TRAJECTORY PLANNING USING OSCILLATORY CHIRP FUNCTIONS APPLIED TO BIPEDAL LOCOMOTION

Fernando Juan Berenguer Císcar
Escuela Superior Politécnica, Universidad Europea de Madrid,C/ Tajo s/n, Villaviciosa de Odón, Spain

Félix Monasterio-Huelin Maciá
E.T.S.I. de Telecomunicación,Universidad Politécnica de Madrid, C/ Ciudad Universitaria s/n, Madrid, Spain

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Abstract: This work presents a method for planning sinusoidal trajectories for an actuated joint, so that the oscillation frequency follows linear profiles, like trapezoidal ones, defined by the user or by a high level planner. The planning method adds a cubic polynomial function for the last segment of the trajectory in order to reach a desired final position of the joint. We apply this planning method to an underactuated bipedal mechanism which gait is generated by the oscillatory movement of its tail. Using linear frequency profiles allow us to modify the speed of the mechanism and to study the efficiency of the system at different speed values.

1 INTRODUCTION

Trapezoidal and linear velocity profiles are widely used in trajectory planning for mobile robots and robot manipulators. The necessary procedure for defining this kind of trajectories can be found in many robotic textbooks (Spong, 1989, Sciavicco, 1996, Craig, 2006). In other robotic areas, as in the case of walking machines and nonholonomic locomotion systems, the robot joints execute oscillatory motions, and some planning methods are based on using sinusoidal trajectories with constant frequencies (Morimoto, 2006, Sfakiotakis, 2006, Murray, 1993). In (Berenguer, 2006) we presented an underactuated bipedal mechanism that is able to walk using only one actuator that moves a tail following a sinusoidal trajectory. The displacement velocity of these systems depends on the oscillation frequencies together with other parameters. The planning method presented here provides continuous sinusoidal joint trajectories that follow desired piecewise-linear frequency profiles and generate smooth variation of the systems speed.

On the other hand, swept sinusoids (chirps) are usually used for identifying and modelling actuators and mechanisms (McClung, 2004, Leavitt, 2006). In this work, we estimate the optimal stride frequency of a biped by means of analyzing the step length at different frequencies, during the execution of a trajectory generated by the proposed planning method.

This paper is organized as follows. Next section presents an initial example problem and shows perhaps a common beginner's mistake in the way of solving this problem. The correct problem's solution is also provided in this section. Section III sets out the planning problem in a general form and presents the proposed solution method. Section IV shows the application of this method to a bipedal model and how we can use the results for analyzing the efficiency of the model at different speeds. Finally, section V presents conclusions and future work.

2 AN INITIAL EXAMPLE

Suppose we want to generate a sinusoidal trajectory with unit amplitude for a robot joint. The joint is initially at its central position (0rad) with zero velocity, and will start to oscillate with an increasing frequency. The joint must reach a frequency of πrad/s at instant t=10s and keep this value during 20 seconds. Finally, the joint must reduce its frequency to zero and achieve the central position again at

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instant t=40s. Figure 1 shows this trapezoidal frequency profile and a continuous joint trajectory q(t) that follows it and finishes at zero position.

As a first solution to this problem, a beginner might propose the expression in (1) for the joint trajectory q(t). This is a sine function and we can check that the function ω(t) follows the trapezoidal profile.

$$q(t) = \sin(\omega(t)t) \quad \text{where}$$
$$\omega(t) = \begin{cases} \pi\dfrac{t}{10} & 0 \le t < 10 \\ \pi & 10 \le t < 30 \\ \pi\dfrac{(40-t)}{10} & 30 < t \le 40 \end{cases} \tag{1}$$

Figure 2 shows this function (1) and of course this is not the correct solution. We can see the reason for this result by analyzing the time derivative $\dot{q}(t)$ given by (2).

$$\dot{q}(t) = \begin{cases} 2\pi\dfrac{t}{10}\cos(\pi\dfrac{t^2}{10}) & 0 \le t < 10 \\ \pi\cos(\pi t) & 10 \le t < 30 \\ \pi\dfrac{(40-2t)}{10}\cos(\pi\dfrac{(40-t)}{10}t) & 30 \le t \le 40 \end{cases} \tag{2}$$

First, at instant t=10s, the left value of $\dot{q}(t)$ is twice as much as the right value, and we can see this effect in the slope change of q(t) in figure 2. We find a similar result at time t=30s, where there is a discontinuity in $\dot{q}(t)$ from π to -2π. It is also unexpected that $\dot{q}(t)$ increases during the last trajectory segment and at t=40s, when ω(t)=0rad/s and we expected zero velocity, its value is -4π rad/s.

The right solution for this example problem is (3), where θ(t) is the phase of the sinusoidal function, and its time derivative $\dot{\theta}(t)$ also follows the trapezoidal profile in figure 1.

$$q(t) = \sin(\theta(t)) \quad \text{where}$$
$$\theta(t) = \begin{cases} \pi\dfrac{t^2}{20} & 0 \le t < 10 \\ \pi t + \pi & 10 \le t < 30 \\ \pi\dfrac{(40-t)^2}{20} & 30 \le t \le 40 \end{cases} \tag{3}$$

Using (3) the final value of q(t) is the desired zero value. In a general problem, a desired final value of q(t) can't be reached if we use only linear functions in the profile and impose time instants and frequency values. We will see that we need another degree of freedom using, for example, a last quadratic function in the profile, to obtain a desired final value for the sinusoidal trajectory.

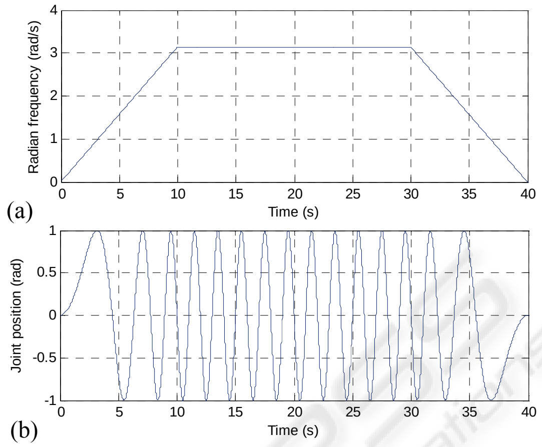

Figure 1: (a) Desired trapezoidal frequency profile and (b) desired trajectory for an actuated robot joint.

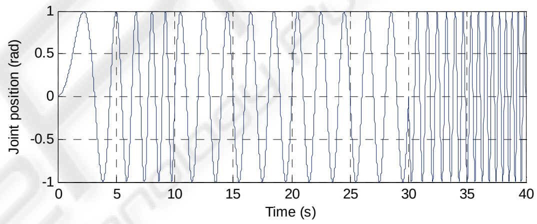

Figure 2: Graphical representation of function (1).

2.1 Basic Definitions

Given a sinusoidal function (sine or cosine) like (4), the argument θ(t) is the instantaneous phase and its time derivative is the instantaneous radian frequency. When θ(t) varies linearly with time, its time derivative is constant and its value is the radian frequency, as in the time interval [10s, 30s] in (3).

$$f(t) = A\sin(\theta(t)) \tag{4}$$

When the phase is quadratic, as in the first and last intervals in (3), the instantaneous radian frequency varies linearly between two values and f(t) is called a chirp function. So, the profile in figure 1 shows the instantaneous radian frequency of a sinusoidal function and represents the concatenation of chirp functions. We will consider here constant frequency sinusoids as a subset of chirp functions, with the same initial and final frequencies.

3 INTERPOLATION OF CHIRP FUNCTIONS

We now present a method for planning trajectories without discontinuities in the joint position and velocity by means of the concatenation of chirp functions. First we obtain the solution without a desired final position, and in section 3.1 we add this constraint to the problem and solve it using a final cubic phase function.

Problem statement: Given a set of N+1 time instants t_i (for i=1 to N+1), and a set of N+1 desired radian frequencies ω_i at each instant t_i, find a set of N chirp functions $f_i(t)$ so that their concatenation represents a continuous trajectory with amplitude A, initial value $q(t_1)=q_1$, and which instantaneous frequency interpolates the frequencies ω_i.

To solve this problem, we will use the function family given by (5).

$$f_i(t) = A\sin(\theta_i(t)) \quad \text{where}$$
$$\theta_i(t) = \begin{cases} 0 & t < t_i \\ a_i(t-t_i)^2 + b_i(t-t_i) + c_i & t_i \le t < t_{i+1} \\ 0 & t_{i+1} \le t \end{cases} \tag{5}$$

The problem centres on finding the coefficients of the phases $\theta_i(t)$, and it is basically the same problem of interpolating trajectories with linear velocity profiles.

The set of conditions that allow us to solve this problem is:

$$\begin{aligned} &\theta_1(t_1) = \arcsin(q_1/A) = c_1 \\ &\theta_i(t_i) = \theta_{i-1}(t_i) = c_i \\ &\dot{\theta}_i(t_i) = \omega_i = b_i \\ &\dot{\theta}_i(t_{i+1}) = \omega_{i+1} = 2a_i(t_{i+1}-t_i) + b_i \end{aligned} \tag{6}$$

From these expressions, the values of a_i and b_i are directly calculated, and the c_i coefficients, that represent the initial phase in each profile's segment, must be calculated iteratively:

$$\begin{aligned} &a_i = \frac{\omega_{i+1}-\omega_i}{2(t_{i+1}-t_i)} \\ &b_i = \omega_i \\ &c_1 = \arcsin(q(t_1)/A) \\ &c_i = a_{i-1}(t_i-t_{i-1})^2 + b_{i-1}(t_i-t_{i-1}) + c_{i-1} \end{aligned} \tag{7}$$

The main problem of this solution is that we can not establish a desired final value of the joint position.

3.1 Concatenation of Chirp Functions with a Final Cubic Phase

Usually, a planned trajectory finishes with zero velocity, and in these cases it is interesting also to reach a desired joint position q_N. This condition adds a new constraint for selecting the last trajectory function $f_N(t)$ and therefore the phase $\theta_N(t)$ needs another degree of freedom, that is, we need to use a cubic function instead of a quadratic one:

$$f_N(t) = A\sin(\theta_N(t)) \quad \text{where}$$
$$\theta_N(t) =$$
$$= \begin{cases} 0 & t < t_N \\ k(t-t_N)^3 + l(t-t_N)^2 + m(t-t_N) + n & t_N \le t < t_{N+1} \\ 0 & t_{N+1} \le t \end{cases} \tag{8}$$

The set of conditions that $\theta_N(t)$ must satisfy is:

$$\begin{aligned} &q_N = A\sin(\theta_N(t_{N+1})) \\ &\theta_N(t_N) = \theta_{N-1}(t_N) \\ &\dot{\theta}_N(t_N) = \omega_N \\ &\dot{\theta}_N(t_{N+1}) = \omega_{N+1} \end{aligned} \tag{9}$$

The first of these conditions has many solutions, so we will find the solution corresponding to an almost linear profile, that is, the final phase value will be the nearest to the final value that we will obtain from (7) for i=N. To obtain the coefficients k, l, m and n, we use the next procedure:

1-. First, we calculate the final phase in the quadratic case using the coefficients a_N, b_N and c_N obtained from (7), and also the integer number of revolutions around the unit circle:

$$\theta_{quad} = a_N(t_{N+1}-t_N)^2 + b_N(t_{N+1}-t_N) + c_N \tag{10}$$

$$r = \text{floor}(\theta_{quad}/2\pi) \tag{11}$$

2-. Next, we select an angle $\alpha \in [0, 2\pi)$, in the same side of the unit circle as θ_{quad}, that satisfies the first condition in (9).

$$\alpha = \begin{cases} \pi - \arcsin(q_N/A) & \cos(\theta_{quad}) < 0 \\ \arcsin(q_N/A) & \cos(\theta_{quad}) \ge 0 \end{cases} \tag{12}$$

If $\alpha<0$, we will add 2π ($\alpha=\alpha+2\pi$).

3-. The desired phase at t_{N+1} is then given by (13).

$$\theta_N(t_{N+1}) = \theta_{N+1} = \alpha + 2r\pi \qquad (13)$$

4-. Finally, we calculate the coefficients by means of (14). These expressions are obtained from (13) and the last three conditions in (9).

$$\begin{aligned}
n &= \theta_{N-1}(t_N) \\
m &= \omega_N \\
l &= \frac{3(\theta_{N+1} - \theta_{N-1}(t_N)) - (2\omega_N + \omega_{N+1})(t_{N+1} - t_N)}{(t_{N+1} - t_N)^2} \\
k &= \frac{-2(\theta_{N+1} - \theta_{N-1}(t_N)) + (\omega_N + \omega_{N+1})(t_{N+1} - t_N)}{(t_{N+1} - t_N)^3}
\end{aligned} \qquad (14)$$

4 APPLICATION TO AN UNDERACTUATED BIPEDAL MECHANISM

We now present experimental results from simulations where we apply this trajectory planning method to a bipedal walking model. This model is described in more detail in (Berenguer, 2007), and we now include a brief description.

4.1 Bipedal Model and Gait Descriptions

The walking model, shown in figure 3, consists of a light body, a tail connected to it and two legs. Each leg is formed by a parallel link mechanism and a flat rectangular foot. The tail, with an almost horizontal displacement, works as a counterbalance and controls the movement of the biped.

The joint connecting the tail to the body is actuated by an electric motor and it is the only actuated degree of freedom. Connecting the body to each leg are the top joints. Their rotation axis is normal to the frontal plane, so they allow the mechanism to raise a foot while both feet remain parallel to the ground. These top joints are passive joints with negligible friction. Finally, each parallel link mechanism has four joints, and we consider that in one of these joints (the ankle joint) there is a spring with friction. Due to the characteristics of the parallel link mechanism, these four joints represent only one passive degree of freedom for each leg of the mechanism. In summary, the model has eleven joints, four passive degrees of freedom and one actuated degree of freedom.

We now describe how the mechanism can walk when the tail moves side to side in an oscillating way. We start by supposing that the biped is at an equilibrium position with the tail in its central position (Fig.4.a). Both ankle springs hold the weight of the mechanism and it stays almost vertical.

When the tail moves to a lateral position of the mechanism, its mass acts as a counterbalance and produces the rise of one of the feet (Fig.4.b). Then only one spring holds the body, so the stance leg falls forward and the swing leg moves forward as a pendulum until the foot contacts the ground (fig.4.c).

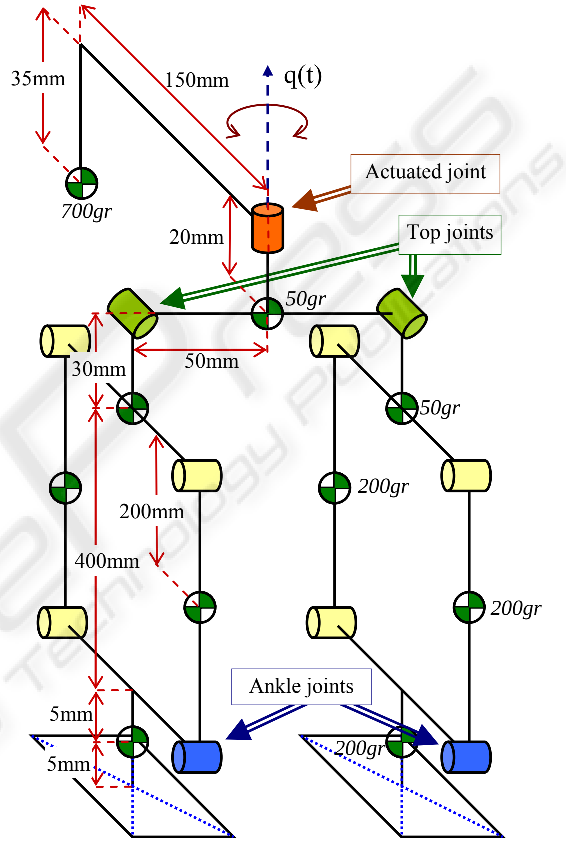

Figure 3: Model of the biped mechanism.

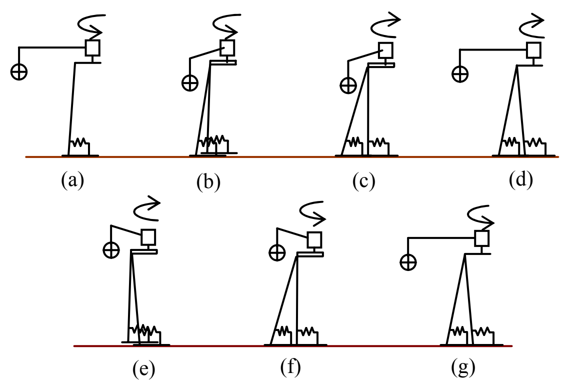

Figure 4: Phases during a stride.

During the new double support phase, the tail moves to the other side and the ankle springs move the body backwards (fig.4.d). When the tail reaches the other side, the second foot rises and a new step is generated (fig.4.e). In this single support phase, the spring of the foot that is in the ground produces enough torque to take the body forward again. This second step finishes with a new contact of the swing leg with the ground (fig.4.f).

Figure 4.g represents the last instant of this initial stride, and the starting point of a new one or the final configuration of a completed trajectory. We can see that if the tail stops, the system will stay in a steady configuration with no energy cost.

4.2 Example of Trajectory Generation

We want to design a trajectory that allows us to evaluate and analyze the biped behaviour at three different oscillation frequencies. The frequency profile must achieve these frequencies in a linear way, and keep them during some periods, so we can suppose a quasi-periodic gait at the end of each constant frequency segment. The desired oscillation amplitude is 1.5 rad and the radian frequencies and time instants are:

$$\begin{aligned} \omega_i[\mathrm{rad/s}] &= \{0 \quad 0.5 \quad 0.5 \quad 1 \quad 1 \quad 1.5 \quad 1.5 \quad 0\} \\ t_i[\mathrm{s}] &= \{0 \quad 20 \quad 70 \quad 90 \quad 140 \quad 160 \quad 200 \quad 300\} \end{aligned} \tag{15}$$

Using (7) we obtain q(t) with a linear profile shown in figure 5.

$$\begin{aligned} &q(t) = 1.5\sin(\theta(t)) \\ &\theta(t) = \theta_i(t) \quad \text{at} \quad t_i \le t < t_{i+1} \\ &\theta_1(t) = 0.0125t^2 \\ &\theta_2(t) = 0.5(t-20)+5 \\ &\theta_3(t) = 0.0125(t-70)^2 + 0.5(t-70)+30 \\ &\theta_4(t) = (t-90)+45 \\ &\theta_5(t) = 0.0125(t-140)^2 + (t-140)+95 \\ &\theta_6(t) = 1.5(t-160)+120 \\ &\theta_7(t) = -0.0075(t-200)^2 + 1.5(t-200)+180 \end{aligned} \tag{16}$$

As we can see in figure 6, this solution provides the final joint position q(300)= -0.76rad. If the desired final tail position is q=0rad, it will be necessary to apply the procedure in section 3.1. The solution in this case is the same as in (16) but with the last phase $\theta_7(t)$ given by:

$$\begin{aligned} \theta_7(t) = &1.062\times10^{-6}(t-200)^3 + \\ &+7.659\times10^{-3}(t-200)^2 + 1.5(t-200)+180 \end{aligned} \tag{17}$$

Figure 5 shows the frequency profiles of both solutions. We can see a most linear segment in the last time interval for the second solution which practically overlaps the first solution's profile. Figure 6 shows both trajectories nearly overlapping during the last time interval, with the same number of oscillations but different final value.

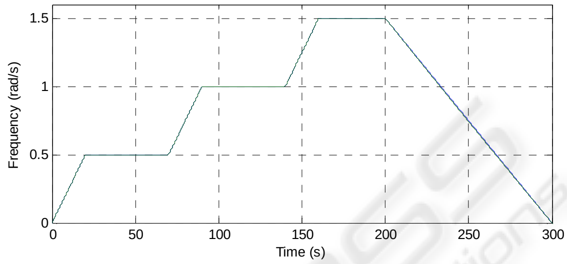

Figure 5: Instantaneous frequency profiles.

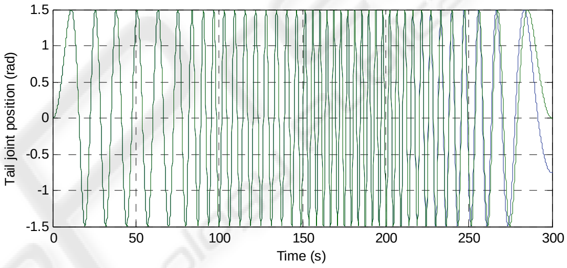

Figure 6: Trajectory of the tail joint.

4.3 Evaluation of the Mechanism Behaviour

The bipedal mechanism walks with a forward speed which is proportional to the stride length and frequency. The stride frequency is the same as the tail oscillation frequency and the stride length depends on the tail frequency and also on other parameters of the model. If we fix the values of these other parameters, the speed and energy consumption of the mechanism will depend in a non linear way on the stride frequency. Using linear frequency profiles we can analyze this dependency and estimate a near optimal joint frequency for an established set of model parameters.

Figure 7 shows the distance covered by the biped and the mechanical energy required by the tail joint during the execution of the last trajectory defined in section 4.2. The relative small amplitude oscillations are due to the forward and backward oscillation of the body during walking. The mechanical energy has been calculated by the integration of the absolute value of the product between the angular velocity of the joint and the required torque. We observe an important difference in the walking speed for the first and second oscillation frequencies, and a much smaller variation for the second and third ones, while the power consumption varies significantly for these last frequencies. So, we find a loss of efficiency when we increment the stride frequency.

Table 1 presents numerical data considering the last stride period just before a change in the oscillation frequency. We suppose that the gait is almost periodic during this last period. Figure 8 shows these values and also an estimation of the same magnitudes at different frequencies. This estimation is obtained from the last profile's segment, which covers all frequencies between 0 and 1.5rad/s, considering each half-oscillation as an approximation of half-period of a sinusoid.

As we can see, speed goes up quickly at low frequencies because stride length also grows. For frequencies greater than 1.04rad/s, stride length decreases with frequency and speed rises more slowly. We also notice that speed and power have similar behaviour (S-curve) before this frequency. After that, the power slope increases whereas speed slope decreases. We consider this 1.04rad/s frequency as a near optimal oscillation frequency for the actuated joint.

Table 1: Stride length, speed and mechanical power at three different stride frequencies during a stride.

Stride frequency (rad/s)	Stride period (s)	Stride length (m)	Speed (m/s) $x10^{-3}$	Power (W) $x10^{-3}$
0.5	12.57	0.0807	6.422	1.386
1.0	6.283	0.1992	31.704	10.707
1.5	4.188	0.1767	42.184	18.694

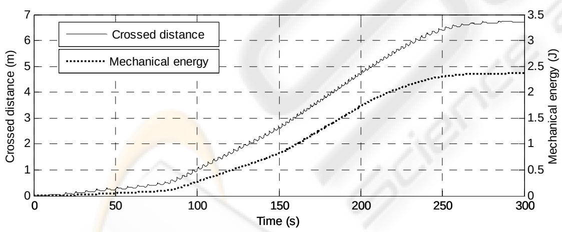

Figure 7: Crossed distance and mechanical energy during the trajectory execution.

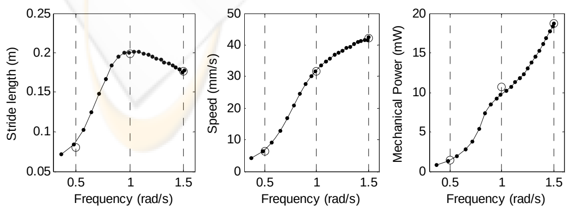

Figure 8: Stride length, mechanism speed and required mechanical power at different joint frequencies.

5 CONCLUSIONS

In this work we propose a method for planning oscillatory trajectories based on the concatenation of chirp functions. By means of adding a final cubic function, the joint can also reach a desired final position following a nearly linear frequency profile. Our aim is to apply this method to a bipedal robot that walks moving a tail in an oscillatory way.

This planning method allows us to study the gait efficiency at different stride frequencies during the design and adjusting phase. On the other hand, the implementation of this planner will allow a real prototype to select the forward speed as a function of the obstacles density, ground inclination or for optimization requirements.

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