### AN INCREMENTAL MAPPING METHOD BASED ON A DEMPSTER-SHAFER FUSION ARCHITECTURE

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Abstract:

Firstly this article presents a multi-level architecture permitting the localization of a mobile platform and secondly an incremental construction of the environment's map. The environment will be modeled by an occupancy grid built with information provided by the stereovision system situated on the platform. The reliability of these data is introduced to the grid by the propagation of uncertainties managed thanks to the theory of the Transferable Belief Model.

### 1 INTRODUCTION

Localization and mapping are fundamental problems for mobile robot autonomous navigation. Indeed, in order to achieve its tasks, the robot has to determine its configuration in its environment. But, if this result is necessary, it is not sufficient. An estimation of the uncertainty and the imprecision of this position should be determined and taken into account by the robot in order to enable it to act in a robust way and to adapt its behaviour according to these two values.

The two notions of uncertainty and imprecision are distinct ones and they must be clearly defined. imprecision results from unavoidable imperfections of the sensors, (ie) the imprecision representing the error associated to the measurement of a value. For example, "the weight of the object is between 1 and 1.5 kg" is an imprecise proposition. On the other hand, the uncertainty represents the belief or the doubt we have on the existence or the validity of a data. This uncertainty comes from the reliability of the observation made by the system: this observation can be uncertain or erroneous. In other words, the uncertainty denotes the truth of a proposition. For example, "John is perhaps in the kitchen" is an uncertain proposition.

In a mobile robotics context, these two notions are paramount. Using several tools and several

localization algorithms, the mobile robot determines its configuration. Knowing an estimation of the uncertainty and the imprecision of this computed localization, it can adopt an adequate behaviour. For example, if one of these two values is too high, it would try to improve the localization estimation by performing a new localization process.

The key tool used in this purpose is the Transferable Belief Model (TBM) (Smets, 1998), a non-probabilistic variant of the Dempster-Shafer theory (Shafer, 1976). Indeed, this theory enables to easily treat uncertainty since it permits to attribute mass not only on single hypothesis, but also on the union of hypotheses. We can thus express ignorance. So it has enabled us to manage and propagate an uncertainty from low-level data (sensor data) in order to get a global uncertainty about the robot localization. We treat the imprecision independently from the uncertainty because their non-correlation have been proved in (Clerentin and all., 2003)

Our dual approach is particularly adapted to the problem of data integration in an occupancy grid, used as part of SLAM paradigm.

We can principally find two types of mapping paradigm to take into account the notion of distance. The first paradigm consists of computing a cartesian representation of the environment which generally used the Extended Kalman filtering (Leonard and Durrant-Whyte., 1992). The second approach based on occupancy grid maps allows to manage the metric maps, which were originally proposed in (Elfes,1987.) and which have been successfully employed in numerous mobile robot systems (Boreinstein and Koren , 1991). In (Fox and all,1999) Dieter Fox introduced a general probabilistic approach simultaneously to provide mapping and localization. A major drawback of occupancy grids is caused by their pure subsymbolic nature: they provide no framework for representing symbolic entities of interest such as doors, desks, etc (Fox and all,1999).

This paper is divided as follows. In a first part, we will detail how our grid occupancy is presented and our uncertain and imprecise sensorial model. Next we will discuss our localization and mapping method based on beacon recognizing . Finally we will present the experimental results.

#### 2 PREAMBLE

## 2.1 Our Grid Occupancy, Its Initialisation

We choose to model the environment of our mobile platform with the occupancy grid tool in 2D. Thus, the error of sensors measure will be implicitly managed since we will not manipulate a point (x,y)but a cell of the grid containing an interval of values ([x],[y]). We choose to center the grid with the initial position of the platform. Then a cell is defined by its position in the grid . A cell also contains information concerning its occupancy degree by some object of the environment. This latter is defined by a mass function relative to the discernment frame  $\Theta 1 = \{yes, no\}$ . These two hypotheses respectively correspond to propositions " yes, this cell is occupied by an object of the environment " and " no, this cell is not occupied ". So the mass function of the cell concerning its occupation is composed of the three values in [0], 1], the mass  $m_{cell}(yes)$  on the hypothesis {yes},  $m_{cell}(no)$  on the hypothesis  $\{no\}$  and  $m_{cell}(\Theta 1)$  on the hypothesis {yes  $\cup no$ } representing the ignorance on its occupancy problem. Initially, we have no a-priori knowledge of the situation. So to model our total ignorance, all the cells are initialized with the neutral mass function, that is to say:  $m_{cell}{yes \cup no} = 1$ and  $m_{cell}{yes} = m_{cell}{no} = 0$ .

# 2.2 Uncertain and Imprecise Sensorial Model

The platform gets a stereovision system composed of two omnidirectionnal sensors (see Figure 1.)

distant of about 50 cm. Every acquisition provides two pictures of the environment .

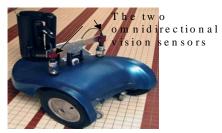


Figure 1: The perception system.

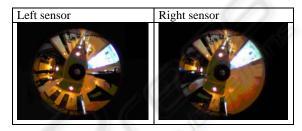


Figure 2: An example of an acquisition.

On Figure 2 all vertical landmarks of the environment like doors or walls project themselves to the center and form some sectors of different gray levels. The positions of these landmarks will permit to fill the occupancy grid and so to build a map . To get this information, we should associate each sector in the first picture with the one that corresponds to it on the second picture. This stage needs some treatments on the primary data.

First of all, on each omnidirectional picture, we define a signal which represents the mean RGB color from a ring localized around the horizon in the field of view. In fact, what we want to detect are the natural vertical beacons of the environment. Omnidirectional vision system project those vertical parts of the environment according to radial straight lines onto the image. During this computation, it is very important that the rings are centered onto the projection of the revolution axis of the mirror. Otherwise, we will not compute the mean RGB color according to the projection of the vertical elements of the environment. This centering task is automatically done with a circular Hough transform (Ballard, 1981). In fact, we look for a circle corresponding to the projection of the black needle situated onto the top of the hyperbolic mirror (see Figure 3) which is situated onto the center of the mirror.

Then, the two 1D mean RGB signals are computed from the ring(Figure 4) and matched together according to a rule of visibility. In fact, if an object is detected from one omnidirectional sensor, it will be visible in a certain area of the other

one, according to the distance between the object and the mobile robot.

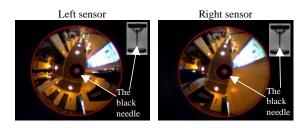


Figure 3: Center location computed with a circular Hough transform.

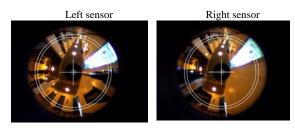


Figure 4: Centered rings to compute the mean RGB signals.

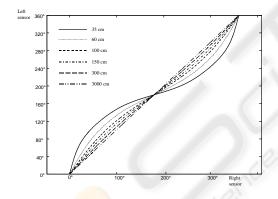


Figure 5: Correspondences between angles from the left sensor to the right sensor for objects situated at different distances from the mobile robot.

Figure 5 shows the correspondences between the angle of the left sensor and the angle of the right sensor according to different distances. We actually notice that the more the object is close to the mobile robot, the more the two angles are different.

The detection algorithm is based upon the derivative of the signal in order to detect sudden changes of color. When we find such value on the left sensor, we look for a similar change in the right sensor signal with a maximum of correlation criteria. In fact, as you can note on the Figure 6, a matching could be close to another one. So, we only keep the

most significant matching according to the correlation value.

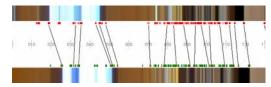


Figure 6: The two extracted mean color signals from omnidirectional pictures (to 0 from  $140^{\circ}$ ) of Figure 4 and the matching between the left sensor (upper) and the right sensor (bottom).

We choose to use this indicator (in [0-1]) which qualify a very good correlation when is near than 1, to build a degree of uncertainty about the matching by the way of three masses (see Figure 7).

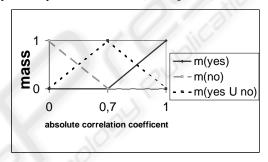


Figure 7: Uncertainty about the matching computed with the correlation coefficient.

When two sectors are matched, we have two pairs of associated angles on the one hand (angles of segments that define borders) and on the other hand a measure of uncertainty on this association. It is directly linked with a landmark which represents it . Therefore the pairs define the position of the landmark and the uncertainty measure its uncertainty on its existence. So this last value is equal to the set of three masses coming from the previous fusion in the discernment frame  $\Theta2=$  {yes the sectors are associated; no they do not correspond}

- the mass on the hypothesis "yes"  $m_{ass}(yes)$
- the mass on the hypothesis "no"  $m_{ass}(no)$

the mass on the hypothesis "I can't decide about this matching"  $m_{ass}(yes \cup no) = m_{ass}(\Theta 2)$ , in other words this mass represents ignorance.

Then the landmark uncertainty is given by the following masses in the discernement frame  $\Theta8=$  {yes the landmark exist; no it don't exist}:

- the mass on the hypothesis "yes"
- $m_{land}(yes) = m_{ass}(yes)$
- the mass on the hypothesis "no"

 $m_{land}(no) = m_{ass}(no)$ 

 the mass on the ignorance hypothesis ie "I can't decide about the existence"

$$m_{land}(\Theta 8) = m_{ass}(\Theta 2)$$

Only sectors that have been associated will be used in the continuation of our survey. The measure of uncertainty  $m_{land}$  qualifies the landmark but also the segment pairs forming borders. Indeed, the existence of a landmark is linked with the existence of the borders.

Our data are uncertain but they are also imprecise because of sensor measurement errors. This imprecision of measure is managed by the way of intervals. The second result of this fusion, that is to say the matching of two angles  $(\alpha, \beta)$ , provides information about extremities  $(x_i, y_i)$  of the vertical landmarks in question(Figure 8) thanks to equations of triangulation (1) and (2). So we transform our data in intervals in order to include this imprecision. We create a error domain empirically around our measures of angle  $(\alpha, \beta)$ . Then the operations (1) and (2) are computed not between reals but on intervals. We obtain the following equations (3) and (4):

$$x_i = \frac{d \times \tan \beta_i}{\tan \beta_i - \tan \alpha_i} \tag{1}$$

$$y_i = \frac{d \times \tan \beta_i \times \tan \alpha_i}{\tan \beta_i - \tan \alpha_i}$$
 (2)

$$[x_i] = \frac{d \times \tan[\beta_i]}{\tan[\beta_i] - \tan[\alpha_i]}$$
(3)

$$[y_i] = \frac{d \times \tan[\beta_i] \times \tan[\alpha_i]}{\tan[\beta_i] - \tan[\alpha_i]}$$
(4)

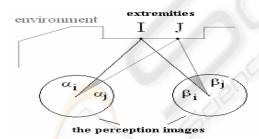


Figure 8 : Sectors matching.

At this level of data exploitation, we have a set of subpaving characterizing the physical extremities of each landmark detected, that is to say the object of which sectors representing it have been matched. These subpavings form the primitive of our sensorial model that we will try to link with the beacons during the time. They are localized by their coordinates ([xi],[yi]) in the frame relative to the platform and they have the same measure of reliability that the landmark ie  $m_{prim} = m_{land}$ .

### 3 LOCALISATION AND MAPPING METHOD

The algorithm consists in matching during the platform displacement the primitives of the sensorial model with information known from the environment that we will call beacons. These matching once achieved will permit both to correct the position of beacons and the estimated position of the platform thanks to the odometry and also to confirm the existence of beacons. In short we will exploit data of these beacons to build our occupancy grid of the surrounding space.

## 3.1 Definition and Initialisation of Beacon

A beacon is defined by a set of coordinates in the reference frame (Xe, Ye) (thus forming a subpaving of localization) and by a degree of uncertainty about its existence composed of three masses as previously shown. This set of masses is established in the discernment frame  $\Theta 3=\{yes, no\}$ . These two hypotheses respectively correspond to propositions "yes, this beacon exists" and "no, this beacon does not exist". So the function mass concerning its existence is composed of the three values, the mass  $m_{bea}(yes)$  on the hypothesis  $\{yes\}$ ,  $m_{bea}(no)$  on the hypothesis  $\{no\}$  and in short  $m_{bea}(\Theta 3)$  on the hypothesis  $\{yes \cap no\}$  representing the ignorance about its existence.

A beacon is born from a primitive observed at instant t that cannot be matched with the existing beacons at this instant. This new observation is a landmark not discovered until now or a false alarm. The only information on the existence of a new beacon comes from the existence of the primitive that gave its birth. Then we choose to give the same measure of uncertainty on the beacon, that is to say  $m_{beat} = m_{prim}$ . Concerning its relative positioning it is equal to the relative localization subpaving of the primitive associated. As thereafter we must associate this beacon to an observation coming from other acquisitions and should use this one in the updating of the occupancy grid. So it is more interesting to manipulate the absolute position . This one is obtained by the change of a frame in relation to the configuration of the platform.

Therefore at each instant, new beacons can appear, and in this case they join the set of the existing beacons to the following acquisition.

## 3.2 The Association Method between Beacons and Primitives

In looking for these matchings, the aim is on the one hand to get the redundant information permitting to increase the degree of certainty on the existence of the beacons and on the other hand to correct their positioning.

So, at any step, we have several beacons that are characterized by the center of their subpaving ([x],[y]). Let us call this point the "beacon center". The uncertainty of each beacon is represented by the mass function  $m_{beal}$ .

In this part, we try to propagate the matchings initialised in the previous paragraph with the observations made during the robot's displacement. In other words, we try to associate beacons with sensed landmarks.

Suppose we manage q beacons at time n. Each beacon is characterized by its "beacon center" (expressed in the reference frame). Let us call this beacon point  $(x_b, y_b)$ . Suppose the robot gets p observations at time n+1. As we have explained in the previous paragraph, we are able to compute each observation localization subpaving  $([x_i], [y_i])$  in the reference frame. So, for each observation, we have to search among the q beacons the one that corresponds to it. In other words, we have to match a beacon center  $(x_b, y_b)$  with an observation subpaving  $([x_i], [y_i])$ . The matching criterion we choose is based on the distance between the beacon center and the center of observation subpaving  $([x_i], [y_i])$ .

So at this level, the problem is to match the p observations obtained at acquisition n+1 with the q beacons that exist at acquisition n. To reach this aim, we use the Transferable Belief Model (Smets, 1998) in the framework of extended open word (Shafer, 1976) because of the introduction of an element noted \* which represents all the hypotheses which are not modeled, in the frame of discernment.

First we treat the most reliable primitives, that is to say the "strong" primitives by order of increasing uncertainty.

For each sensed primitive Pj ( $j \in [1..p]$ ), we apply the following algorithm:

- The frame of discernment  $\Theta_i$  is composed of:
  - the q beacons represented by the hypothesis Qi ( $i \in [1..q]$ ). Qi means "the primitive Pj is matched with the beacon Qi")
  - and the element \* which means "the primitive Pj cannot be matched with one of the q beacons".
  - So:  $\Theta_i = \{Q_1, Q_2, ..., *\}$
- The matching criterion is the distance between the center of the subpaving of observation Pj and one of the beacon centers of Qi

- Considering the basic probability assignment (BPA) shown Figure 9, for each beacon Qi we compute:
  - $-m_i(Qi)$  the mass associated with the proposition " $P_i$  is matched with Qi".
  - $m_i(\neg Qi)$  the mass associated with the proposition " $P_i$  is not matched with  $Q_i$ ".
  - $-m_i(\Theta_j)$  the mass representing the ignorance concerning the observation Pi.
- The BPA is shown on Figure 9.

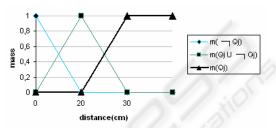


Figure 9: BPA of the matching criterion.

 After the treatment of all the q beacons, we have q triplets:

$$m_1(Q_1)$$
  $m_1(\neg Q_1)$   $m_1(\Theta_j)$   
 $-m_2(Q_2)$   $m_2(\neg Q_2)$   $m_2(\Theta_j)$   
 $-m_2(Q_q)$   $m_q(\neg Q_q)$   $m_q(\Theta_j)$ 

– We fuse these triplets using the disjunctive conjunctive operator built by Dubois And Prade (Dubois and Prade, 1998). Indeed, this operator allows a natural conflict management, ideally adapted for our problem. In our case, the conflict comes from the existence of several potential candidates for the matching, that is to say some near beacons can correspond to a sensed landmark. With this operator, the conflict is distributed on the union of the hypotheses which generate this conflict.

For example, on Figure 10, the beacon center  $P_1$  and  $P_2$  are candidates for a matching with the primitive subpaving ([x], [y]). So  $m_1(P_1)$  is high (the expert concerning  $P_1$  says that  $P_1$  can be matched with ([x], [y])) and  $m_2(P_2)$  is high too. If the fusion is performed with the classical Smets operator, these two high values produce some high conflict. But, with the Dubois and Prade operator, the conflict generated by the fusion of  $m_1(P_1)$  and  $m_2(P_2)$  is rejected on  $m_{12}(P_1 \cup P_2)$ . This means that both  $P_1$  and  $P_2$  are candidates for the matching.

- So, after the fusion of the q triplets with this operator, we get a mass on each single hypothesis  $m_{match}(Qi)$ ,  $i \in [1..q]$ , on all the unions of hypotheses  $m_{match}(Qi \cup Qj... \cup Qq)$ , on the star hypothesis  $m_{match}(*)$  and on the ignorance  $m_{match}(\Theta_i)$ .
- The final decision is the hypothesis which has the maximal pignistic probability (Smets, 1998). If it is the \* hypothesis, no matching is achieved. This

situation can correspond to two cases: either the primitive Pj is an outlier, or Pj can be used to initiate a new beacon since any existing track can be associated to it.

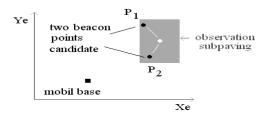


Figure 10: An example of two beacons that generate some conflict.

Once a matching is achieved, the uncertainty of the concerned beacon has to be updated. This uncertainty is denoted by the mass function  $m_{bea}$  defined on the frame of discernment  $\Theta_3$ . This updating has to take the reliability of the matched primitive (mass function  $m_{prim}$ ) and also the uncertainty of the matching into account. This matching uncertainty is deduced from the pignistic probability of the selected matched primitive by the mass function m2 shown on Figure 11. For example, if the pignistic probability is equal to 0.75, the matching uncertainty is denoted by the following mass function  $m_2$ :  $m_2(\text{yes})=m_2(\{\text{yes}, \text{no}\})=0.5$ ;  $m_2(\text{no})=0.$ 

Finally, the beacon uncertainty at time t (denoted by the mass function  $m_{beat}$ ) is updated by fusing the beacon uncertainty at the previous time t-1, the primitive uncertainty  $(m_{prim})$  and the matching uncertainty  $(m_2)$ :  $m_{beat} = m_{beat-1} \cap m_{prim} \cap m_2$ , where  $\cap$  is the fusion operator of Smets.

Let us recall that this mass function is composed of three values:  $m_{beat}(yes)$ ,  $m_{beat}(no)$ ,  $m_{beat}(\Theta_3)$ .

## 3.3 The Management of non Associated Beacons

Concerning the beacons that have not been matched at this instant, our first reflection was the following. As no observation can be associated, it implies that our beacon has not been detected to this acquisition. Therefore the first idea was to put in doubt its existence in decreasing its degree of existence. But even if the beacon is no more visible from instant t for example because the mobile platform is moving, the object nevertheless exists. It is necessary not to lose this information at the level of the map. Then we decide not to modify the degree of existence of a beacon which was not matched.

#### 3.4 The Consequences on our Grid

A new beacon or a beacon that have been associated to an observation provide two kinds of information both on the occupied space and on the empty space of our grid. Let us examine the case of one of these beacons at time t to explain this phenomenon. As we have already said, this beacon has a measure of uncertainty on its existence. It is defined by the mass function  $m_{beat}$ .

#### 3.4.1. The Occupied Space

The existence of a beacon is directly bound to the occupation of cells containing its localization subpaving. Therefore the degree of occupation of these cells must take the degree of existence of the beacon into account. It is achieved thanks to the fusion with the operator of Smets of these two mass functions. So if the mass function of the beacon indicates rather a certain existence then this fusion will increase the degree of occupation of concerned cells. On the contrary, if it indicates an existence which is somewhat unreliable, the fusion will reverberate this doubt on these same cells. A cell is concerned by the fusion if its intersection with the localization subpaving is not empty, they appear in gray on Figure 12a.

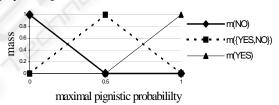


Figure 11: Mass function m2 of the matching uncertainty.

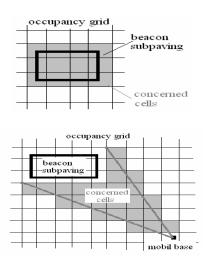


Figure 12: a) Occupied Space, b) Free Space.

The fusion is the following:  $m_{\text{cell t}} = m_{\text{cell t-1}} \cap m_{\text{bea t}}$ 

#### 3.4.2 The Free Space

On the other hand, since this beacon has been associated to an observation, it implies that the space between the point of observation in this case the mobile platform and the beacon does not contain any obstacle. This space is therefore free. But it is free in relation with the existence of the beacon.

This operation is achieved in the same way as previously, that is to say merging with the operator of Smets. But this time, we fuse its current occupation degree with a mass function  $m_3$  built as being the "contrary" of the mass function  $m_{bea}$ . Because the more the beacon is denoted by a high mass on the hypothesis {yes, I exist}, the more the mass on the hypothesis {no, this cell is not occupied} for the cell of the free space (Figure 12 b) must increase. This function  $m_3$  is the next one:

 $m_3$ { no} =  $m_{bea\,t}$  {yes},  $m_3$ { yes  $\cup$  no} =  $m_{bea\,t}$  { yes  $\cup$  no} and  $m_3$ {yes} =  $m_{bea\,t}$  { no}.

And this fusion is given by the following expression:  $m_{cell\ t} = m_{cell\ t-1} \cap m_3$ 

To resume we get a set of beacons and a occupancy grid of obstacles of the surrounding space.

### 3.5 The Correction Method

Now we use these data to correct the position of beacons at first and then the estimated position of the mobile platform. These stages are under development. We currently use the correction modules presented below that will be to improve in future works. The beacons are characterized by an error domain of center (x,y). We notice that this center, disposed on the grid, is surrounded with the cells of different occupation levels. To take account of the information we modify the position of the beacon. In fact, we choose the center of gravity of a window  $5 \times 5$  cells around the center pondered by their respective mass  $m_{cell}(yes)$ , as the point that now characterizes the position of the beacon.

Let us remember that the configuration of the mobile platform is estimated with odometric information. Or we know the classical phenomena of cumulative error if no correction method is achieved (Delafosse and all, 2005). Our correction module is based on the cumulative error minimization. We limit the real possible positions of the platform to centers of cells of a window 3 x 3 around the position estimated by odometry. The kept position among the nine will be the p position that minimizes

the accumulated sum of distances between beacons and primitives observed since the p position.

### 4 EXPERIMENTAL RESULTS

We present experimental results obtained in a structured indoor environment on Figure 13. The platform is stopped to every stereoscopic acquisition achieved every 30 cms. The managed trajectory is a large boucle represented in yellow on the Figure 14. The natural landmarks mainly observed are the framings of doors, corners, walls and pillars. In Figure 14 we present the obtained map building. The blue cells correspond to the empty space and the red one to the occupied space. The intermediate colors highlight the merging of the occupied and free state. We can notice that the method is robust since most observable landmarks are integrated to the map according to the real map presented in grey on the graphs. We can easily detect for example the corners of the "cross" hall and the free space between each others. We can also notice the certainty of the free space is clearly represented by the color purple. Our approach complementary to the probabilistic one, form an alternative to the SLAM paradigm based on the occupancy grid. We can observe the correlation between the uncertainty of a landmark position estimation and the updating cell values.

### 5 CONCLUSION

In this article we have presented an architecture of fusion and integration of data for the SLAM paradigm. It is based on a representation of occupancy grid type. The originality of the proposition is on the one hand the propagation of uncertainties on several levels of treatments and on the other hand the management uncoupled of imprecision and uncertainty. The association of these two concepts permits an important reliability in the process of new primitive integrations in the map. This step is crucial since it conditions the global consistency of the cartographic representation on an important number of acquisitions. Moreover our approach permits to solve the problem of "primitive number explosion" which generally implies a divergence of the SLAM process. Besides the precision obtained on the position estimation of observable landmarks is relatively important. So the «symbolic» approach presented constitutes an interesting alternative to methods classically used in this domain that are generally probabilistic.

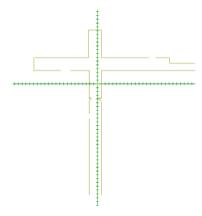


Figure 13: The experimental environment (scale 1m x 1m). We focus on the part of the corridor which represente a cross.

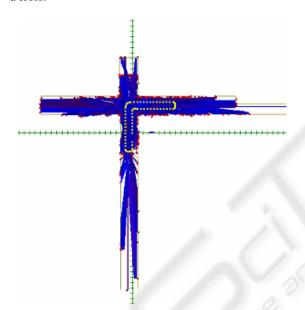


Figure 14: The resulting map.

### REFERENCES

- Ballard D. H., "Generalizing the Hough transform to detect arbitrary shapes." Pattern Recognition, v. 13, n. 2, pp. 111-122, 1981.
- Boreinstein J., Koren Y., 1991. "Histogrammic in-motion mapping for mobile robot obstacle avoidance", IEEE Trans. On rob. and auto., Vol. 7, N°4, pp. 1688-1693
- Clerentin A., Delahoche L., Brassart E., Drocourt C., 2003. "Uncertainty and error treatment in a dynamic localization system", proc. of the 11th Int. Conf. on Advanced Robotics (ICAR03), pp.1314-1319, Coimbra, Portugal.
- Delafosse M., Clerentin C., Delahoche L., Brassart E., 2005. "Uncertainty and Imprecision Modeling for the Mobile Robot Localization Problem" IEEE

- international conference on robotics and automation ICRA 05 , Barcelona, Spain.
- Dubois D., Prade H., Sandri S., 1998. Possibilistic logic with fuzzy constants and fuzzily restricted quantifiers.
  In: Logic Programming and Soft Computing, (Martin, T.P. et Arcelli-Fontana, F., Eds.), Research Studies Press, Baldock, England, 69-90.
- Elfes A. 1987. "Sonar-based real world mapping and navigation", IEEE Journal of robotics and automation, Vol. RA-3, N°3, pp. 249-265, June 1987
- Fox D., Burgard W., Thrun S., 1999. "Probabilistic Methods for Mobile Robot Mapping", Proc. of the IJCAI-99 Workshop on Adaptive Spatial Representations of Dynamic Environments.
- Leonard J.J., Durrant-Whyte H.F., 1992. "Dynamic map building for an autonomous mobile robot", The International Journal of Robotics Research, Vol. 11 n°4, August
- Royère C, 2002. "Contribution à la résolution du conflit dans le cadre de la théorie de l'évidence", thèse de doctorat de l'Université de Technologie de Compiègne.
- Shafer GA, 1976. A mathematical theory of evidence, Princeton: university press.
- Smets Ph ,1998. "The Transferable Belief Model for Quantified Belief Representation.", Handbook of Defeasible Reasoning and Uncertainty Management Systems, Kluwer Ed, pp 267-301.