every T seconds, too. The whole system composed by a z.o.h, the system and the sampler is equivalent to a discrete time system.

Following (Monaco and Normand-Cyrot, 1985; Monaco and Normand-Cyrot, 2001), it is possible to characterize the discrete time system derived by a continuous time nonlinear system. Sampling the system in Equation 8 with a sampling time T, the discrete time dynamics becomes

$$\begin{aligned} x(k+1) &= x(k) + T\left(f + \sum_{i=1}^{m} u_i(k) g_i\right)\Bigg|_{x(k)} + \\ &+ \frac{T^2}{2}\left(L_{f+\sum_{i=1}^{m} u_i[k] g_i}\right)^2 (Id)\Bigg|_{x(k)} + \ldots \\ &= F^T(x(k), u(k)) \end{aligned} \tag{9}$$

where $L_{(\cdot)}$ denotes the Lie derivative. It is possible to see that, this series is locally convergent choosing an appropriate T. See (Monaco and Normand-Cyrot, 1985) for details.

The problem here is the analytical expression of $F^T(x(k), u(k))$. It can happen that it is not possible to compute it. Otherwise, if from the series of Equation 9 it is possible to derive an analytical expression for its limit function, the system of Equation 8 is said to be *exactly discretizable* and its limit function is called an *exact sampled representation* of 8. If, better, the series results to be finite, in the sense that all the terms from a certain index on goes to zero, a *finite sampled representation* is obtained.

Finite sampled representations are transformed, under coordinates changes, into exact sampled ones ((Monaco and Normand-Cyrot, 2001)). As obvious, finite discretizability is not a coordinate free property while exact discretizability is. Note that the existence of an exact sampled representation corresponds to analytical integrability.

The possibility of getting an exact sampled representation is related to the existence of a state feedback which allow that ((Di Giamberardino et al., 1996b)).

A nonholonomic system as the one of Equation 8, can be transformed into a chained form system by means of a coordinate change.

This leads to a useful property for discretization pointed out in (Monaco and Normand-Cyrot, 1992). In fact it can be seen that a quite large class of nonholonomic systems admit exact sampled models (polynomial state equations). Among them, one finds the chained form systems which can be associated to many mechanical systems by means of state feedbacks and coordinates changes.

3.2 The Camera-Cycle Case

Suppose the controlled inputs are piecewise constant, such that

$$\begin{cases} v(t) = v_k \\ \omega(t) = \omega_k \end{cases} \quad t \in [kT, (k+1)T) \tag{10}$$

where $k = 0, 1, ..$ and T is the sampling period.

Since the controls are constant, it is possible to integrate the system in Equation 7 in a linear fashion. It yields to

$$\begin{aligned} \begin{bmatrix} h_1(k+1) \\ h_2(k+1) \end{bmatrix} &= A(\omega_k) \begin{bmatrix} h_1(k) \\ h_2(k) \end{bmatrix} \\ \begin{bmatrix} h_3(k+1) \\ h_4(k+1) \end{bmatrix} &= A(\omega_k)^T \begin{bmatrix} h_3(k) \\ h_4(k) \end{bmatrix} + B(\omega_k)_d v_k \end{aligned} \tag{11}$$

where

$$\begin{aligned} A(\omega_k) &= \begin{bmatrix} \cos\omega_k T & -\sin\omega_k T \\ \sin\omega_k T & \cos\omega_k T \end{bmatrix} \\ B(\omega_k) &= \begin{bmatrix} \sin(\omega_k T)/\omega_k \\ (\cos(\omega_k T) - 1)/\omega_k \end{bmatrix} \end{aligned} \tag{12}$$

If one considers the angular velocity input as a time varying parameter, the system in Equation 11 become a linear time varying system. Such a property allows an easy way to compute the evolution of the system. Precisely, its evolution becomes

$$\begin{bmatrix} h_1(k) \\ h_2(k) \end{bmatrix} = \prod_{i=0}^{k-1} A(\omega_{k-i-1}) \begin{bmatrix} h_1(0) \\ h_2(0) \end{bmatrix}$$

$$\begin{bmatrix} h_3(k) \\ h_4(k) \end{bmatrix} = \prod_{i=0}^{k-1} A(\omega_{k-i-1})^T \begin{bmatrix} h_3(0) \\ h_4(0) \end{bmatrix} +$$

$$+ \sum_{j=0}^{k-2} \left(\prod_{i=j+1}^{k-1} A(\omega_{k-i})^T \right) B(\omega_j)\, {}_d v_j + B(\omega_{k-1})\, {}_d v_{k-1} \tag{13}$$

where the sequences $\{v_k\}$ and $\{\omega_k\}$ are the control inputs. This structure will be useful in the sequel for the control law computation.

4 CONTROLLING A DISCRETE TIME NONHOLONOMIC SYSTEM

Interestingly, difficult continuous control problems may benefit of a preliminary sampling procedure of the dynamics, so approaching the problem in the discrete time domain instead of in the continuous one.

Starting from a discrete time system representation, it is possible to compute a control strategy that solves steering problems of nonholonomic systems. In (Monaco and Normand-Cyrot, 1992) it has been proposed to use a *multirate digital control* for solving nonholonomic control problems, and in several works its effectiveness has been shown (for example (Chelouah et al., 1993; Di Giamberardino et al., 1996a; Di Giamberardino et al., 1996b; Di Giamberardino, 2001)).

4.1 Camera-cycle Multirate Control

The system under study is the one in Equation 11 and the form of its state evolution in Equation 13.

The problem to face is to steer the system from the initial state $h_0 = [1,0,0,0]^T$ (obviously corresponds to the origin of the configuration space of the unicycle) to a desired state ${}^d h$, using a multirate controller.

If r is the number of sampling periods chosen, setting the angular velocity constant over all the motion, one gets for thestate evolution

$$\begin{bmatrix} h_1(r) \\ h_2(r) \end{bmatrix} = A^r(\bar{\omega}) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} h_3(r) \\ h_4(r) \end{bmatrix} = A^{r-1}(\bar{\omega}) B(\bar{\omega})\, {}_d v_0 + \tag{14}$$

$$+ A^{r-2}(\bar{\omega}) B(\bar{\omega})\, {}_d v_1 + \ldots + B(\bar{\omega})\, {}_d v_{r-1}$$

At this point, given a desired state, one just need to compute the controls. The angular velocity $\bar{\omega}$ is firstly calculated such that:

$$\begin{bmatrix} {}^d h_1 \\ {}^d h_2 \end{bmatrix} = A^r(\bar{\omega}) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{15}$$

Once $\bar{\omega}$ is chosen, the linear velocity values ${}_d v_0, \ldots, {}_d v_{r-1}$ can be calculated solving the linear system

$$\begin{bmatrix} {}^d h_3 \\ {}^d h_4 \end{bmatrix} = R \begin{bmatrix} {}_d v_0 \\ {}_d v_1 \\ \ldots \\ {}_d v_{r-1} \end{bmatrix} \tag{16}$$

$$R = \begin{bmatrix} A^{k-1}(\bar{\omega}) B(\bar{\omega}) & \ldots & B(\bar{\omega}) \end{bmatrix} \tag{17}$$

which is easily derived from the second two equations of 14.

Note that, for steering from the initial state to any other state configuration, at least $r = 2$ steps are needed, except for the configuration that present the same orientation of the initial one. More precisely, it can be seen that if this occurs, the angular velocity $\bar{\omega}$ is equal to zero or Π/T and the matrix R in Equation 16 become singular. Exactly that matrix shows the reachability space of the discrete time system: if $\bar{\omega} \neq \{0, \Pi/T\}$ then the vector B is rotated r-times and the whole configuration space is spanned.

Furthermore, if 2 is the minimum multirate order to guarantee the reachability of any point of the configuration, one can choose a multirate order such that $r > 2$ and the further degrees of freedom in the controls can be used to accomplish the task obtaining a smoother trajectory or avoiding some obstacles, for instance. Note that it can be achieved solving a quadratic programming problem as

$$\min_{V \in \mathbb{V}} \frac{1}{2} V^T \Sigma V + \Gamma V \tag{18}$$

where Σ and Γ are two weighting matrixes, such that the robot reaches the desired pose, granting some optimal objectives. Other constraints can be easily added to take account of further mobile robot movements requirements.

4.2 Closing the Loop with the Planning Strategy

Let $[{}^d\theta, {}^dh_3, {}^dh_4]^T$ be the desired system state and mark the actual one with the subscript k . Note that, in the control law development, the orientation of the mobile robot is used, instead of the first two components of the system of Equation 11.

Summarize the algorithm steps as

0. Set $r_k = r$

1. Choose $\bar{\omega} = ({}^d\theta - \theta_k)/r_kT$

2. Compute the control sequence V as $\min_V V^TV$ such that $RV = \begin{bmatrix} {}^dh_3 \\ {}^dh_4 \end{bmatrix}$ with the same notation of Equations 16 and 18

3. If $r_k > 2$ then $r_k = r_k - 1$ and go to step 1. Otherwise, the algorithm stops.

The choice of the cost function shown leads to a planned path length minimization. If the orientation error of the point 1 is equal to zero, it needs to be perturbed in order to guarantee some solution admissibility to the programming problem of point 2.

Furthermore, since the kinematic controlled model derives directly from an homography, it is possible use the homographies compositional property to easily update the desired pose, from the actual one, at every control computation step. Exactly, since

$$ {}^dH_0 = {}^dH_{r-1}{}^{r-1}H_{r-2} \ldots {}^kH_{k-1} \ldots {}^1H_0 \qquad (19) $$

it is possible to easily update the desired pose as needed for the close loop control strategy.

A Simulated path is presented in Figures 2: the ideal simulated steer execution is perturbed by the presence of some additive noise in the controls. This simulate the effect of some non ideal controller behavior (wheel slipping, actuators dynamics, ...). The constrained quadratic problems involved in the controls computation are solved using an implementation of the algorithm presented in (Coleman and Li, 1996)

5 CONCLUSION

In this paper, a kinematic model for a system composed by a mobile robot and a camera, has been presented. Since such a model is exactly discretizable, it has been possile to propose a multirate digital control strategy able to steer the system to a desired pose in an exact way. The effectiveness of the control scheme adopted has been verified by simulations.

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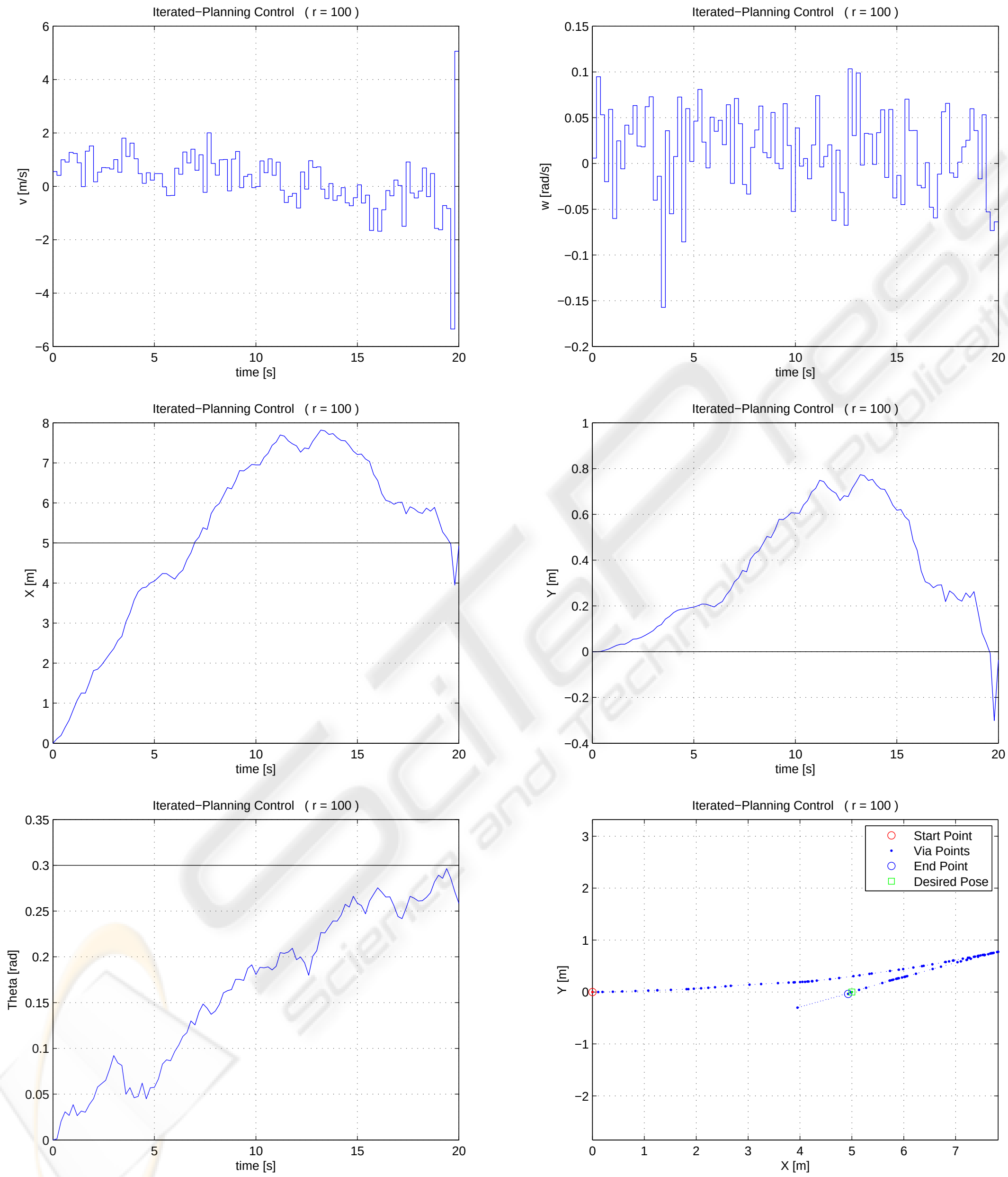

Figure 2: Multirate control simulation ($d = 1m$). Additive random noise on controls (gaussian with std.dev. 0.5 and 0.05, for v and ω respectively).