APPLICATIONS OF A MODEL BASED PREDICTIVE CONTROL TO HEAT-EXCHANGERS

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Abstract: Model based predictive control (MBPC) is an optimization-based approach that has been successfully applied to a wide variety of control problems. When MBPC is employed on nonlinear processes, the application of this typical linear controller is limited to relatively small operating regions. The accuracy of the model has significant effect on the performance of the closed loop system. Hence, the capabilities of MBPC will degrade as the operating level moves away from its original design level of operation. This paper presents an MBPC algorithm which uses on-line simulation and rule-based control. The basic idea is the on-line simulation of the future behaviour of control system, by using a few control sequences and based on nonlinear analytical model equations. Finally, the simulations are used to obtain the 'optimal' control signal. These issues will be discussed and nonlinear modelling and control of a single-pass, concentric-tube, counter flow or parallel flow heat exchanger will be presented as an example.

1 INTRODUCTION

Model Based Predictive Control (MBPC) refers to a class of algorithms that utilize an explicit process model to compute the control signal by minimizing an objective function (Comacho, 1999). The performance objective typically penalizes predicted future errors and manipulated variable movement subject to various constraints. The ideas appearing in greater or lesser degree in all the predictive control family are basically:

   -explicit use of a model to predict the process output in the future;

   -on line optimization of a cost objective function over a future horizon;

   -receding strategy, so that at each instant, the horizon is displaced towards the future, which involves the application of the first control signal of the sequence calculated at each step.

   Performance of MBPC could become unacceptable due to a very inaccurate model, thus requiring a more accurate model. This task is an instance of closed-loop identification and adaptive control. Here it is important to remember that the model is only used as an instrument in creating the best combined performance of the controller and the actual system, so the model does not necessarily need to be a good open-loop model of the system. The performance measure should be able to capture as much of the closed loop behavior as possible.

   Let's consider that it is possible to compute:
- the predictions of output over a finite horizon (N);
- the cost of an objective function,
   for each possible sequence:

$$u(.) = \{u(t), u(t+1),.., u(t+N)\} \qquad (1)$$

and then to choose the first element of the optimal control sequence. For a first look, the advantages of the proposed algorithm (Balan, 2001) include the following:

   -the minimum of objective function is global;

   -it is not necessary to invert a matrix, so potential difficulties are avoided;

   -it can be applied to nonlinear processes if a nonlinear model is available;

   -the constraints (linear or nonlinear) can easily be implemented.

   The drawback of this scheme is a very long computational time, because there are possibly a lot of sequences. For example, if u(t) is applied to the

process using a "p" bits numerical-analog converter (DAC), the number of sequences is 2^{p*N}.Therefore, the number of sequences must be reduced.

In the next sections, these issues will be discussed and nonlinear modelling and control of a single-pass, concentric-tube, counter flow heat exchanger will be presented as an example.

2 THE MODEL OF THE HEAT-EXCHANGER

Heat exchangers are devices that facilitate heat transfer between two or more fluids at different temperatures. Usually, MBPC uses a linear model and an on-line least square algorithm (RLS) to determine the parameters. Heat exchangers are nonlinear processes. To apply the standard MBPC algorithms it is possible to use multiple model adaptive control approach (MMAC) which uses a bank of models to capture the possible input-output behavior of processes (Dougherty, 2003). Other solutions are based on neural networks and fuzzy logic (Fischer, 1998), (Fink, 2001).

In this paper it is used an example from (Ozisik, 1985): a heat exchanger with hot fluid -engine oil at 80ºC, cold fluid - water at 20º C, by using a single-pass counter flow (or parallel flow for some experiments) concentric-tube. Other data and notations: length (L): 60m, heat transfer coefficients (k_1=1000 W/(m^2 ºC), k_2=80 W/(m^2 ºC)), the temperature profile of fluids and wall ($\theta_1(z,t)$, $\theta_2(z,t)$, $\theta_w(z,t)$), specific heat (c_1, c_2, c_w), cross-sectional area for fluids flow and wall (S_1, S_2, S_w), density of fluids and wall (ρ_1, ρ_2, ρ_w), flow speed of fluids (v_1, v_2), transfer area (S) (fig. 1).

If physical properties (density, heat capacity, heat transfer coefficients, flow speed) are assumed constant, the heat exchanger model is described using a shell energy balance as (Douglas, 1972):

-hot fluid:

$$c_1\rho_1 S_1 \frac{\partial \theta_1(z,t)}{\partial t} - c_1\rho_1 v_1 S_1 \frac{\partial \theta_1(z,t)}{\partial z} = \frac{k_1 S}{L}[\theta_w(z,t) - \theta_1(z,t)] \quad (2)$$

-cold fluid:

$$c_2\rho_2 S_2 \frac{\partial \theta_2(z,t)}{\partial t} + c_2\rho_2 v_2 S_2 \frac{\partial \theta_1(z,t)}{\partial z} = \frac{k_2 S}{L}[\theta_w(z,t) - \theta_2(z,t)] \quad (3)$$

-wall:

$$c_w\rho_w S_w \frac{\partial \theta_w(z,t)}{\partial t} = \frac{S}{L}[k_1\theta_1(z,t) + k_2\theta_2(z,t) - (k_1+k_2)\theta_w(z,t)] \quad (4)$$

Using general notation $\theta_{a(i,j)}$ with a=1 (hot fluid), a=2 (cold fluid), a=w (wall), i, j discrete elements in space respectively time, the discrete equations corresponding to partial differential equations (2),(3),(4) are:
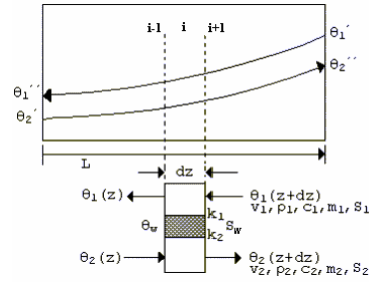

Figure 1: Temperature distributions.

$$\theta_1(i,j+1) = \theta_1(i,j)\left[1 - v_1\frac{\Delta t}{\Delta z} - \frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\right] + $$
$$+ v_1\frac{\Delta t}{\Delta z}\theta_1(i+1,j) + \frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\theta_w(i,j) \quad (5)$$

$$\theta_2(i,j+1) = \theta_2(i,j)\left[1 + v_2\frac{\Delta t}{\Delta z} - \frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\right] - $$
$$- v_2\frac{\Delta t}{\Delta z}\theta_2(i+1,j) + \frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\theta_w(i,j) \quad (6)$$

$$\theta_w(i,j+1) = \theta_w(i,j) + $$
$$+ \frac{S\Delta t}{L}[k_1\theta_1(i,j) + k_2\theta_2(i,j) + (k_1+k_2)\theta_w(i,j)] \quad (7)$$

In a control application, these equations can not be used directly because v_1 and v_2 are not constant in time. Let's consider next assumptions:

-the speed of fluids is limited:

$$v_{1(min)} < v_1 < v_{1(max)};$$
$$v_{2(min)} < v_2 < v_{2(max)}; v_{max} = \max(v_{1(max)}, v_{2(max)}) \quad (8)$$

- the fluids speed is only time-function:

$$v_1 = v_1(t), \ dv_1/dz = 0, \ v_2 = v_2(t), \ dv_2/dz = 0 \quad (9)$$

- the length of heat exchanger is divided in n intervals: $L = n\Delta z;$ (10)

- in an interval Δt, the fluids cover only a part of Δz: $n_v v_{max}\Delta t = \Delta z$; $\Delta t < L/(nn_v v_{max})$ (11)

- two variables Δz_1, Δz_2 are using to totalize the small fluid displacements:

$$\Delta z_1(t+\Delta t) = \Delta z_1(t) + v_1\Delta t ;$$
$$\Delta z_2(t+\Delta t) = \Delta z_2(t) + v_2\Delta t \quad (12)$$

- in simulations, the displacements of the fluids become effective only if $\Delta z_1 > \Delta z$ or/and $\Delta z_2 > \Delta z$; in these cases:

$$\Delta z_1 \leftarrow \Delta z_1 - \Delta z \text{ or/and } \Delta z_2 \leftarrow \Delta z_2 - \Delta z \quad (13)$$

In other words, in simulations, the continue moves of fluids are replaced with small discrete displacements. As a result, the heat exchanger model is described by equations:

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$$\theta_1(i,j+1) = \theta_1(i,j)\left[1 - \frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\right] + \frac{k_1 S\Delta t}{Lc_1\rho_1 S_1}\theta_w(i,j) \quad (14)$$

$$\theta_2(i,j+1) = \theta_2(i,j)\left[1 - \frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\right] + \frac{k_2 S\Delta t}{Lc_2\rho_2 S_2}\theta_w(i,j) \quad (15)$$

$$\theta_w(i,j+1) = \theta_w(i,j) +$$
$$+ \frac{S\Delta t}{L}\left[k_1\theta_1(i,j) + k_2\theta_2(i,j) + (k_1+k_2)\theta_w(i,j)\right] \quad (16)$$

In a practical implementation, there are used equations (12), (13), (14), (15), (16).

It is important the number and position of temperature sensors. Here, it is considered that only the inlet and outlet temperatures (hot fluid, cold fluid and wall) and the flow rate of fluids are measured. The temperatures inside heat exchanger are estimated. The quality of heat exchange depends especially by the heat transfer coefficients. These parameters depend by temperatures, accumulation of deposits of one kind or another on heat transfer surface, shape of tube, etc. The temperature distributions inside heat exchanger (process and model) are presented in fig. 2 using notations $\theta_a(i,j)$. for process and $M\theta_a(i,j)$ for the model.
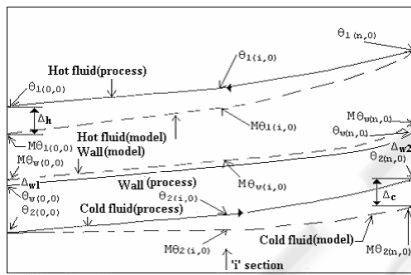

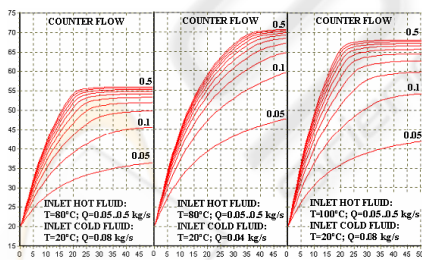Figure 2: Process and model (counter flow) – diagrams.

Figure 3: Step reply- counter flow.

To underline the main characteristics of the heat exchangers that are used in simulations, there are presented the step replies in some cases (counter flow - fig. 3; parallel flow – fig. 4). First, the temperatures of fluids are 20º C, than it is changed the inlet temperature of hot fluid (input of the process). There are different conditions for inlet temperatures and flow rate fluids. Flow rate of hot

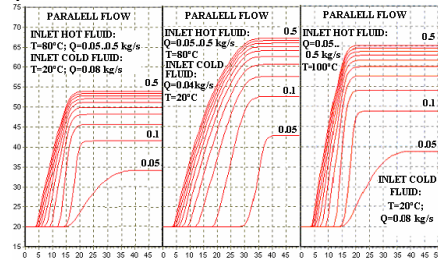fluid is a parameter and permits to obtain a family of step replies.
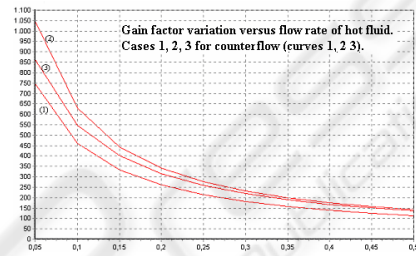

Figure 4: Step reply- parallel flow.

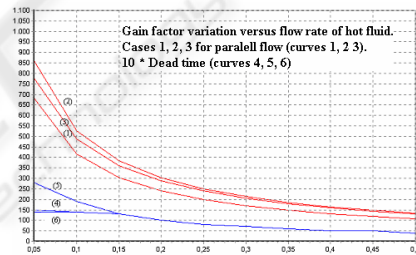Figure 5: Counter flow- gain factor.

Figure 6: Parallel flow- gain, dead time.

Figures 5 and 6 present the dependence of gain factor and dead time by flow rate. These simulations underline the non-linear features of processes and, for parallel flow, a dead time, which is dependent especially by flow rate of hot fluid.

3 CONTROL ALGORITHM

A model based adaptive-predictive algorithm which uses on line simulation and rule based control, designed for linear processes, is developed in (Balan, 2001), (Balan, 2005). This algorithm can be applied with some modifies to nonlinear processes. The nonlinear equations of the process can be used directly in the control algorithm. The predictions of system output are calculated by integrating the nonlinear ordinary differential equations of the model over the prediction horizon, by using a few

control sequences (Balan, 2005). For a first stage, are used, the next four control sequences:

$$u_1(t) = \{u_{\min}, u_{\min}, .., u_{\min}\}$$
$$u_2(t) = \{u_{\max}, u_{\min}, .., u_{\min}\}$$
$$u_3(t) = \{u_{\min}, u_{\max}, .., u_{\max}\}$$
$$u_4(t) = \{u_{\max}, u_{\max}, .., u_{\max}\} \qquad (17)$$

where u_{min} and u_{max} are the limits of the control signal, limits imposed by the practical constraints. These values can depend on context and can be functions of time. There are two pair sequences: $(u_1(t), u_2(t))$ and $(u_3(t), u_4(t))$ which are different through the preponderance of u_{min} or u_{max} in the future control signal. The pair sequences are different only through the first term.

Using these sequences results four output sequences $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$. The control signal is computed using a set of rules based on the extreme values y_{max0}, y_{max1}, y_{min0}, y_{min1} (fig. 7- d is dead time, $t_1=N$, y_r is setpoint) of the output predictions. In the followings, considering processes with positive sign, it can be put in evidence four usual cases:

Case 1: If $y_{max0}{<}y_r$ (corresponding to $u_1(t)$ sequence) and $y_{max1}{>}y_r$ (corresponding to $u_2(t)$ sequence) Then (using a linear interpolation):

$$u(t) = \frac{u_{\max} - u_{\min}}{y_{\max 1} - y_{\max 0}} y_r + \frac{u_{\min} y_{\max 1} - u_{\max} y_{\max 0}}{y_{\max 1} - y_{\max 0}} \qquad (18)$$

Case 2: If $y_{min0}{<}y_r$ (corresponding to $u_3(t)$ sequence) and $y_{min1}{>}y_r$ (corresponding to $u_4(t)$ sequence) Then (using a linear interpolation):

$$u(t) = \frac{u_{\max} - u_{\min}}{y_{\min 1} - y_{\min 0}} y_r + \frac{u_{\min} y_{\min 1} - u_{\max} y_{\min 0}}{y_{\min 1} - y_{\min 0}} \qquad (19)$$

Case 3: If: $y_{max0}{>}y_r$ Then $u(t_0)=u_{min}$ (20)

Case 4: If: $y_{max1}{<}y_r$ Then $u(t_0)=u_{max}$ (21)

In fig. 7, every output prediction curve is marked with a number which correspond to the number of control sequence from relations (17). Similar to case 3 and case 4, there are two similarly cases if $dy/dt{<}0$ for $t{<}t_0$. If the algorithm uses only these 6 rules, the variance of u(t) will be large (Balan, 2001).

So, in the second stage, depended by behaviour of the control system, are used next methods:

-an algorithm that modifies the limits of control signal:

$$u_{min} \leq u_{minst}(t) \leq u(t) \leq u_{maxst}(t) \leq u_{max}$$
$$\Delta u_{min} \leq \Delta u \leq \Delta u_{max} \qquad (22)$$

For example:

$$u_{\min st}(t) = f_1(u_{\min st}(t-1), u_{\max st}(t-1), y(t), y_r(t)) \qquad (23)$$
$$u_{\max st}(t) = f_2(u_{\min st}(t-1), u_{\max st}(t-1), y(t), y_r(t)) \qquad (24)$$

where f_1, f_2 are functions which decrease or increase (depended by behavior of the control system) the difference between $u_{maxst}(t)$ and $u_{minst}(t)$.
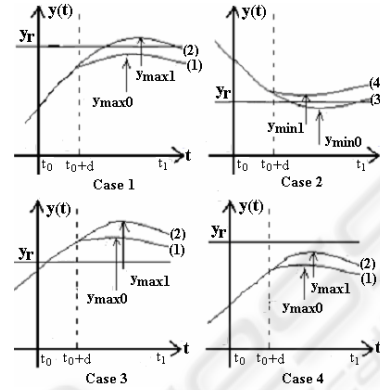

Figure 7: Examples of output predictions.

In relations (18)..(21), the values of u_{max}, u_{min} are replaced with $u_{minst}(t)$, $u_{maxst}(t)$. In the following, if is necessary, the next relations are used:

$$u_{\min st}(t) = u_{\min st}(t-1) + k_{st}(u_{st} - u_{\min st}(t-1)) \qquad (25)$$
$$u_{\max st}(t) = u_{\max st}(t-1) - k_{st}(u_{\max st}(t-1) - u_{st}) \qquad (26)$$

where k_{st} is a weight parameter and u_{st} is the estimated value of control signal in steady state. But in some circumstancing (perturbations, inaccurate model) the limits of control signal must increase. Also, it is necessary to limit the minimum value of $u_{maxst}(t)-u_{minst}(t){>}d_{ust}{>}0$, where d_{ust} is a parameter of the control algorithm.

-using the "variable setpoint" (Balan, 2001):

$$y_{r1}(t) = y_r(t) + k_{ref}[y(t) - y_r(t)] \qquad (27)$$

where k_{ref} is a weight factor

-using a filter to compute control signal (especially in steady state regime).

This paper will be tackled only the case when the main aim is to control the temperature of outlet cold fluid. To do this, it is used the flow rate of hot fluid (controller's output). There are possible other objectives for example to maximize the heat transfer between fluids. First, there was used an adaptive-predictive algorithm based on on-line simulation and a linear model (Balan, 2001). The parameters of model were identified on-line using least square algorithm. This method could be applied, with poor results, only for counter flow heat exchanger. It is

necessary to consider the non-linear features of heat exchanger and to use a model of the heat exchanger based on the finite difference method. It is supposed that initially the heat transfer coefficients are unknown and than they are identified on-line. In simulations, there are used three sets of finite difference equations: process equations, model equations, on line simulation equations.

The behaviour of heat exchanger depends by some types of parameters:

1. Construction parameters: length of tube, surface of heat transfer, diameters of tubes, etc. These parameters can be considered constants.

2. Fluids parameters: density, specific heat etc. These parameters depend by temperature and other conditions.

3. Parameters that determine the quality of heat exchange, especially the heat transfer coefficients. These parameters depend by temperatures, accumulation of deposits of one kind or another on heat transfer surface, shape of tube, etc.

At every sample period, it is possible to compute Δ_h, Δ_c, Δ_{w1}, Δ_{w2}, the temperature prediction errors of outlet hot fluid, outlet cold fluid, wall (fig. 2).

These predictions can be used to correct the temperature distributions inside the model of heat exchanger, using translations and rotations of distributions. Also, prediction errors can be used to modify the parameters of the model using an algorithm based on rules. The control scheme is presented in fig. 8.
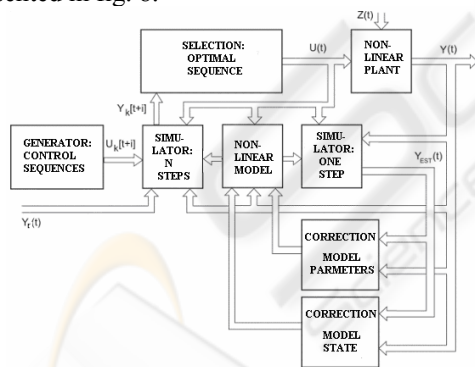

Figure 8: Control scheme.

4 APPLICATIONS WITH HEAT EXCHANGERS

The next applications show the main features of the algorithm applied to heat exchanger. The set point has a variable shape (42°C, 47°C, 52°C, 47°C, 42°C..). The limits of u(t) (hot fluid flow rate) are: $0.05 \leq u(t) \leq 0.5$ [kg/s]. The flow rate of cold fluid is

constant (0.08 kg/s). The temperatures of cold fluid ($20°$) and hot fluid ($80°$) are constant. Some experiments with variable flow rate or/and variable temperature of cold fluid are presented in (Balan, 2001).

First, it is used an accurate model (Fig. 9, fig. 10). If the algorithm uses only 1..6 rules, the variance of $u(t)$ will be large. To reduce this variance, a solution is to use a funnel zone for control signal, based on inequality (22).

In steady-state regime, control signal is computed using average of past and new values. The algorithm do not use directly an integral component. In figure 9, steps 50..80, the algorithm tries to reduce the error as fast as possible. As a result, a damped oscillation appears. To avoid this behavior, a solution is to use a reference trajectory.
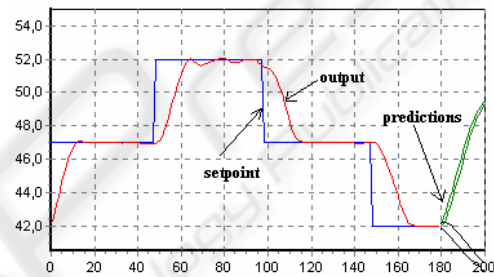
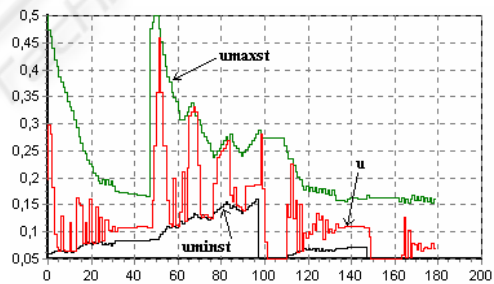

Figure 9: Setpoint, output (accurate model).

Figure 10: Controller output (accurate model).

In figure 11, 12 it is presented an adaptive case; the heat transfer coefficients depend by temperature:

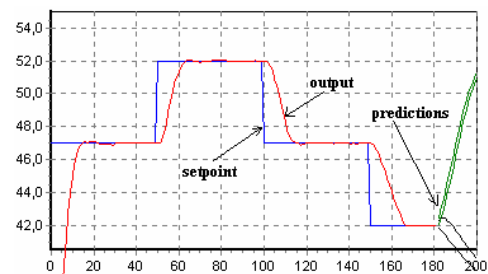$$k = k_0 \left(1 + \theta / 200\right) \qquad (28)$$

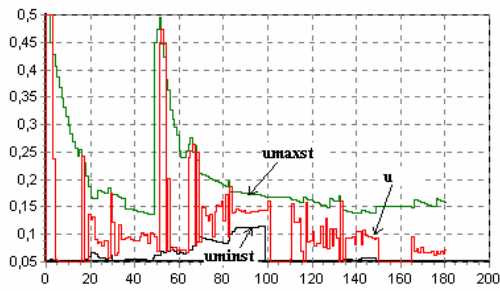Figure 11: Setpoint, output (adaptive case).

Figure 12: Controller output (adaptive case).

Initial the temperature of cold and hot fluids is $20°$. The evolution of the estimations of heat transfer coefficients is presented in figure 13. To obtain these estimations, both rotations and translations of temperature distributions and rule based correction of heat transfer coefficients are used. In figure 14 it is used the same conditions for heat transfer coefficients, but it is not used this approach.
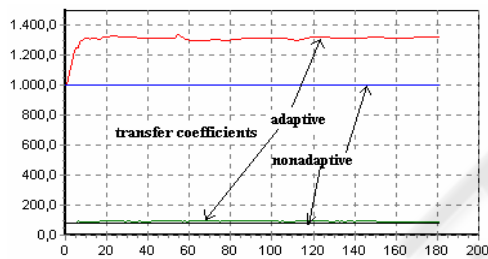

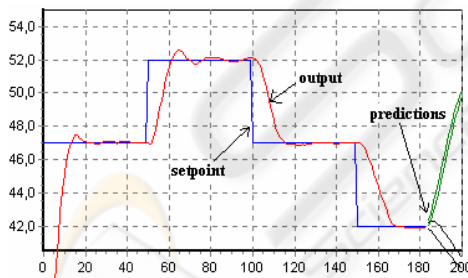Figure 13: Parameters identification.

Figure 14: Setpoint, output (adaptive case).

As a result, the quality of control algorithm decreases.

5 CONCLUSION

The paper presents a simple and intuitive algorithm applied in the case of a non linear process: heat exchanger. A non-linear model of the process, based on finite difference method, is used. This approach is a numerical alternative to usual criteria equations; offer a way to ensure the accuracy of a best-fit heat exchanger selection, and point out that the fluids properties must not be mathematically emphases. Using the process model and a reduce number of the sequences control, it is simulated the future behaviour of the process and based on a set of rules it is chosen the signal control considered optimum at the actual moment. Of course there are some difficulties such as the proof of the stability, the way of choosing of the control sequences and the set of rules which will lead to a better result, choosing some parameters etc. Although, taking into account the simplicity of this algorithm the obtained results in the case of the presented examples by nonlinear systems are remarkable. A demo application that implements the proposed algorithm can be downloaded (see web link). In the future, starting from the proposed algorithm, the work will focus on: the optimal chosen of the control parameters, the study of other set of control sequences, the study of other set of control rules, adaptive case and practical implementation.

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