COORDINATED MOTION CONTROL OF MULTIPLE ROBOTS

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Abstract: In this paper a set of robot coordination approaches is presented. Described method 0s are based on formation function concept. Accuracy of different approaches is compared using formation function time graphs. Virtual structure method is analyzed, then virtual structure expanded with behavioral formation feedback is presented. Finally leader-follower scheme is described. Presented methods are illustrated by simulation results. Differentially driven nonholonomic mobile robots were used in simulations.

1 INTRODUCTION

Multiple robot coordination is currently one of the most investigated area of robotics. Great development in computer sciences, multi-agent systems and availability of low-cost, effective and compact digital equipment caused that many researchers focused their attention on this subject. Multi-agent robotic systems have wide range of applications: service robots, transportation systems, mapping, surveillance, security and many others.

Multi-robot coordination methods can be conventionally partitioned into three classes of approaches: virtual structure approach (Egerstedt and Hu, 2001), (P. Ogren, 2002), (W. Kang, 2000), (Kar-Han Tan, 1997), behavioral approach (Esposito and Kumar, 2000), (J. R. Lawton and Beard, 2000), (Yamaguchi, 1998), (Yamaguchi, 1999), (Kowalczyk and Kozlowski, 2005) and leader follower scheme (R. Fierro and Ostrowski, 2001), (J. Spletzer and Kumar, 2001) (sometimes treated as a combination of first two approaches). Each of them is more or less suitable for particular application. There exist some solutions with characteristic features of more than one approach (B. J. Young and Kelsey, 2001).

In virtual structure methods control is centralized. It is suitable for the tasks that require high precision coordinated motion of few robots, i.e. when it is necessary to transport one huge object by the formation of robots. Centralized architecture of the control cause that system is not scalable. Adding new agents causes more intensive utilization of the main controller. This method requires also high-speed communication between main controller and agents. For virtual structure method it is usually relatively easy to analyze and proof stability of the system mathematically.

In behavioral method control is entirely distributed. It is not necessary to use communication; however, using it may increase efficiency. Behavioral methods were inspired by observations in biology and physics. Control is decentralized and in result system is easy scalable. Stability analysis is difficult or even impossible. These methods are not suitable for high-precision motion tasks, but they are very effective for applications that can be decomposed into many independent subtasks. In opposition to virtual structure methods behavioral methods are fault-tolerant.

Leader-follower methods own some features from virtual structure and behavioral methods. Communication can be used to make control more effective, but it is not necessary. Control is distributed and in result easy scalable, however, there is hierarchical dependency between robots and as a result system is not as fault-tolerant as in behavioral approach. Stability

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roof is usually possible for leader-follower methods. Typical applications for leader-follower methods are spacecraft and aircraft formations. It can be also used in mapping and exploration of the terrain.

Paper is organized as follows. In section 2 we describe feedback linearization of differentially-driven mobile robot. In section 3 virtual structure approach is presented. In section 4 virtual structure approach is expanded with formation keeping behavior. In section 5 leader-follower scheme is presented. In section 6 we conclude the paper. Simulation results are included in sections 3-5.

2 FEEDBACK LINEARIZATION

Most of formation control methods require robots to be fully actuated or transformed into fully actuated. Model of such robot is given:

$$\ddot{P}_i = u_i, \tag{1}$$

where P_i is the position vector of the i-th robot, $P_i \in R^2$, u_i - control force vector exerted on the i-th robot, $u_i \in R^2$, $i = 1, 2, \ldots, N$, N - number of robots.

Two-wheel differentially-driven mobile robot can be transformed into fully actuated using feedback linearization. The same technique can be applied also to other kind of mobile platforms. This causes that formation control can be implemented independently from motion controller and architecture of the robots.

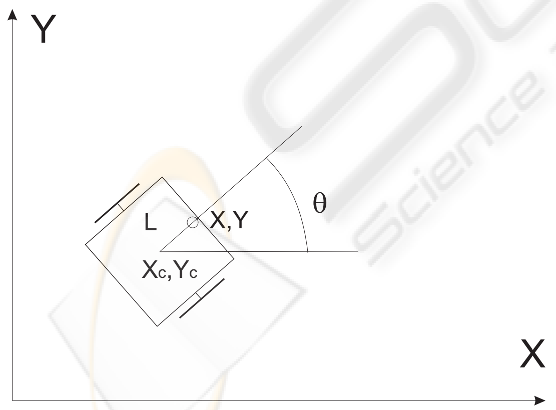

Figure 1: Nonholonomic differentially driven wheeled mobile robot (index designating number of the robot was omitted for clarity).

The motion of the i-th robot is given by:

$$\begin{bmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \\ \dot{\theta}_i \\ \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} v_i \cdot \cos(\theta_i) \\ v_i \cdot \sin(\theta_i) \\ \omega_i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_i} & 0 \\ 0 & \frac{1}{J_i} \end{bmatrix} \begin{bmatrix} F_i \\ \tau_i \end{bmatrix}, \tag{2}$$

where $[x_{ci}, y_{ci}]^T$ - position of the midpoint of the wheel axis, θ_i - orientation of the robot, v_i - linear velocity, ω_i - angular velocity, m_i - mass of the robot, J_i - moment of inertia of the robot, F_i - control force and τ_i - control moment of force.

Dynamics of this kind of robot can be linearized if robot's position output is chosen suitably. As shown in (J. R. Lawton and Beard, 2002) a good choice is position of the point located in a distance L_i along the line that is perpendicular to the wheel axis and intersects with the point $[x_{ci} \;\; y_{ci}]^T$ (Fig. 1). Selected output can be described as follows:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} + L_i \cdot \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix} \tag{3}$$

Differentiating above equation twice we obtain:

$$\begin{aligned} \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} &= \begin{bmatrix} -v_i\omega_i \sin(\theta_i) - L_i\omega_i^2 \cos(\theta_i) \\ v_i\omega_i \cos(\theta_i) - L_i\omega_i^2 \sin(\theta_i) \end{bmatrix} \\ &+ \begin{bmatrix} \frac{1}{m_i}\cos(\theta_i) & -\frac{L_i}{J_i}\sin(\theta_i) \\ \frac{1}{m_i}\sin(\theta_i) & \frac{L_i}{J_i}\cos(\theta_i) \end{bmatrix} \begin{bmatrix} F_i \\ \tau_i \end{bmatrix} \end{aligned} \tag{4}$$

Since

$$\det \begin{bmatrix} \frac{1}{m_i}\cos(\theta_i) & -\frac{L_i}{J_i}\sin(\theta_i) \\ \frac{1}{m_i}\sin(\theta_i) & \frac{L_i}{J_i}\cos(\theta_i) \end{bmatrix} = \frac{L_i}{m_i J_i} \neq 0 \tag{5}$$

the system with output $[x_i \;\; y_i]^T$ can be output feedback linearized.

The output feedback linearizing control law is

$$\begin{aligned} \begin{bmatrix} F_i \\ \tau_i \end{bmatrix} &= \begin{bmatrix} \frac{1}{m_i}\cos(\theta_i) & -\frac{L_i}{J_i}\sin(\theta_i) \\ \frac{1}{m_i}\sin(\theta_i) & \frac{L_i}{J_i}\cos(\theta_i) \end{bmatrix}^{-1} \\ &\cdot \left(u_i - \begin{bmatrix} -v_i\omega_i \sin(\theta_i) - L_i\omega_i^2 \cos(\theta_i) \\ v_i\omega_i \cos(\theta_i) - L_i\omega_i^2 \sin(\theta_i) \end{bmatrix} \right) \end{aligned} \tag{6}$$

Substituting above result into Eq. (5) and simplifying we obtain feedback linearized robot model given by Eq. (1).

3 VIRTUAL STRUCTURE

In this section virtual structure method is presented. Concept of formation function that was introduced in (P. Ogren, 2002) is used. Virtual structure is suitable for applications that require very precise, coordinated motion of formation of robots. As mentioned in the introduction this approach has some disadvantages, however, in some applications it is the only suitable method.

In Fig. 2 formation of four robots tracks desired trajectory. All robots keep their relative positions P_i, $i = 1,\ldots,4$ to the current point of desired trajectory P_{trj}.

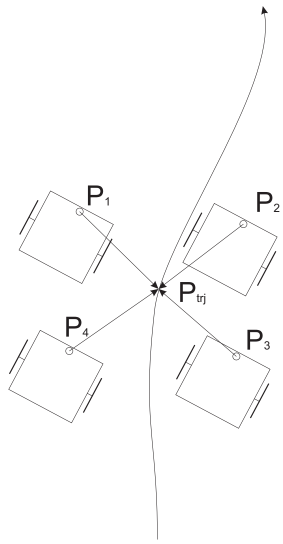

Figure 2: Virtual structure approach - robots of the formation tracks desired trajectory with offsets given by constant vectors (offset vectors).

The formation function is as follows:

$$F = \sum_{i=1}^{N} \|(P_i - P_{i\,of}) - P_{trj}\|^2, \qquad (7)$$

where $P_{trj} = [x_{trj} \;\; y_{trj}]^T$ is the current point on the trajectory to be tracked by the formation, $P_{i\,of} = [x_{i\,of} \;\; y_{i\,of}]^T$ is the offset vector for i-th robot. Function F is positive definite, equal to zero only when all robots of the formation match their desired positions.

Control of the i-th robot is proposed as follows:

$$u_i = -K_P \begin{bmatrix} \frac{\partial F}{\partial x_i} \\ \frac{\partial F}{\partial y_i} \end{bmatrix} - K_V \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix}, \qquad (8)$$

where K_P and K_V are positive gains that determine characteristics of the control. Second term in Eq. (8) represents dumping.

In Fig. 3 trajectories of centers of masses of four robots are shown. They follow formation trajectory that starts in $(0,0)$ position and ends in $(-1.1, 4.5)$. Offset vectors for robots $1,\ldots,4$ are as follows: $(0.25, 0.25)$, $(-0.25, 0.25)$, $(-0.25, -0.25)$ and $(0.25, -0.25)$. Initial orientations of robots are: $\theta_1 = -\frac{\pi}{2}$, $\theta_2 = \pi$, $\theta_3 = \frac{\pi}{2}$ and $\theta_4 = 0$. The values of control gains are: $K_P = 30$ and $K_V = 10$. In Fig. 4 starting segments of robots trajectories are shown. Initially all robots of the formation change their orientations to $\theta_i \approx 1.88 rad$ ($i = 1,\ldots,4$) to track the desired trajectory.

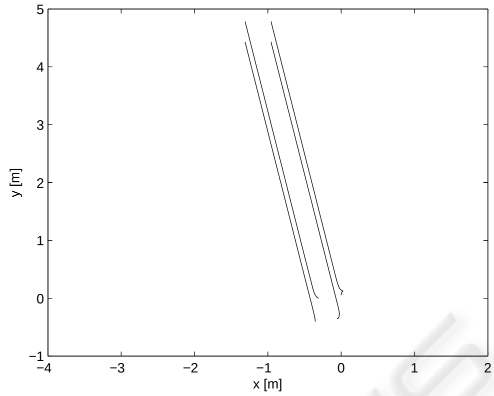

Figure 3: Formation of four robots tracks desired trajectory using virtual structure control method.

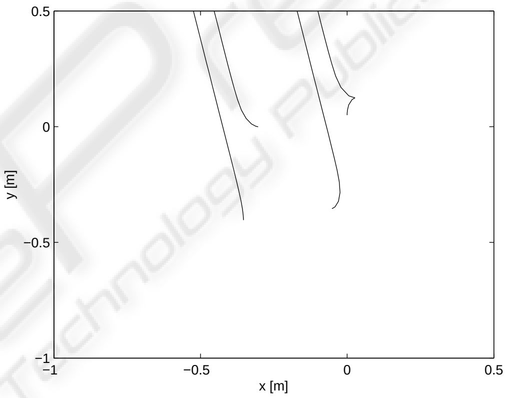

Figure 4: Starting segment of trajectories shown in Fig. 3.

In Fig. 5 the graph of formation function as a function of time is shown. It can be used to evaluate the control because value of formation function represents formation error. As one can see in Fig. 5, after transient state (about 1.5s), formation error stabilizes below $0.9 m^2$.

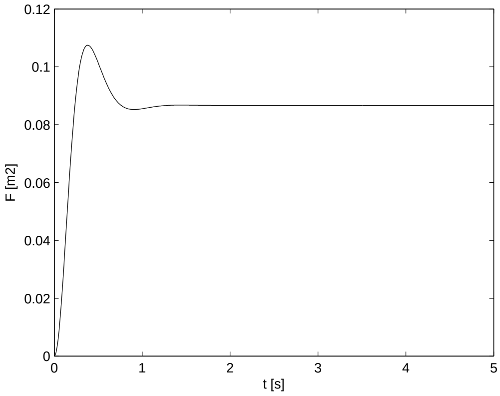

Figure 5: Formation function (time graph) for virtual structure control.

4 VIRTUAL STRUCTURE WITH FORMATION KEEPING BEHAVIOR

In this section we present virtual structure method expanded with behavioral component. This component provide formation feedback that cause formation to slow down when one of robots slows down or when it stops. In such case two concurrent goals occur: trajectory tracking and formation keeping. Presented method does not avoid collisions between robots. The control algorithm try to fulfill both of them. Tuning control gains one can set more to track the trajectory or to keep the formation.

In Fig. 6 formation of four robots tracks desired trajectory. All robots keep their relative positions P_i, to the current position of desired trajectory P_{trj}. Additionally robots keep positions relatively to their neighbors.

The formation function is given as follows:

$$F = F_1 + F_2, \tag{9}$$

where F_1 is given by Eq. (7) and F_2 is as follows:

$$F_2 = \sum_{i=1}^{N} \Big[\|(P_i - P_{i\,of}) - (P_k - P_{k\,of})\|^2 + \|(P_i - P_{i\,of}) - (P_j - P_{j\,of})\|^2 \Big], \tag{10}$$

where $P_{k\,of} = [x_{k\,of} \;\; y_{k\,of}]^T$ and $P_{j\,of} = [x_{j\,of} \;\; y_{j\,of}]^T$ are offset vectors to k-th and j-th neighbor robot; k and j are $\{4,2\}$ for robot 1, $\{1,3\}$ for robot 2, $\{2,4\}$ for robot 3 and $\{3,1\}$ for robot 4. Component F_2 of the formation function represents coupling between robots and formation feedback.

Control of the i-th robot is given as follows:

$$u_i = -K_P \begin{bmatrix} \frac{\partial F_1}{\partial x_i} \\ \frac{\partial F_1}{\partial y_i} \end{bmatrix} - K_F \begin{bmatrix} \frac{\partial F_2}{\partial x_i} \\ \frac{\partial F_2}{\partial y_i} \end{bmatrix} - K_V \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix}, \tag{11}$$

where K_F is a positive factor representing the strength of the formation feedback.

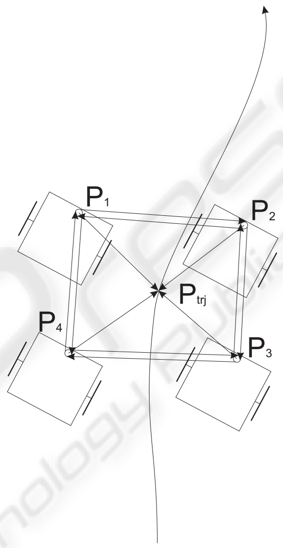

Figure 6: Virtual structure expanded with formation feedback behavior; positions of robots depend not only on the desired formation trajectory but also on positions of other robots of the formation.

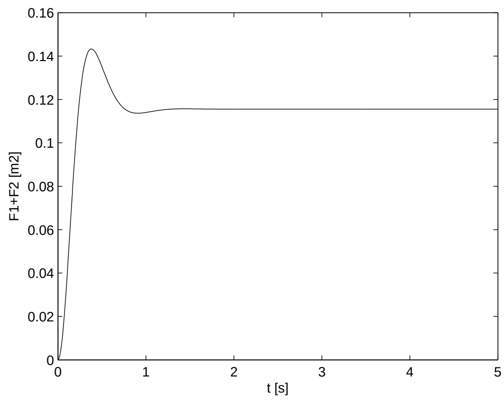

Figure 7: Formation function (time graph) for virtual structure with formation keeping behavior.

In Fig. 7 graph of formation function for formation of robots that executes the same task as in section 3 is shown. The values of control gains are: $K_P = 30$,

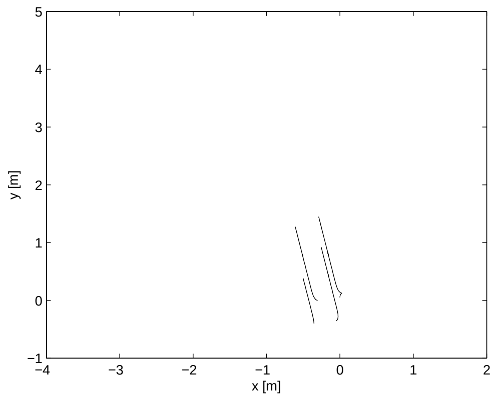

Figure 8: Formation of four robots tracks desired trajectory using virtual structure control method. Left-down robot fails after 1 second. Other robots slow down (in case without robot failure the formation goes to position around (-1.1, 4.5), like in case shown in Fig.3).

$K_F = 30$ and $K_V = 10$. It is very likely that in real application F_2 component of the formation function will be much greater due to time delay of sensor measurements and communication. Especially for large formation of robots disturbances of the motion of one robot will be transferred through the formation structure and affect motion of other robots. In this method worse accuracy is the cost paid for failure immunity.

In Fig. 8 simulation results for the case when one of robots fails is presented. Trajectory tracking and formation keeping are performed simultaneously. The priority of the goal depends on K_P/K_F ratio.

5 LEADER-FOLLOWER

In this section method based on leader-follower concept is presented. In most known leader-follower methods nonholonomic mobile robots are used. Robot called leader tracks a desired trajectory. Other robots keep desired separation and bearing to the leader. Dependencies between robots in large formations may be complex: some of them are followers and are followed by other robots at the same time.

Solution shown in this section is not typical leader-follower scheme. As methods described in previous sections this control is based on formation function.

Leader follower approach, in its simplest form, may be treated as a kind of virtual structure method. In Fig. 9 formation of four robots is shown. This control differs from virtual structure method only with reference point for formation in fact. In the pure virtual structure it is the point of the virtual trajectory. In leader-follower scheme it is position of leader robot.

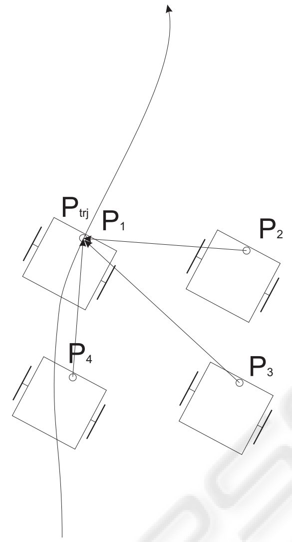

Figure 9: Leader-follower approach; one robot tracks desired formation, other robots keep relative position to the leader.

The formation function is given by the following equation:

$$F = \sum_{i=2}^{N} \|P_i - P_{1iof}\|^2, \tag{12}$$

where $P_{1iof} = [x_{1iof} \;\; y_{1iof}]$ is offset vector between i-th robot leader (robot number 1).

Control of the i-th robot is given by Eq. (8).

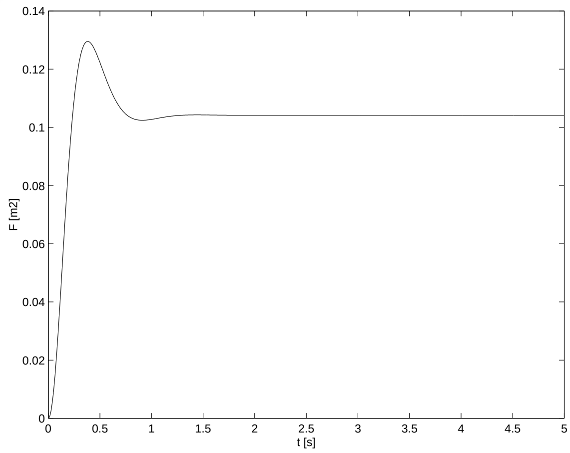

Figure 10: Formation function (time graph) for leader follower scheme.

In Fig. 10 the graph of formation function for four robots that execute the same task as in section 3 is shown.

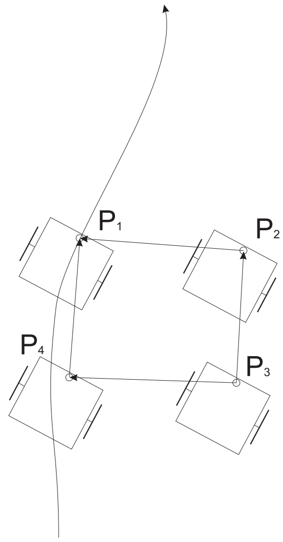

Figure 11: Leader-follower approach; one robot tracks desired formation, two other robots keep relative position to the leader, fourth robot keep relative position to followers.

In case shown in Fig. 11 the dependency between robots is constructed in a different way. The leader is followed by two robots, fourth robot keep relative position to followers and in fact they are leaders for this robot. Based on this concept very complex formations of robots with hierarchical structure may be built.

6 CONCLUSIONS

Three control methods for robot formation coordination were presented: virtual structure, virtual structure expanded with behavioral formation feedback and leader-follower scheme. Their accuracies were compared on basis of formation function graphs. Presented methods will be verified experimentally in our future work.

REFERENCES

B. J. Young, R. W. B. and Kelsey, J. M. (2001). Coordinated control of multiple robots using formation feedback. Technical report, MAGICC Lab.

Egerstedt, M. and Hu, X. (2001). Formation constrained multi-agent control. In *Proceedings of the 2001 IEEE International Conference on Robotics and Automation*, pages 3961–3966, Seoul, Korea.

Esposito, J. M. and Kumar, V. (2000). A formalism for parallel composition of reactive and deliberative control objectives for mobile robots. Technical report, Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia.

J. R. Lawton, B. J. Y. and Beard, R. W. (2000). A decentralized approach to elementary formation maneuvers. In *Proceedings of the 2000 IEEE International Conference on Robotics and Automation*, pages 2728–2733, San Francisco, CA.

J. R. Lawton, B. Y. and Beard, R. (2002). A decentralized approach to elementary formation manouvers.

J. Spletzer, A. K. Das, R. F. C. J. T. and Kumar, V. (2001). Cooperative localization and control for multi-robot manipulation. Technical report, GRASP Laboratory - University of Pennsylvania, Philadelphia.

Kar-Han Tan, M. A. L. (1997). Virtual structures for high-precision cooperative mobile robotic control. Technical report, Computer Science Department, University of California, Los Angeles.

Kowalczyk, W. and Kozlowski, K. (2005). Artificial potential based motion control for a large formation of mobile robots. In *ICRA '2005*, Barcelona, Spain.

P. Ogren, M. E. n. X. H. (2002). A control lyapunov function approach to multi-agent coordination. In *IEEE Transactions on Robotics and Automation, vol. 18, no. 5*, pages 847–851.

R. Fierro, A. K. Das, V. K. and Ostrowski, J. P. (2001). Hybrid control of formation of robots. In *Proceedings of the 2001 IEEE International Conference on Robotics and Automation*, pages 3672–3677, Seoul, Korea.

W. Kang, N. Xi, A. S. (2000). Formation control of autonomous agents in 3d workspace. In *Proceedings of the 2000 IEEE International Conference on Robotics and Automation*, pages 1755–1760, San Francisco, CA.

Yamaguchi, H. (1998). A cooperative hunting behavior by mobile robot troops. In *ICRA '98*, pages 3204–3209, Leuven, Belgium.

Yamaguchi, H. (1999). A cooperative hunting behavior by mobile-robot troops. In *The International Journal of Robotics Research, Vol, 18, No. 8*, pages 931–940.