BOUNDARY CONTROL OF A CHANNEL
Last Improvements

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Abstract: Different improvements have been developed in regards to the stability and the control of two-by-two non linear systems of conservation laws, and in particular for the Saint-Venant equations and the control of flow and water level on irrigation channel. One stability result based on the Riemann coordinates is presented here and sufficient conditions are given to insure the Cauchy convergence. Another result still based on the Riemann approach is presented too, in the linear case, to improve the feedback control based on the Riemann invariants.

1 INTRODUCTION

In this paper, we are concerned with the stability of the non linear Saint-Venant equations, a two-by-two systems of conservation laws, that are described by hyperbolic partial differential equations, with one independent time variable $t \in [0,\infty)$ and one independent space variable, $x \in [0,L]$. For such systems, the considered boundary control problem is the problem of designing feedback control actions at the boundaries (i.e. at $x = 0$ and $x = L$) in order to ensure that the smooth solution of the Cauchy problem converge to a desired steady state.
This problem has been previously considered in the literature ((Litrico et al., 2005)), and in our previous papers (Coron et al., 2002). Those results have been improved in (Dos Santos et al., 2007) in order to take account of non homogeneous terms (like perturbations, slope or frictions) adding an integral part to the Riemann control developed.
Recently, the non linear problem of the stability of systems of two conservation laws perturbed by non homogeneous terms has been investigated (Prieur et al., 2006), (Dos Santos and Prieur, 2007), using the state evolution of the Riemann coordinates.
This paper aim is to shortly present both last results develop on (Dos Santos and Prieur, 2007), (Dos Santos et al., 2007) and to illustrate them with simulations and experimentations based on a river data and the Valence micro-channel respectively.
After a short presentation of the shallow water equations, the first problem is stated, the tools presented, and the stability result established. The second result is developed in the same way in the fourth section and the simulations results are produced as well as the experimentations ones in the last part.

2 DESCRIPTION OF THE MODEL: SAINT-VENANT EQUATIONS

We consider a reach of an open channel as represented in Figure 1.
We assume that the channel is prismatic with a constant rectangular section. Note that in our configuration, the slope could be non null as well as the friction effects.
The flow dynamics are described by a system of two laws of conservation (Saint-Venant or shallow water equations), namely a law of mass conservation and a law of momentum conservation

$$\partial_t H + \partial_x (Q/B) = 0, \qquad (1)$$

$$\partial_t Q + \partial_x \left(\frac{Q^2}{BH} + \frac{1}{2} gBH^2\right) = gBH(I - J), \qquad (2)$$

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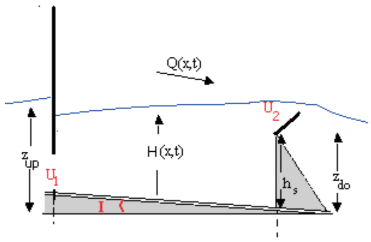

Figure 1: Scheme of a channel: one reach with an overflow gate.

where $H(t,x)$ stands for the water level and $Q(t,x)$ the water flows in the reach while g denotes the gravitation constant ($m.s^{-2}$). I is the bottom slope ($m.m^{-1}$), B is the channel width (m) and J is the slope's friction ($m.m^{-1}$).
The slope's friction J is expressed with the Manning-Strickler expression, (n_M is the Manning coefficient ($s.m^{-1/3}$) and the Strickler coefficient is $K = \frac{1}{n_M}$ ($m^{1/3}.s^{-1}$)),

$$J(Q,H) = \frac{n_M^2 Q^2}{S(H)^2 R(H)^{4/3}},$$

where $S(H)$ is the wet surface (m^2) and $P(H)$ the wet perimeter (m): $S(H) = BH, P(H) = B + 2H$, $R(H)$ is the hydraulic radius (m), $R(H) = S(H)/P(H)$.

The control actions are the positions U_0 and U_L of the two spillways located at the extremities of the pool and related to the state variables H and Q by the following expressions.
Two cases may occur for the gate equations at $x = 0$ and $x = L$:

- a submerged underflow gate:

$$Q(x_i,t) \quad = \quad U_i B \mu_i \sqrt{2g(H_1(x_i,t) - H_2(x_i,t))} \tag{3}$$

- or a submerged overflow gate:

$$H_1(x_i,t) \quad = \quad \left(\frac{Q^2(x_i,t)}{2gB^2\mu_i^2}\right)^{1/3} + h_{s,i} + U_i, \tag{4}$$

where $H_1(x,t)$ is the water level before the gate, $H_2(x,t)$ is the water level after the gate and $h_{s,i}$ is the height of the fixed part of the overflow gate n°i (Fig. (1)) and μ_i is the water flow coefficient of the gate n°i located at $x = x_i$.
Note that the system (1)-(2) is strictly hyperbolic, i.e. its Jacobian matrix has two non-zero real distinct eigenvalues:

$$\lambda_1(H,V) = \frac{Q}{BH} + \sqrt{gH},\ \lambda_2(H,V) = \frac{Q}{BH} - \sqrt{gH}.$$

They are generally called *characteristic velocities*.
The flow is said to be *fluvial* (or subcritical) when the characteristic velocities have opposite signs:

$$\lambda_2(H,V) < 0 < \lambda_1(H,V).$$

Different stability results have been given for the linearized system (Coron et al., 2007)-(Dos Santos et al., 2007) and the non-linear one (Prieur et al., 2006)-(Dos Santos and Prieur, 2007) using the properties of Riemann coordinates. Those results are quickly resumed in both following sections.

3 FIRST RESULT: INTEGRAL ACTIONS AND LYAPUNOV STABILITY ANALYSIS

3.1 Linearized System

An equilibrium (H_e, Q_e) is a constant solution of the equations (1)-(2) , i.e. $H(t,x) = H_e$, $Q(t,x) = Q_e$ $\forall t$ and $\forall x$ which satisfies the relation:

$$J(H_e, Q_e) = I. \tag{5}$$

A linearized model is used to describe the variations around this equilibrium. The following notations are introduced:

$$h(t,x) \hat{=} H(t,x) - H_e(x),\ q(t,x) \hat{=} Q(t,x) - Q_e(x).$$

The linearized model around the equilibrium (H_e, Q_e) is then written as

$$\partial_t h(t,x) + \partial_x q(t,x) = \quad 0 \tag{6}$$

$$\partial_t q(t,x) + cd\partial_x h(t,x) + (c-d)\partial_x q(t,x) \quad = \quad -\gamma h(t,x) - \delta q(t,x), \tag{7}$$

with:

$$c = \sqrt{gBH_e} + \frac{Q_e}{H_e\sqrt{B}},\ d = \sqrt{gBH_e} - \frac{Q_e}{H_e\sqrt{B}}$$

$$\gamma = gBH_e\frac{\partial J}{\partial H}(H_e,Q_e),\ \delta = gBH_e\frac{\partial J}{\partial Q}(H_e,Q_e).$$

In the special case where the channel is horizontal ($I = 0$) and the friction slope is negligible ($n \approx 0$), we observe that $\gamma = \delta = 0$ and that this linearized system is exactly in the form of the following linear hyperbolic system:

$$\partial_t h(t,x) + \partial_x q(t,x) = 0 \tag{8}$$

$$\partial_t q(t,x) + cd\partial_x h(t,x) + (c-d)\partial_x q(t,x) = 0. \tag{9}$$

It is therefore legitimate to apply the control with integral actions that have been analyzed in (Coron et al., 2007) to open channels having small bottom and friction slopes.

3.2 Riemann Coordinates and Stability Conditions

In order to solve this boundary control problem, the *Riemann coordinates* (see e.g. (Renardy and Rogers, 1993) p. 79) defined by the following change of coordinates are introduced :

$$a(t,x) = q(t,x) + dh(t,x) \quad (10)$$
$$b(t,x) = q(t,x) - ch(t,x) \quad (11)$$

With these coordinates, the system (8)-(9) is written under the following diagonal form :

$$\partial_t a(t,x) + c\partial_x a(t,x) = 0 \quad (12)$$
$$\partial_t b(t,x) - d\partial_x b(t,x) = 0 \quad (13)$$

In the Riemann coordinates, the control problem can be restated as the problem of determining the control actions in such a way that the solutions $a(t,x)$, $b(t,x)$ converge towards zero.

The boundary control laws $u_0(t)$ and $u_L(t)$ are defined such that the boundary conditions (3)-(4) expressed in the Riemann coordinates satisfy the linear relations (Coron et al., 2007) augmented with appropriate integrals as follows:

$$a_0(t) + k_0 b_0(t) + m_0 y_0(t) = 0 \quad (14)$$
$$b_L(t) + k_L a_L(t) + m_L y_L(t) = 0 \quad (15)$$

where k_0, k_L and m_0, m_L are constant design parameters that have to be tuned to guarantee the stability. The integral y_0 on the flow q at the boundary $x = 0$ and the integral y_L on the other state h at the boundary $x = L$ are defined as:

$$y_0(t) = \int_0^t q_0(s)ds = \int_0^t \frac{ca_0(s) + db_0(s)}{c+d} ds$$
$$y_L(t) = \int_0^t h_L(s)ds = \int_0^t \frac{a_L(s) - b_L(s)}{c+d} ds.$$

Using Lyapunov theory, one can prove this theorem:

Theorem 1 *Let m_0, m_L and k_0, k_L four constants such that the six following inequalities hold:*

$$m_0 > 0, \qquad m_L < 0, \quad (16)$$
$$|k_0| < 1, \qquad |k_0 k_L| < 1, \quad (17)$$
$$|k_L| < \frac{c}{d} \qquad \frac{d}{c} < 1, \quad (18)$$

Then there exist five positive constants A, B, μ, N_0 and N_L such that, for every solution $(a(t,x), b(t,x))$, $t \geqslant 0$, $x \in [0,L]$, of (12), (13), (14) and (15) the following function:

$$U(t) = \frac{A}{c}\int_0^L a^2(t,x)e^{-\mu x/c}dx + \frac{c+d}{2}N_0 y_0^2(t) + \frac{B}{d}\int_0^L b^2(t,x)e^{\mu x/d}dx + \frac{c+d}{2}N_L y_L^2(t)$$

satisfies:

$$\dot{U} \leq -\mu U.$$

Remark 1 *As it has been mentioned above, in our previous paper (Coron et al., 2007) the special case with $m_0 = m_L = 0$ in the boundary conditions (14)-(15) and $N_0 = 0$, $N_L = 0$ has been treated. We have shown that inequality $|k_0 k_L| < 1$ is sufficient to have $\dot{U} < -\mu U$ for some $\mu > 0$ along the system trajectories and ensure the convergence of $a(t,x)$ and $b(t,x)$ to zero.*

In the fifth section, we shall illustrate the efficiency of the control with simulations on a realistic model of a waterway and with experimental results on a real life laboratory plant.

4 SECOND RESULT: STABILITY OF THE NON-LINEAR SAINT-VENANT EQUATIONS

Previous result delead with the stability of two conservation laws systems, which can be written as (8)-(9) (Coron et al., 2007), i.e. for homogeneous hyperbolic systems. The stability condition depends thus of the spectral radius of the Jacobian matrix linked. In (Prieur et al., 2006), those results have been extended to the non homogeneous system, with an additional condition on the size of the non homogeneous terms. Here, we proposed a new result that improve the sufficient stability condition $|k_0 k_L| < 1$ (Dos Santos and Prieur, 2007).

4.1 Statement

In order to introduce the problem under consideration in this work, we need some additional notations:

- The usual euclidian norm $|\cdot|$ in $\mathbb{R}$ is denoted by $|\cdot|$. The ball centered in $0 \in \mathbb{R}$ with radius $\varepsilon > 0$ is denoted $B(\varepsilon)$;
- Given Φ continuous on $[0,L]$ and Ψ continuously differentiable on $[0,L]$, we denote

$$|\Phi|_{C^0(0,L)} = \max_{x\in[0,L]}|\Phi(x)|\,,$$
$$|\Psi|_{C^1(0,L)} = |\Psi|_{C^0(0,L)} + |\Psi\,'|_{C^0(0,L)}\,;$$

- the set of continuously differentiable functions $\Psi^{\#}$: $[0,L] \to \mathbb{R}$ satisfying the compatibility assumption $\mathcal{C}$ and $|\Psi^{\#}|_{C^1(0,L)} \leq \varepsilon$ is denoted $B_{\mathcal{C}}(\varepsilon)$.

For constant control actions $U_0(t) = \bar{U}_0$ and $U_L(t) = \bar{U}_L$, a *steady-state solution* is a constant solution $(H,Q)(t,x) = (\bar{H},\bar{Q})(x)$ for all $t \in [0,+\infty)$, for all $x \in [0,L]$ which satisfies (1)-(2) and the boundary conditions (3)-(4).

At time $t \geq 0$, *the output* of the system (1)-(2) is given by the following

$$y(t) = (H_0(t), H_L(t)) \tag{19}$$

The problem under consideration in this work is the following: *Given a steady-state $(\bar{H},\bar{Q})^T$, called the set point, we consider the problem of the local exponential stabilization of (1)-(2) by means of a boundary output feedback controller, i.e. we want to compute a boundary output feedback controller $y \mapsto (U_0(y), U_L(y))$ such that, for any smooth small enough (in C^1-norm) initial condition $H^\#$ and $Q^\#$ satisfying our compatibility conditions, the PDE (1)-(2) with the boundary conditions (3)-(4) and the initial condition*

$$(H,Q)(x,0) = (H^\#, Q^\#)(x) \ ,\forall x \in [0,L]. \tag{20}$$

has a unique smooth solution converging exponentially fast (in C^1-norm) towards $(\bar{H},\bar{Q})^T$.

4.2 Stability Result

First note that the system (1)-(2) is strictly hyperbolic, i.e. the Jacobian matrix of this system has two non-zero real distinct eigenvalues:

$$\lambda_1(H,Q) = \frac{Q}{BH} + \sqrt{gH},\ \lambda_2(H,Q) = \frac{Q}{BH} - \sqrt{gH}.$$

They are generally called *characteristic velocities*.

The flow is said to be *fluvial* (or subcritical) when the characteristic velocities have opposite signs:

$$\lambda_2(H,Q) < 0 < \lambda_1(H,Q).$$

Under constant boundary conditions $Q(0,t) = \bar{Q}_0$ and $H(L,t) = \bar{H}_L$, for all t, there exists a steady state solution $x \mapsto (\bar{Q},\bar{H})$ satisfying

$$\partial_x \bar{Q}(x) = 0, \qquad \partial_x \bar{H}(x) = -g\bar{H}\frac{I-\bar{J}}{\bar{\lambda}_1\bar{\lambda}_2}, \tag{21}$$

with $\bar{\lambda}_1 = \lambda_1(\bar{H},\bar{Q})$, and $\bar{\lambda}_2 = \lambda_2(\bar{H},\bar{Q})$.

Let t_1 and t_2 be the time instants defined by

$$x_1(t_1) = L \ ,\ x_2(t_2) = 0, \tag{22}$$

where x_i, $i = 1,2$, are the solution of the Cauchy problem

$$\dot{x}_i(t) = \lambda_i(\bar{H},\bar{Q}),\ x_1(0) = 0,\ x_2(L) = 0.$$

To state our stability result, we need to introduce the following notations

$$\bar{a} = (\frac{\bar{Q}}{BH} + 2\sqrt{gH})\ ,\ \bar{b} = (\frac{\bar{Q}}{BH} - 2\sqrt{gH}).$$

We can explicit functions f_1 and f_2, and expressions ℓ_1 and ℓ_2 depending on the equilibrium and on the perturbations such that, for all $i \in \{1,2\}$,

$$\ell_i = f_i(\bar{\lambda}_i, \bar{a}, \bar{b}, I, n_M)\ . \tag{23}$$

Due to space limitation, the explicit expression of ℓ_1 and ℓ_2 is omitted, it is developed in (Dos Santos and Prieur, 2007).

The boundary conditions are written as follows:

$$a(t,0) + k_0 b(t,0) \;=\; 0 \tag{24}$$
$$b(t,L) + k_L a(t,L) \;=\; 0, \tag{25}$$

where k_0, k_L are constant design parameters that have to be tuned to guarantee the stability.

We are now in position to state our stability result, here in the case of a reach bounded by two underflow gates:

Theorem 2 *Let t_1, t_2, ℓ_1 and ℓ_2 be defined by (22), and (23) respectively.*

If the bottom slope function I, the slope's friction function J are sufficiently small in C^1 norm, then we have

$$\max(t_1\ell_1, t_2\ell_2) < 1\ , \tag{26}$$

In that case, there exist k_0 and k_L such that

$$|\, k_0 k_L \,| + t_2 \,|\, k_0 \,|\, \ell_2 + t_1\ell_1 \;<\; 1, \tag{27}$$
$$|\, k_0 k_L \,| + t_1 \,|\, k_L \,|\, \ell_1 + t_2\ell_2 \;<\; 1. \tag{28}$$

The following boundary output feedback controller

$$U_0 \;=\; H_0 \frac{\frac{\bar{Q}_0}{B\bar{H}_0} - 2\sqrt{g}\alpha_0\left(\sqrt{H_0} - \sqrt{\bar{H}_0}\right)}{\mu_0\sqrt{2g(z_{up} - H(0,t))}}, \tag{29}$$

$$U_L \;=\; H_L \frac{\frac{\bar{Q}_L}{B\bar{H}_L} + 2\sqrt{g}\alpha_L\left(\sqrt{H_L} - \sqrt{\bar{H}_L}\right)}{\mu_L\sqrt{2g(H(L,t) - z_{do})}}, \tag{30}$$

where $H_0 = H(t,0)$, $H_L = H(t,L)$, $\alpha_0 = \frac{1-k_0}{1+k_0}$, and $\alpha_L = \frac{1-k_L}{1+k_L}$ make the closed loop system locally exponentially stable, i.e. there exist $\varepsilon_0 > 0$, $C > 0$ and $\mu > 0$ such that, for all initial conditions $(H^\#, Q^\#)$: $[0,L] \to (0,+\infty)$ continuously differentiable, satisfying some compatibility conditions and the inequality

$$|(H^\#, Q^\#) - (\bar{H},\bar{Q})|_{C^1(0,L)} \leq \varepsilon,$$

there exists a unique C^1 solution of the Saint-Venant equations (1)-(2), with the boundary conditions (3)-(4) and the initial condition (20), defined for all $(x,t) \in [0,L] \times [0,+\infty)$. Moreover it satisfies, $\forall t \geq 0$,

$$|(H,Q) - (\bar{H},\bar{Q})|_{C^1(0,L)} \leq C_1 e^{-\mu\, t}|(H^\#, Q^\#)|_{C^1(0,L)}\ .$$

This result is proved using Riemann coordinates formalism, the Saint-Venant equations are rewritten in Riemann coordinates. Due to the slope's friction J and the bottom slope I, it gives rise to a system of conservation laws with non-homogeneous terms. The evolution of the Riemann coordinates along the characteristic curves are estimated. This estimation could be possible as soon as the non-homogenous terms are sufficiently small. A sufficient condition in terms of the boundary conditions for the asymptotic stability of the Riemann coordinates is given. This necessary condition is written as (26) in terms of the variables H and Q.
This result is illustrated in the following part.

5 NUMERICAL SIMULATIONS AND EXPERIMENTS

In this section we applied both result on numerical simulations of a river and on an experimental setup. In both cases, the assumption (26) is satisfied, thus we succeed to design an stabilizing boundary output feedback controller. Let us note that if the inequalities (27) and (28) hold then we have

$$|\, k_0 k_L \,| < \min(1 - t_1\ell_1, 1 - t_2\ell_2). \quad (31)$$

In the same way, conditions (16)-(18) are satisfied.

5.1 Simulation Results on a River

To illustrate our results, simulations have been realized with the realistic data of a river, on the software SIC developed by the CEMAGREF. Physical parameters of this river are given in Table 1, and the gates are overflow ones.

Table 1: Parameters of one reach of the river.

parameters	$B(m)$	$L(m)$	μ
values	3	2272	0.6
parameters	slope $I(m^1.s^{-1})$	K $(m^{1/3}.s^{-1})$	
values	$1.8046e^{-4}$	60	

One series of simulations is described (Fig. 2) the initial condition are the following:
$Q_e(0) = 2m^3.s^{-1}$, $Q_e(L) = 0.7m^3.s^{-1}$, $H_e(0) = 1.41m$, $z_e(L) = 1.8m$.
The steady state to reach is defined by:
$\bar{Q}(0) = 2m^3.s^{-1}$, $\bar{Q}(L) = 0.7m^3.s^{-1}$, $\bar{H}(0) = 1.85m$, $\bar{H}(L) = 2.26m$.

Using (31), we note that the tuning parameters should satisfy $k_0k_L < k_0k_{L\,max} = 0.1682$.

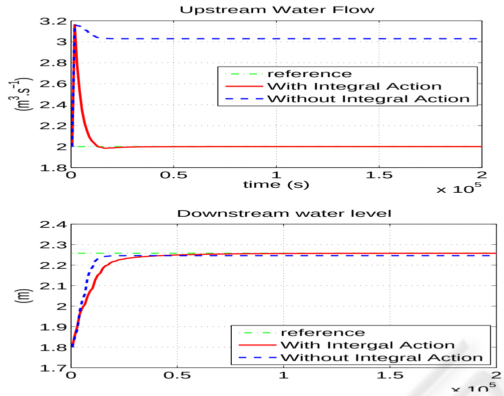

Figure 2: Water flow at upstream and level at downstream.

Two simulations are pictured, with the following values $k_0k_L = 0.0039$, and

1. $m_0 = 0 = m_L$,
2. $m_0 = -0.0001$ $m_L = 0.001$.

Other simulations with higher values of k_0k_L ($k_0k_L > k_0k_{L\,max}$) diverge in the sense that the water flow and level do not converge to the steady state required or oscillate.
All the simulations shows the well suitability of the two stability tests (27)-(28) and of the condition(31), the **three** have to be verified to insure the stability of the system.
The stability hypothesis (16)-(18) linked to the integral actions are checked, even if it is applied to the non linear system.

5.2 Experimental Results on a Micro-channel

An experimental validation has been performed on the Valence micro-channel (Tab.2). This pilot channel is located in Valence (France). It is operated under the responsibility of the LCIS[1] laboratory. This experimental channel (total length=8 meters) has an adjustable slope and a rectangular cross-section (width=0.1 meter). The channel is ended at downstream by a variable overflow spillway and furnished with three underflow control gates (Fig. (3)).

Table 2: Parameters of the channel of Valence.

parameters	$B(m)$	$L\ (m)$	$K\ (m^{1/3}.s^{-1})$
values	0.1	7	97
parameters	μ_{U_0}	μ_{U_L}	slope $(m.m^{-1})$
values	0.6	0.73	$1.6\,^0/_{00}$

[1] Laboratoire de Conception et d'Intégration des Systèmes

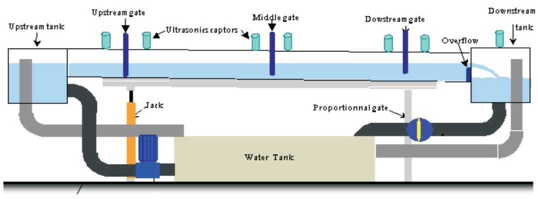

Figure 3: Pilot channel of Valence.

One experimentation has been chosen to illustrate this approach.
Note that water flow is deduced from the gate equations, and has not been measured directly. The data pictured below have been filtered to get a better idea of the experimentation results.

In each experiment, the system is initially in open loop at a steady state:
$Q_e(0) = 2.5dm^3.s^{-1}$, $H_e(0) = 1.1dm$, $H_e(L) = 1.26dm$.
The loop is closed at time $t = 50sec$ with a new set point given by:
$\overline{Q}(0) = 2dm^3.s^{-1}$, $\overline{H}(0) = 1.3dm$, $\overline{H}(L) = 1.43dm$.
Two experimentations are pictured in Fig. (4), with $\max k_0 k_L = 0.888$ and the following values:

1. $k_0 k_L = 0.247$ & $m_0 = 0$, $m_L = 0$,
2. $k_0 k_L = 0.247$ & $m_0 = -0.002$, $m_L = 0.001$.

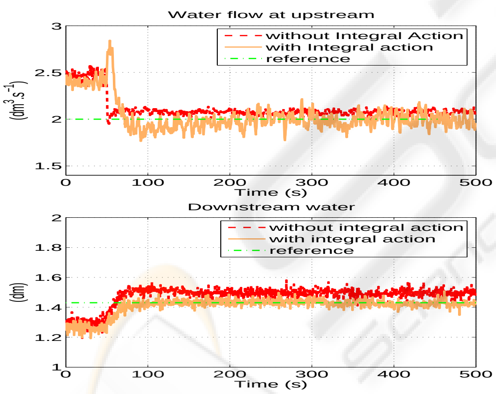

Figure 4: Water flow at upstream and level at downstream.

To conclude this part, let notice that for the micro-channel, both tests (27)-(28) are quiet equivalent (is not the case for rivers like the Sambre in Belgium). In all the cases, one conclusion is the same, the stability of the system is insured if both tests (27)-(28) and the condition (31) are realized.
Exact convergence is ensured if the integral part of the control is added even if it is applied to the real and so the non linear system.

6 CONCLUSION

In this paper, a boundary control law with integral action is proposed, as a new stability condition depending on the Riemann coordinates. Simulations and experimentations realized strengthen on the fact that the stability conditions (16)-(18) can be developed to fit to the non linear case. Improvements will be the development of the works on (Dos Santos et al., 2007) to non linear system of conservation laws, and/or to couple both previous results and generalize them to greater dimension systems.

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