INTERCEPTION AND COOPERATIVE RENDEZVOUS BETWEEN AUTONOMOUS VEHICLES

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Abstract: The rendezvous problem between autonomous vehicles is formulated as an optimal cooperative control problem with terminal constraints. A major approach to the solution of optimal control problems is to seek solutions which satisfy the first order necessary conditions for an optimum. Such an approach is based on a Hamiltonian formulation, which leads to a difficult two-point boundary-value problem. In this paper, a different approach is used in which the control history is found directly by a genetic algorithm search method. The main advantage of the method is that it does not require the development of a Hamiltonian formulation and consequently, it eliminates the need to deal with an adjoint problem. This method has been applied to the solution of interception and rendezvous problems in an underwater environment, where the direction of the thrust vector is used as the control. The method is first tested on an interception chaser-target problem where the passive target vehicle moves along a straight line at constant speed. We then treat a cooperative rendezvous problem between two active autonomous vehicles. The effects of gravity, thrust and viscous drag are considered and the rendezvous location is treated as a terminal constraint.

1 INTRODUCTION

In an active-passive rendezvous problem between two vehicles, the passive or target vehicle does not apply any control maneuvers along its trajectory. The active or chaser vehicle is controlled or guided such as to meet the passive vehicle at a later time, matching both the location and the velocity of the target vehicle. In a cooperative rendezvous problem, the two vehicles are active and maneuver such as to meet at a later time, at the same location with the same velocity. The rendezvous problem consists of finding the control sequences or the guidance laws that are required in order to bring the two vehicles to a final state of rendezvous.

An optimal control problem consists of finding the control histories (control as a function of time) and the state variables of the dynamical system such as to minimize a performance index. The differential equations of motion of the vehicles are then treated as dynamical constraints. A possible approach to the solution of the rendezvous problem is to formulate it as an optimal control problem in which it is required to find the controls such as to minimize the differences between the final locations and final velocities of the vehicles. The methods of approach for solving optimal control problems include the classical indirect methods and the more recent direct methods. The indirect methods are based on the calculus of variations and its extension to the maximum principle of Pontryagin, which is based on a Hamiltonian formulation. These methods use necessary first order conditions for an optimum, they introduce adjoint variables and require the solution of a two-point boundary value problem (TPBVP) for the state and adjoint variables. Usually, the state variables are subjected to initial conditions and the adjoint variables to terminal or final conditions. Two-point boundary value problems (TPBVP) are much more difficult to solve than initial value problems (IVP). For this reason, direct methods of solution have been developed which avoid completely the Hamiltonian formulation. For

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example, a possible approach is to reformulate the optimal control problem as a nonlinear programming (NLP) problem by direct transcription of the dynamical equations at prescribed discrete points or collocation points. This method was originally developed by Dickmanns and Well (Dickmanns, 1975.) and used by Hargraves and Paris (Hargraves, 1987) to solve several atmospheric trajectory optimization problems.

Another class of direct methods is based on biologically inspired methods of optimization. These include evolutionary methods such as genetic algorithms, particle swarm optimization methods and ant colony optimization algorithms. PSO) mimics the social behavior of a swarm of insects, see for example (Venter, 2002), (Crispin,2005). Genetic Algorithms (GAs) (Goldberg, 1989) are a powerful alternative method for solving optimal control problems, see also (Crispin, 2006 and 2007). GAs use a stochastic search method and are robust when compared to gradient methods. They are based on a directed random search which can explore a large region of the design space without conducting an exhaustive search. This increases the probability of finding a global optimum solution to the problem. They can handle continuous or discontinuous variables since they use binary coding. They require only values of the objective function but no values of the derivatives. However, GAs do not guarantee convergence to the global optimum. If the algorithm converges too fast, the probability of exploring some regions of the design space will decrease. Methods have been developed for preventing the algorithm from converging to a local optimum. These include fitness scaling, increased probability of mutation, redefinition of the fitness function and other methods that can help maintain the diversity of the population during the genetic search.

2 COOPERATIVE RENDEZVOUS AS AN OPTIMAL CONTROL PROBLEM

We study trajectories of vehicles moving in an incompressible viscous fluid in a 2-dimensional domain. The motion is described in a cartesian system of coordinates (x,y), where x is positive to the right and y is positive downwards in the direction of gravity. The vehicle weight acts downward, in the positive y direction. The vehicle has a propulsion system that delivers a thrust of constant magnitude. The thrust is always tangent to the trajectory. The vehicle is controlled by varying the thrust direction. Since the fluid is viscous, a drag force acts on the vehicle, in the opposite direction of the velocity. The control variable of the problem is the thrust direction $\gamma(t)$. The angle $\gamma(t)$ is measured positive clockwise from the horizontal direction (positive x direction).

The rendezvous problem is formulated as an optimal control problem, in which it is required to determine the control functions, or control histories $\gamma_1(t)$ and $\gamma_2(t)$ of the two vehicles, such that they will meet at a prescribed location at the final time t_f. Since GAs deal with discrete variables, we discretize the values of $\gamma(t)$. We assume that the mass of the vehicles is constant. The motion of the vehicle is governed by Newton's second law and the kinematic relations between velocity and distance:

$$d(mV)/dt = mg + T + D \tag{2.1}$$

$$dx/dt = V\cos\gamma \tag{2.2}$$

$$dy/dt = V\sin\gamma \tag{2.3}$$

where D is the drag force acting on the body, V is the velocity vector, T is the thrust vector and g is the acceleration of gravity. Since we assumed m is constant,

$$dV/dt = g + T/m + D/m \tag{2.4}$$

Writing this equation for the components of the forces along the tangent to the vehicle's path, we get:

$$dV/dt = g\sin\gamma + T/m - D/m \tag{2.5}$$

The drag D is given by:

$$D = \frac{1}{2}\rho V^2 S C_D \tag{2.6}$$

where ρ is the fluid density, S a typical cross-section area of the vehicle and C_D the drag coefficient, which depends on the Reynolds number $Re = \rho V d/\mu$, where d is a typical dimension of the vehicle and μ the fluid viscosity.

Substituting the drag from Eq.(2.6) and writing $T = amg$, where a is the thrust to weight ratio T/mg, Eq.(2.5) becomes:

$$dV/dt = g\sin\gamma + ag - \rho V^2 S C_D/2m \tag{2.7}$$

Introducing a characteristic length L_c, time t_c and speed v_c as

$$L_c = 2m/\rho S C_D, \;\; t_c = \sqrt{L_c/g}, \;\; v_c = \sqrt{g L_c} \tag{2.8}$$

the following nondimensional variables can be defined:

$$x = L_c\bar{x}, \quad y = L_c\bar{y}$$

$$t = (L_c/g)^{1/2}\,\bar{t}, \quad V = (gL_c)^{1/2}\,\overline{V} \tag{2.9}$$

Substituting in Eq.(2.7), we have:

$$d\overline{V}/d\bar{t} = a + \sin\gamma(t) - \overline{V}^2 \tag{2.10}$$

Similarly, the other equations of motion can be written in nondimensional form as

$$d\bar{x}/d\bar{t} = \overline{V}\cos\gamma(t) \tag{2.11}$$

$$d\bar{y}/d\bar{t} = \overline{V}\sin\gamma(t) \tag{2.12}$$

For each vehicle the initial conditions are:

$$V(0) = V_0,\; x(0) = x_0,\; y(0) = y_0 \tag{2.13}$$

In rendezvous problems, terminal constraints on the final location can also be required

$$\bar{x}(\bar{t}_f) = \bar{x}_f = x_f/L_c$$

$$\bar{y}(\bar{t}_f) = \bar{y}_f = y_f/L_c \tag{2.14}$$

where the nondimensional final time is given by

$$\overline{t_f} = t_f/\sqrt{L_c/g}$$

We now define a rendezvous problem between two vehicles. We denote the variables of the first vehicle by a subscript 1 and those of the second vehicle by a subscript 2. We will now drop the bar notation indicating nondimensional variables. The two vehicles might have different thrust to weight ratios, which are denoted by a_1 and a_2, respectively. The equations of motion for the system of two vehicles are:

$$dV_1/dt = a_1 + \sin\gamma_1(t) - V_1^2 \tag{2.15}$$

$$dx_1/dt = V_1\cos\gamma_1(t) \tag{2.16}$$

$$dy_1/dt = V_1\sin\gamma_1(t) \tag{2.17}$$

$$dV_2/dt = a_2 + \sin\gamma_2(t) - V_2^2 \tag{2.18}$$

$$dx_2/dt = V_2\cos\gamma_2(t) \tag{2.19}$$

$$dy_2/dt = V_2\sin\gamma_2(t) \tag{2.20}$$

The vehicles can start the motion from different locations and at different speeds. The initial conditions are given by:

$$V_1(0) = V_{10},\; x_1(0) = x_{10},\; y_1(0) = y_{10} \tag{2.21}$$

$$V_2(0) = V_{20},\; x_2(0) = x_{20},\; y_2(0) = y_{20} \tag{2.22}$$

The cooperative rendezvous problem consists of finding the control functions $\gamma_1(t)$ and $\gamma_2(t)$ such as the two vehicles arrive at a given terminal location (x_f, y_f) and at the same speed in the given time t_f. The terminal constraints are then given by:

$$x_1(t_f) = x_f,\; x_2(t_f) = x_f$$

$$y_1(t_f) = y_f,\; y_2(t_f) = y_f \tag{2.23}$$

$$V_1(t_f) = V_2(t_f)$$

We can also define an interception problem, of the target-chaser type, in which one vehicle is passive and the chaser vehicle maneuvers such as to match the location of the target vehicle, but not its velocity. Consistent with the above terminal constraints, we define the following objective function for the optimal control problem:

$$f(x_j(t_f), V_j(t_f)) = \sum_{j=1}^{N_v} \left\| x_j(t_f) - x_f \right\|^2 + \Delta V_j^2(t_f) = \min \tag{2.24}$$

where N_v is the number of vehicles, $x_f = (x_f, y_f)$ is the prescribed interception or rendezvous point and $\Delta V_j^2(t_f)$ is the square of the difference between the magnitudes of the velocities of the vehicles. If we define the norm as a Euclidean distance, we can write the following objective function for the case of two vehicles:

$$f[x_1(t_f), x_2(t_f), y_1(t_f), y_2(t_f), V_1(t_f), V_2(t_f)] =$$

$$= (x_1(t_f) - x_f)^2 + (x_2(t_f) - x_f)^2 + (y_1(t_f) - y_f)^2$$

$$+ (y_2(t_f) - y_f)^2 + (V_1(t_f) - V_2(t_f))^2 = \min \tag{2.25}$$

We use standard numerical methods for integrating the differential equations. The time interval t_f is divided into N time steps of duration $\Delta t = t_f/N$. The discrete time is $t_i = i\Delta t$. We used a second-order Runge-Kutta method with fixed time step. We also tried a fourth-order Runge-Kutta method and a variable time step and found that the results were not sensitive to the method of integration. The control function $\gamma(t)$ is discretized to $\gamma(i) = \gamma(t_i)$ according to the number of time steps N used for the numerical integration. Depending on the accuracy of the desired solution, we can choose the number of bits n^i for encoding the value of the control $\gamma(i)$ at each time step i. The size n^i used for encoding $\gamma(i)$ and the number of time steps N will have an influence on the computational time. Therefore n^i and N must be chosen carefully, in order to obtain an accurate enough solution in a reasonable time. The total length of the chromosome is given by:

$$L_{ch} = n^i N N_v \tag{2.26}$$

For this problem, we were able to increase the rate of convergence of the algorithm by introducing heuristic arguments. For instance, having noticed that $\gamma(t)$ is a monotonically decreasing function of time, we were able to speed up the algorithm by choosing a function with such a property, a priori. Therefore, instead of waiting for the algorithm to converge towards a monotonous $\gamma(t)$, we can sort the values of γ of each individual solution in decreasing order, before calculating its fitness. We also use smoothing of the control function by fitting a third or fourth-order polynomial to the discrete values of γ. The values of the polynomial at the N discrete time points are then used as the current values of γ and are used in the integration of the differential equations.

An appropriate range for γ is $\gamma \in [0, \pi/2]$. We choose $N = 30$ as a reasonable number of time steps. We now need to choose the parameters associated with the Genetic Algorithm. First, we select the lengths of the "genes" for encoding the discrete values of γ. A choice of $n^i = 8$ bits for $\forall i \in [0, N-1]$ was made. The interval between two consecutive possible values of γ is given by:

$$\Delta\gamma = (\gamma_{max} - \gamma_{min})/(2^n - 1) \approx 0.0062\,\text{rad} = 0.35\,\text{deg}$$

For two vehicles and 30 time steps, the length of a chromosome is then given by:

$$L_{ch} = n^i N N_v = 480\,\text{bits}$$

A reasonable size for the population of solutions is typically in the range $n_{\text{pop}} \in [50, 200]$. For this problem, there is no need for a particularly large population, so we select $n_{\text{pop}} = 50$. The probability of mutation is set to a value of 5 percent $p_{\text{mut}} = 0.05$.

3 CHASER-TARGET INTERCEPTION

We study a chaser-target interception problem between two vehicles. In this case the first vehicle is active and the second is passive and moves along a straight line at a constant depth $y_2 = y_f$, constant speed V_2 and constant angle $\gamma_2 = 0$. The two vehicles start from different points and the interception occurs at the known depth of the target vehicle $y_2 = y_f$. The horizontal distance x_f to the interception point is free. We present the case where the target moves at moderate speed and can be captured by the active chaser vehicle. Since this is an interception problem, we do not require matching between the final velocities.

$$a_1 = a = 0.05, \quad a_2 = 5a = 0.25$$

$$t_0 = 0, t_f = 5$$

$$\gamma_1 \in [0, \pi/2], \quad \gamma_2 = 0, \quad y_f = 2 \tag{3.1}$$

The initial conditions are:

$$x_1(0) = x_2(0) = 0, \; y_1(0) = 0$$

$$y_2(0) = y_f = 2, \; V_1(0) = 0$$

$$V_2(0) = V_2 = \sqrt{a_2 + \sin\gamma_2} = \sqrt{a_2} = 0.5 \tag{3.2}$$

In order to match the final locations, the following objective or fitness function is defined:

$$f[x_1(t_f), x_2(t_f), y_1(t_f)] =$$

$$= (x_1(t_f) - x_2(t_f))^2 + (y_1(t_f) - y_f)^2 = \min \tag{3.3}$$

The parameters for this test case are summarized in the following table and the results are given in Figs. 1-3.

N_v	n_{pop}	n^i	N
2	50	8	30
p_{mut}	N_{gen}	$\gamma_{1\min}$	$\gamma_{1\max}$
0.05	50	0	$\pi/2$
a	t_0	t_f	(x_{01}, y_{01})
0.05	0	5	$(0,0)$
(x_{02}, y_{02})	(V_{01}, V_{02})	x_f	y_f
$(0,2)$	$(0, \sqrt{5a})$	free	2

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We next treat a rendezvous between two vehicles. The two vehicles start from different points and rendezvous at point (x_f, y_f) in a given time t_f. The vehicles have the same thrust to weight ratio $a_1 = a_2 = a$.

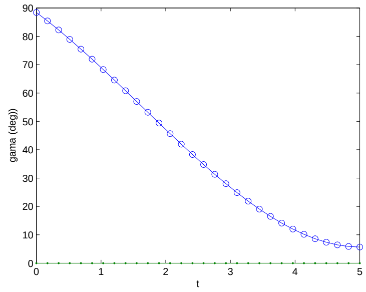

Figure 1: Control function $\gamma_1(t)$ and $\gamma_2 = 0$ for a chaser-target interception with prescribed target depth.

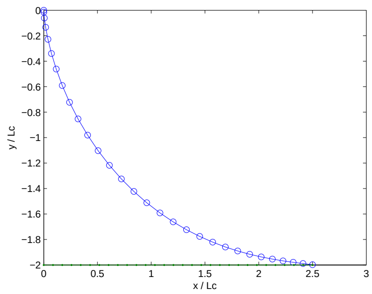

Figure 2: Trajectories for a chaser-target interception with prescribed target depth. The sign of y was reversed for plotting.

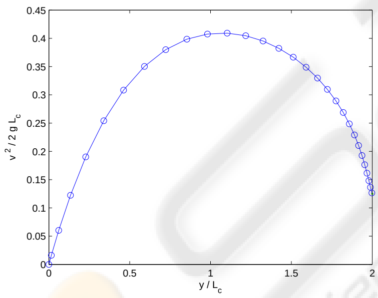

Figure 3: Kinetic energy as a function of depth for a chaser-target interception with prescribed target depth.

$$a_1 = a = 0.05, \quad a_2 = a = 0.05$$

$$t_0 = 0,\ t_f = 5,\ x_f = 2.8,\ y_f = 2$$

$$\gamma_1 \in [0, \pi/2] \quad \gamma_2 \in [0, \pi/2] \tag{4.1}$$

with initial conditions:

$$x_1(0) = 0,\ y_1(0) = 0,\ x_2(0) = 0.5,\ y_2(0) = 0$$

$$V_1(0) = 0,\ V_2(0) = 0 \tag{4.2}$$

In a rendezvous problem, the objective or fitness function is given by:

$$f[x_1(t_f), x_2(t_f), y_1(t_f), y_2(t_f), V_1(t_f), V_2(t_f)] =$$

$$= (x_1(t_f) - x_f)^2 + (y_1(t_f) - y_f)^2 + (x_2(t_f) - x_f)^2$$

$$+ (y_2(t_f) - y_f)^2 + (V_1(t_f) - V_2(t_f))^2 = \min \tag{4.3}$$

The parameters for this test case are summarized in the following table and the results are presented in Figs.4-5.

N_v	n_{pop}	n^i	N
2	50	8	30
p_{mut}	N_{gen}	$\gamma_{1\min}$	$\gamma_{1\max}$
0.05	200	0	$\pi/2$
a	t_0	t_f	(x_{01}, y_{01})
0.05	0	5	$(0, 0)$
(x_{02}, y_{02})	(V_{01}, V_{02})	x_f	y_f
$(0.5, 0)$	$(0, 0)$	2.8	2

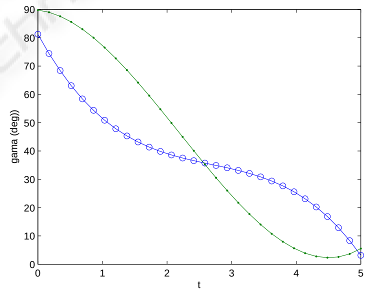

Figure 4: Control functions $\gamma_1(t)$ and $\gamma_2(t)$ for rendezvous between two vehicles with prescribed terminal point.

5 CONCLUSION

The rendezvous problem between two active autonomous vehicles moving in an underwater environment has been treated using an optimal control formulation with terminal constraints. The two vehicles have fixed thrust propulsion system and use the direction of the velocity vector for steering and control. We use a genetic algorithm to determine directly the control histories of both vehicles by evolving populations of possible solutions. An interception problem, where one vehicle moves along a straight line with constant

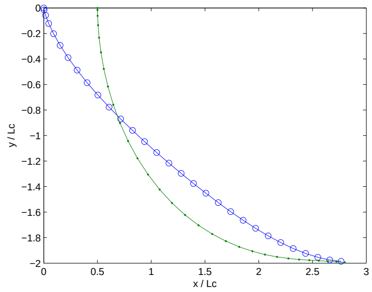

Figure 5: Trajectories for rendezvous between two vehicles with prescribed terminal point.

velocity and the second vehicle acts as a chaser, maneuvering such as to capture the target in a given time, has also been treated as a test problem. It was found that the chaser can capture the target within the prescribed time as long as the target speed is below a critical speed. We then treated the rendezvous problem between two active vehicles where both the final positions and velocities are matched. As the initial horizontal distance between the two vehicles is increased, it becomes more difficult to solve the problem and the genetic algorithm requires more generations to converge to a near optimal solution.

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