COMPARYING A TABU SEARCH PROCESS
Using and Not Using and Intensification Strategy to Solve the Vehicle Routing Problem

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Abstract: In this paper we propose a Tabu Search algorithm to solve the Vehicle Routing Problem. The Vehicle Routing Problem are usually defined as the problem that concerns in creation of least cost routs to serve a set of clients by a fleet of vehicles. We develop an intensifications strategy to diversify the neighbours generated and to increase the neighbourhood size. We had done experiments using and not using the intensification strategy to compare the performance of the search. The experiments we had done showed that an intensification strategy allows an increase on the solutions quality.

1 INTRODUCTION

The Vehicle Routing Problem (VRP) is a NP-Hard problem (Lenstra and Rinooy Kan, 1981) that is usually dealt within the logistic context (Ho and Haugland, 2004; Xu and Kelly, 1996). It can be described as a set of customers that have to be served by a fleet of vehicles, satisfying some constraints (Laporte, 1992; Xu and Kelly, 1996). The Transport is one of the most costly activities in logistic, typically varying in one or two thirds of the total costs (Ballou, 2001). Therefore, the necessity of improving the efficiency of this activity has great importance. A small percentage saved with this activity could result in a substantial saving total (Bodin, Golden and Assad, 1983). There many variants and constraints that can be considered, i.e. it can be considered that the fleet may be heterogeneous the vehicles must execute collections and deliveries, there may exist more than one depot, etc. In this paper we are dealing with the classic version of this problem, were just the vehicle capacity constraint are considered.

2 THE VEHICLE ROUTING PROBLEM

A classical definition is presented in Barbarasolgu and Ozgur (Barbarasolgu and Ozgur, 1999). The VRP is defined in a complete, undirected graph G=(V.A) where a fleet of Nv vehicle of homogeneous capacity is located. All remaining vertices are customers to be served. A non-negative matrix $C=(c_{ij})$ is defined on A representing the distance between the vertices. The costs are the same in both directions. A non-negative demand di, is associated with each vertex representing the customer demand at vi. The routes must start and finish at the depot. The clients must be visited just once, by only one vehicle and the total demand of the route can't exceed the capacity Qv of the vehicle. In some cases, there is a limitation on the total route duration. In this case, t_{ij} is defined to represent travel time for each (vi,vj), ti represents the service time at any vertex vi and is required that the total time duration of any route should not exceed Tv. A typical formulation based on Barbarasoglu and Ozgur ones (Barbarasolgu and Ozgur, 1999) are used in this paper:

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$$\text{Minimize} \sum_i \sum_j \sum_v c_{ij} X_{ij}^v \tag{1}$$

Subject:

$$\sum_i \sum_v X_{ij}^v = 1 \text{ for all j} \tag{2}$$

$$\sum_j \sum_v X_{ij}^v = 1 \text{ for all i} \tag{3}$$

$$\sum_i X_{ip}^v - \sum_j X_{pj}^v = 0 \text{ for all p,v} \tag{4}$$

$$\sum_i d_i \left(\sum_j X_{ij}^v \right) <= Q_v \text{ for all v} \tag{5}$$

$$\sum_{j=1}^{n} X_{0j}^v <= 1 \text{ for all v} \tag{6}$$

$$\sum_{i=1}^{n} X_{i0}^v <= 1 \text{ for all v} \tag{7}$$

$$X_{ij}^v \in Z \text{ for all i,j and v} \tag{8}$$

Where X_{ij}^v are binary variables indicating if arc(vi,vj) is traversed by vehicle v. The objective function of distance/cost/time is expressed by eq. (1). Constraints in eqs (2) and (3) together state that each demand vertex is served by exactly one vehicle. The eq. (4) guarantees that a vehicle leaves the demand vertex as soon as it has served the vertex. Vehicle capacity is expressed by (5) where Q_v is the capacity. Constraints (6) and (7) express that vehicle availability can't be exceeded. The sub tour elimination constraints are given in eq.(8) where Z can be defined by:

$$Z = \left\{ \left(X_{ij}^v \right) : \sum_{i \in B} \sum_{j \in B} X_{ij}^v <= |B| - 1 \right. \tag{8}$$

$$\left. \text{for } B \subseteq V / \{0\}; |B| >= 2 \right\}$$

3 RESOLUTIONS METHODS

Since VRP is Np-Hard to obtain good solutions in an acceptable time, heuristics are used and this is the reason why the majority of researchers and scientists direct their efforts in heuristics development (Thangiah and Petrovik, 1997; Nelson et al, 1985; Xu and Kelly, 1996). Osman and Laporte (Osman and Laporte, 1996) define heuristic as a technique, which seeks good solutions at a reasonable computational cost without being able to guarantee the optimality. Laporte et al (Laporte et al, 2000) define two main groups of heuristics: classical heuristics, developed mostly between 1960 and 1990, and metaheuristics. The classical heuristics are divided in three groups: constructor methods, two-phase methods and improvement methods. Since 1990, the metaheuristics have been applied to the VRP problem. To Osman and Laporte (Osman and Laporte, 1996) a metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space in order to find efficiently near-optimal solutions. Several metaheuristics have been proposed to solve the VRP problem. Among these ones, Tabu Search are considered the best metaheuristic for VRP. To review some works with Tabu Search and others metaheuristics some readings are suggested (Cordeau et al, 2002; Tarantilis et al, 2005).

3.1 Tabu Search

It was proposed by Glover (Glover, 1989) and had its concepts detailed in Glover and Laguna (Glover and Laguna, 1997). It's a technique to solve optimization combinatorial problems (Glover, 1989) that consists in an iterative routine to construct neighborhoods emphasizing the prohibition of stopping in an optimum local. The process that Tabu Search searches for the best solution is through an aggressive exploration (Glover and Laguna, 1997), choosing the best movement for each iteration, not depending on if this movement improves or not the value of the actual solution. In Tabu Search development, intensification and diversification strategies are alternated through the tabu attributes analysis. Diversification strategies direct the search to new regions, aiming to reach whole search space while the intensification strategies reinforce the search in the neighborhood of a solution historically good (Glover and Laguna, 1997). The stop criterion makes it possible to stop the search. It can be defined as the interaction where the best results were found or as the maximum number of iteration without an improvement in the value of the objective function. The tabu list is a structure that keeps some solution's attributes that are considered tabu. The objective of this list is to forbid the use of some solutions during some defined time.

4 VRP APPLICATION

The application developed are divided into three modules:

a) Net Generation module: This module generates the nets that will be used in the application using vertices coordinates and demands given.
b) Initial Solution module: This module generates the initial solutions of the nets. The initial solutions are created thought the use of an algorithm implementing the Nearest Insertion heuristic (Tyagi, 1968; Cook et al, 1998).
c) Tabu Search module: This module performs the tabu search algorithm. The Tabu Search elements that were used are now detailed. The stop criterion adopted is the maximum number of iterations without any improvement in the value of the objective value. The tabu list keeps all the routes and the cost of the solution, forbidden these routes to be used together during the tabu tenure defined. And elite solution list is used to keep the best results that were found during the search. It was proposed an intensification strategy to be used every time when the search executes 15 iterations without an improvement in the objective function value. In this strategy we visit every solution that is in elite list generating a big neighbourhood for each one. There were defined two movements to neighbourhood generation. V1, that makes the exchange of vertices and V2, that makes the relocation of vertices. In V1, one route r1 is selected and than one vertex of this route is chosen. We try to exchange this vertex with every vertex of all the other routes. The exchange is done if the addition of the two new demands doesn't exceed the vehicle's capacity of both routes. This procedure is done for every vertex of the route r1. To every exchange that is made, one neighbour is generated. In V2, we select one route and choose one vertex and then we try to reallocate it into all others routes, if it doesn't exceed the vehicle capacity of the route. When a vertex can be insert into a route, we try to insert it into all possible positions inside this route. To every position that a vertex is inserted, one neighbour is generated.
When these movements are used in the search with intensification, they are called V1' and V2' because with intensification, not only one route is selected like in V1 and V2, but also all routes of the solution are chosen. Aiming increase the neighbourhood size and the diversification between the solutions, we proposed to use the movements alone and together.

5 COMPUTACIONAL EXPERIENCE

The computational experiments were conducted on problems 1, 2, 3, 4 and 5 of Christofides Mingozzi and Toth (Christofides Mingozzi and Toth, 1979). These problems contain 50, 75, 100, 150 and 199 vertices and one depot respectively and they are frequently used in papers for tests purposes. The objective of the experiments was to compare the search process using and not using intensification strategy using the different movements proposed. There were proposed 9 values to Nbmax {100, 250, 500, 750, 1000, 1250, 1500, 1750, and 2000} and 6 values to Tabu List size {10, 25, 50, 75, 100, 200}. For every problem, the experiments were divided into 6 groups according with the used movements. Table 1 shows these groups.

Table 1: Groups of experiments divided by movements.

Search mode	Used movement
Using Intensification	V1
Using Intensification	V2
Using Intensification	V1,V2
Not Using Intensification	V1 + V1'
Not Using Intensification	V2 + V2'
Not Using Intensification	V1V2 + V1' V2'

There were generated 54 experiments for each group combining all values proposed to Nbmax with all Tabu List size. So, for each problem there were generated 162 experiments using intensification and 162 experiments not using it. Two types of analyses were done. In one type it was evaluated the best result obtained for a fixed value of Nbmax used with all Tabu List size and in other type it was evaluated the best results obtained for a fixed size of Tabu List used with all values proposed to Nbmax. Analyses had also been made comparing the best result found for each group of experiment, in this case comparing the quality of the different movements.

5.1 Analysing the Nbmax Variation for Each Tabu List Size

By analysing the results in this perspective it will be evaluate the variation of the Nbmax for each Tabu List size. The objective is verified if big values of Nbmax can improved the quality of Tabu Search process. We create a "lower average" for the average from results obtained with Nbmax = 100 and Nbmax = 250 and an "upper average" for the average from results obtained with Nbmax = 1750 and 2000. For all problems, analysing each one of the 6 groups of experiments done, the "upper average" were always lesser than the "lower average", indicating that big values of Nbmax can improve the search quality. Figure 1 shows an example of the graphics generated with the results of the search process. It's clear to see that an increase in Nbmax value can improve the search quality, by decreasing the results costs of the solutions.

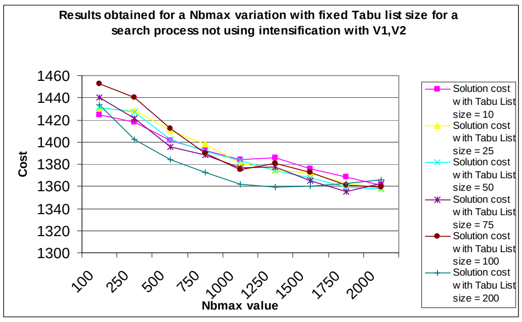

Figure 1: Costs obtained from Nbmax variation with different Tabu List size for problem 5 using V1 and V2 without intensification.

Tables 2 to 6 show the number of best results that were found in each Nbmax value:

Table 2: Quantity and localization of the best Results found for problem 1.

	500	750	1000	1250	1500	1750	2000
Int.	1	1	3	3	3	4	3
No Int.	0	0	1	2	4	6	5

Table 3: Quantity and localization of the best Results found for problem 2.

	500	750	1000	1250	1500	1750	2000
Int.	1	0	1	3	4	2	7
No Int.	0	0	1	2	3	6	6

Table 4: Quantity and localization of the best Results found for problem 3.

	500	750	1000	1250	1500	1750	2000
Int.	0	1	0	2	1	4	10
No Int.	0	1	0	1	2	1	12

Table 5: Quantity and localization of the best Results found for problem 4.

	500	750	1000	1250	1500	1750	2000
Int.	0	0	1	2	3	1	11
No Int.	0	0	1	4	1	2	10

Table 6: Quantity and localization of the best Results found for problem 5.

	500	750	1000	1250	1500	1750	2000
Int.	0	0	2	3	0	1	12
No Int.	0	0	0	1	1	5	11

These tables shown that most best results were found when the search used big values of Nbmax.

5.2 Analysing the Tabu List Size Variation for Each Nbmax Value

By analysing the results in this perspective it will be evaluate the variation of the tabu list size for each Nbmax value. The objective is verified if big Tabu list size can improved the quality of Tabu Search process. For problem 1, not using intensification, 66,66% of the results were found with Tabu list size >=75. Using intensification it was 48,14%. For problem 2 the percentage were 55,55% and 59,25%, not using and using intensification. For problem 3 the percentage were 77,77% and 96,29% not using and using intensification. For problem 4 these percentage were 74,05% and 77,77% and for problem 5 they were 63,88% and 92,59%. So by analysing these results we can see that big tabu list size can improve the quality of the search process.

5.3 Comparing the Search Process using and not using Intensification

This analysis intend to compare the search process using and not using the intensification strategy to see if it can improve the results generated.
Figure 2 shows an example of the graphics done with the results obtained in both search process to compare the quality of the different search process.

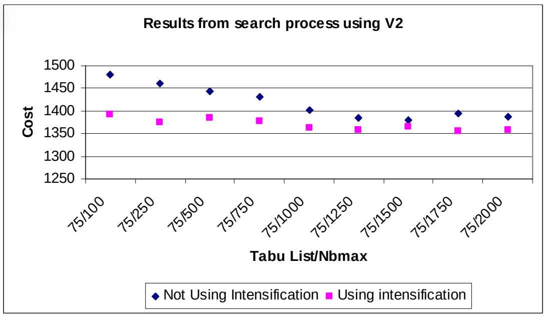

Figure 2: Costs obtained from both search process for Tabu List size = 75 and using V2 for problem 5.

This figure shows that an intensification strategy increase all results of the search process using V2 for problem 5. A comparison with the results generated by the search process using and not using intensification was done. Figures 3 to 7 shows the percentage of results that were improved with the intensification strategy. Figure 3 shows that for problem 1, from 162 results that were generated 97 were improved with intensification.

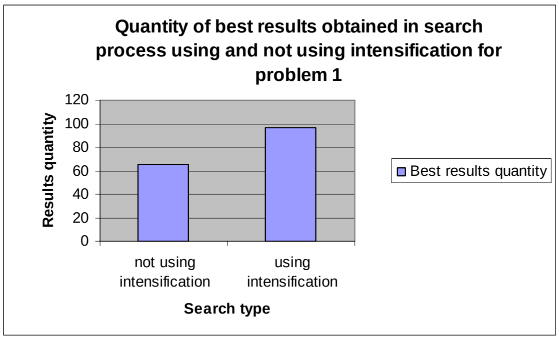

Figure 3: Improve caused by the intensification search for problem 1.

For problem 2, from 162 results, 121 were improved using intensification strategy.

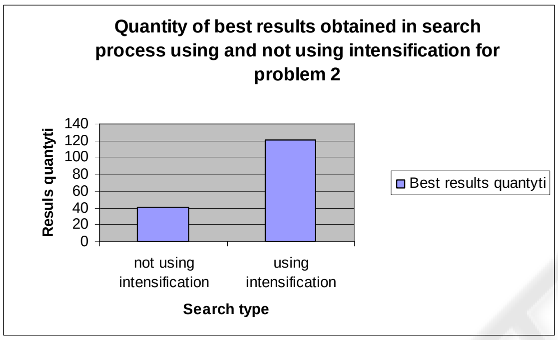

Figure 4: Improve caused by the intensification search for problem 1.

For problem 3, from 162 results 102 were improved using intensification strategy.

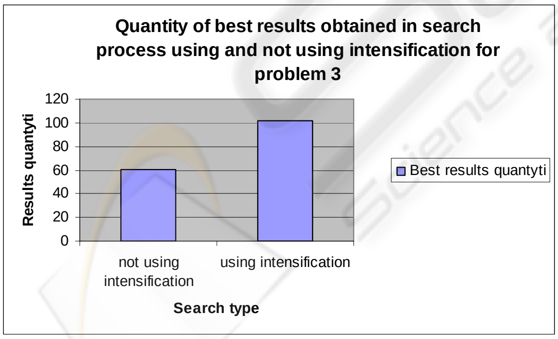

Figure 5: Improve caused by the intensification search for problem 3.

For problem 4, from 162 results 117 were improved using intensification strategy.

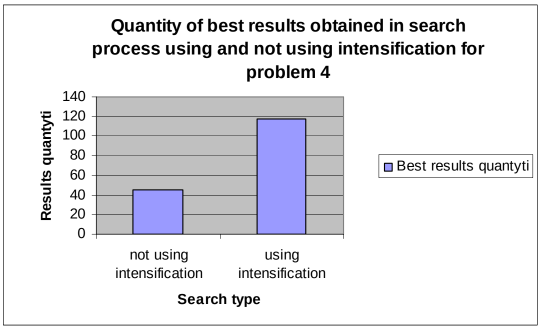

Figure 6: Improve caused by the intensification search for problem 4.

For problem 5, from 162 results 135 were improved using intensification strategy.

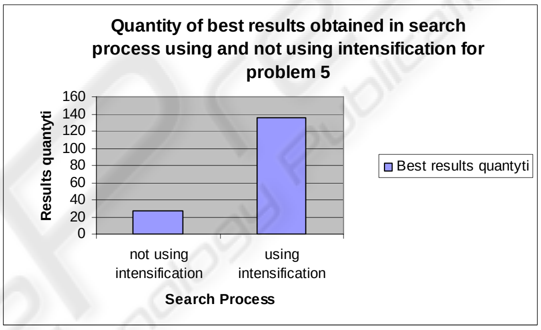

Figure 7: Improve caused by the intensification search for problem 5.

The figures show that an increase in solution quality of, at least, 50% happens when intensification strategy is used
The average results and the standard deviation are shown for problem 1, 2, 3, 4 and 5 in tables 7 to 11.

Table 7: Average results and standard deviation for problem 1.

Problem 1				
	Average		Standard Deviation	
	Not Using Intensive		Using Intensive	
V1	657,55	650,95	24,79	28,30
V2	582,08	570,30	21,16	22,21
V1,V2	537,36	542,01	8,56	13,62

Table 8: Average results and standard deviation for problem 2.

Problem 2				
	Average		Standard Deviation	
	Not Using Intensive.		Using Intensive	
V1	951,07	943,02	21,68	25,84
V2	895,75	883,03	20,67	21,71
V1,V2	867,96	863,06	13,00	14,70

Table 9: Average results and standard deviation for problem 3.

Problem 3				
	Average		Standard Deviation	
	Not Using Intensive.		Using Intensive	
V1	954,01	948,12	19,60	16,95
V2	903,63	901,17	14,50	17,84
V1,V2	879,90	870,10	15,05	20,73

Table 10: Average results and standard deviation for problem 4.

Problem 4				
	Average		Standard Deviation	
	Not Using Intensive.		Using Intensive	
V1	1215,35	1210,79	12,79	10,70
V2	1124,93	1118,83	15,66	12,52
V1,V2	1087,72	1079,54	18,15	16,70

Table 11: Average results and standard deviation for problem 5.

Problem 5				
	Average		Standard Deviation	
	Not Using Intensive.		Using Intensive	
V1	1569,11	1561,59	10,12	16,22
V2	1421,41	1393,56	34,40	12,75
V1,V2	1387,93	1377,82	26,64	16,37

From the results presented in tables 7 to 11 we can see that the results generated by the movements grouped are better than the results obtained using the movements alone. The reason for this is that when movements are used together the size of the neighbourhood generated is bigger than the neighbourhood generated by V1 or V2 alone. The movements together also cause an increase of the diversification of the solutions. And when the search generates more results, it is doing a deeper search in the space. Of course, as was shown, the intensification strategy helps the search to produce more qualified results.

When comparing the V1 and V2 movements, we can see that V2 produce results more qualified. If we analyse the policy behind the movement, we can say that V2 is more flexible than V1. V1 needs that two constraints are satisfied to generate one neighbour. While in V2, just one demand capacity must be verified (the capacity of the vehicle that serve the route where the vertex are being allocated) in V1, both routes must be verified to see if the vehicles capacities aren't exceeded.

Next table shows the best results obtained for each problem. All the best results were obtained during the search using movements V1 and V2 together and with the intensification strategy.

Table 12: Best results obtained.

Problem	Best Result
1	525,42
2	847,82
3	837,79
4	1061,07
5	1352,74

5.4 Comparisons

Aiming to evaluate the quality of the application developed, some papers were selected from the literature to compare the results. There were selected some classical heuristic and some papers that also used Tabu Search to solve the VRP. The paper selected were: {WL} Willard (1989), {PF}Pureza and França (1991), {OM1} Osman (1991), {OM2} Osman (1993), {RG} Rego (1998), {GHL} Gendreau, Hertz and Laporte (1994), {BO} Barbarasoglu and Ozgur (1999), {XK} Xu and Kelly (1996), {TV} Toth and Vigo (2003), {CW} Clarke and Wright (1964), {GM} Gillet and Miller (1974), {MJ} Mole and Jamenson (1976), {CMT} Christofides, Mingozzi and Toth (1979).

Table 13 shows the comparison done with the results from the papers. The results were obtained in Barbarasolgu and Ozgur (Barbarasolgu and Ozgur, 1999) and in Gendreau, Hertz and Laporte (Gendreau, Hertz and Laporte, 1994). In the first columns the paper used is shown. Columns 2 and 4 present the best results from the paper and columns 3 and 5 shows the difference in percentage from the results obtained in this paper to the paper compared. This difference was called "gap". The (+) indicate that our result is that percentage more than the result from the paper. The (-) indicate that our results is that percentage minor than the result from the paper.

Table 13: Best Results and gap for problem 1 and 2.

	Problem 1		Problem 2	
	Best	%Gap	Best	%Gap
WL	588	11,91(-)	893	5,33(-)
RG	557,86	6,17(-)	847	0,10(+)
PF	536	2,10(-)	842	0,69(+)
OM1	524,61	0,15(+)	844	0,45(+)
OM2	524,61	0,15(+)	844	0,45(+)
GHL	524,61	0,15(+)	835,77	1,42(+)
BO	524,61	0,15(+)	836,71	1,31(+)
XK	524,61	0,15(+)	835,26	1,48(+)
TV	524,61	0,15(+)	838,60	1,09(+)
CW	578,56	10,11(-)	888,04	4,74(-)
GM	546	3,92(-)	865	2,03(-)
MJ	575	9,44(-)	910	7,33(-)
CMT	534	1,63(-)	871	2,73(-)

Table 14: Best Results and gap for problem 3 and 4.

	Problem 3		Problem 4	
	Best	%Gap	Best	%Gap
WL	906	8,14(-)	-	-
RG	832,04	0,69(+)	1074,21	1,31(+)
PF	851	1,58(-)	1081	1,88(-)
OM1	835	0,33(+)	1052	0,85(+)
OM2	838	0,03(-)	1044,35	1,58(+)
GHL	829,45	1,00(+)	1036,16	2,35(+)
BO	828,72	1,08(+)	1043,89	1,62(+)
XK	826,14	1,39(+)	1029,56	2,97(+)
TV	828,56	1,10(+)	1028,42	3,08(+)
CW	878,70	4,88(-)	1204	13,47(-)
GM	862	2,89(-)	1079	1,69(-)
MJ	882	5,28(-)	1259	18,65(-)
CMT	851	1,58(-)	1093	3,01(-)

Table 15: Best Results and gap for problem 5.

	Problem 5	
	Best	%Gap
WL	-	-
RG	1352,88	0,014(-)
PF	-	-
OM1	1354	0,09(-)
OM2	1334,55	1,34(+)
GHL	1322,65	2,22(+)
BO	1306,16	3,44(+)
XK	1298,58	4,00(+)
TV	1291,45	4,53(+)
CW	1540	13,84(-)
GM	1389	2,68(-)
MJ	1545	14,21(-)
CMT	1418	4,82(-)

By analysing these tables we can see that our application produce more qualified results than all the classical heuristics used in comparison because our result was better than all of the heuristic results. Comparing with other tabu search algorithm, we can say that our algorithm is very competitive. It dominates at least 2 results from the 9 used for each problem. Moreover, the results generated were less than 5% of the other results for all cases. And in 25 cases out of 45 this percentage is minor than 2%.

6 FINAL CONSIDERATIONS

In this paper it was proposed an application using Tabu Search to solve the vehicle routing problem. This application was divided into 3 modules: a net generation module, an initial solution module and tabu search module. We used two movements based in relocation of vertices and exchange of vertices to create the neighbourhood. We use the movements alone and together, intending to diversify the solutions. We used an elite list solution to keep the best results found during the search. We propose an intensification strategy to use every time the search executes 15 iterations without improvement in objective value. We proposed some experiments to test if the solution quality increase or not with the increase in Nbmax value and in Tabu List size. We also compare the search process using and not using intensifications intending to see if this solution´s quality is improved with the Intensification strategy. The experiments showed that big values to Nbmax and Tabu list size could improve the results. From the experiments we also can see that an intensification strategy can improve the quality of the search.

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