FEASIBILITY OF SUBSPACE IDENTIFICATION FOR BIPEDS
An Innovative Approach for Kino-Dynamic Systems

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Abstract: Different approaches have been briefly overviewed which have been used in stability of biped robots. Current implementations either mimic human behavior or use heuristic control. This paper suggests the use of model-free crisp control in operational space configuration for better control and understanding of kino-dynamic systems and biped robots.

1 INTRODUCTION

Designing a control strategy for a biped robot can be quite tedious as dynamics involved are non-linear, multi-variable, naturally unstable and foot-ground interaction is limited (Wolkotte (2003); Kim et al. (2004); Caballero et al. (2004)). These all problems suggest that controller should be sophisticated enough to cater for all these factors. This is why most implementations don't use classical control techniques but rely on techniques which mimic human behavior or are based on heuristic control (Pratt (2000)).

In order to examine crisp control and a mathematical solution for biped stability using other than above mentioned techniques, subspace identification is proposed, which then can be coupled with post-modern control techniques such as $\mathscr{H}_\infty$ to design a model-free control system. In theory such controllers are developed but has never been used for kino-dynamic systems (Favoreel et al. (1998); Woodley et al. (2001a)).

In the paper, previous implementations of biped robots are mentioned in section 2. Model-free and model based implementations are briefly discussed in section 3. Subspace identification and its model-free implementations are discussed in section 4. Proposed implemetation is mentioned in section 5. Section 6 discusses the results of using subspace identification technique in biped robot leg.

2 PREVIOUS IMPLEMENTATIONS

An overview of literature suggests that history of biped robots has only handful of milestones. Quasi-dynamic walking gait on bipeds was achieved in 1980 by Kato et al. using artificial muscles (Kato et al. (1983)). In 1983, Raibert demonstrated a planar one-legged hopping robot that could hop at desired velocity and jump over small obstacles (Raibert (1986)). In 1990, McGeer demonstrated first passive walking for robots that could walk down a slop without any active elements (McGeer (1990)). In 1997, Honda introduced its biped robot P2 which set a new trend in bipeds. Latest from Honda, ASIMO, has state-of-the-art technology in this field (Sakagami et al. (2002); Hirai et al. (1998); Lim and Takanishi (2005)). Control systems employed in the development of bipeds can be divided into different categories.

Most of the robots fall into the category which employs simple models, which can be calculated by Newtonian mechanics, others are based on walking and running dynamics (Kajita et al. (1992); Schwind (1998)). These models are because of the inspiration from biometrics (McMahon (1984); Alexander (1996)). This technique is best used when trajectory is given. It can be subdivided into further two types. First one are the bipeds which are clone of ASIMO and others are based on intuitive control. The best ef-

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fort in this technique has come from Pratt and Pratt, and most impressive implementations in this type of control framework also came from the same group (Pratt and Pratt (1998, 1999); Pratt (2000)).

Other type of controllers are based on "neural" oscillators or pattern generators (Taga (1995)). There are studies which suggest that vertebrates have some kind of pattern generation mechanism which enables them to walk dynamically. Generators can be hand-tuned to construct a detailed feedback response for dynamic walking. Last type of controllers are the ones which are based on machine learning.

3 METHODS TO USE EXPERIMENTAL DATA

To design a control system, equations for a biped robot can be calculated from Newtonian mechanics. It is shown in (Pratt (2000)) that equation of motion of a massless leg with a torso having mass m, can be written as:

$$ml^2\ddot{\theta}_1 = mgl\sin\theta_1 - 2ml\dot{l}\dot{\theta}_1 - J\ddot{\theta}_b \qquad (1)$$

here θ_1 is the angle between the normal axis to ground and axis going through the CoP (center of pressure) in foot and center of mass of the torso, J is the rotational moment of inertia, and θ_b is the angle between torso axis and the leg. Equation 1 suggests that there are three ways to change rotational dynamics about center of pressure. First method is to change the position of the body, which will change θ_1 and location for center of pressure. This method is the most effective one. Second method is to change the inertial momentum J and third method is to change the length l. Effect because of the last two quantities is not much when compared with effect due to change in location of center of pressure.

As we are more interested in exploring a more robust and generic solution for kino-dynamic systems, techniques to use experimental data to determine system equations will be discussed. There are four methods to use experimental data as shown in table 1 (Woodley (2001)). Mainly, choice depends on application. For real-time systems which are easy to model, indirect control is a better choice. The system then adapts itself and updates its model parameters according to the conditions. Normally on-line model based design is referred as indirect control. If a system is hard to model from first principles (as Newton's laws of motion) or there are time varying nonlinearities then direct adaptive control would suite the application. Examples of plants which are difficult to model are arc furnaces (Wilson (1997); Staib and Staib (1992)) and helicopter rotors (Lohar (2000); Tischler et al. (1994)). Biped robots on the other hand can be modeled but they exhibit time varying nonlinearities (Wolkotte (2003); Kim et al. (2004); Caballero et al. (2004)).

4 SYSTEM IDENTIFICATION

There are many system identification techniques. The list starts with classical prediction error (PE) and its variants; auto regression with exogenous input (ARX), output error (OE), auto regression moving average with exogenous input (ARMAX), and Box Jenkins (BJ) (Norton (1986); Ljung (1999)).

4.1 Subspace Identification

Aside from classic system identification methods, there are subspace identification methods, which gained a lot of popularity in recent years (Morari and Lee (1999)).

If plant's input and output values at discrete times are given by (Overschee and Moor (1996)):

$$\left(\begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{n-1} \end{bmatrix}, \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{bmatrix} \right)$$

Hankel matrices for past and future inputs are written as

$$U_p \triangleq \begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \cdots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{bmatrix} \in \mathbb{R}^{im\times j}$$

$$U_f \triangleq \begin{bmatrix} u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \vdots & \vdots & \cdots & \vdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{bmatrix} \in \mathbb{R}^{im\times j}$$

Similarly Hankel matrices for past and future outputs can be written as $Y_p \in \mathbb{R}^{il\times j}$ and $Y_f \in \mathbb{R}^{il\times j}$ respectively. Let us define W_p as

$$W_p \triangleq \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$$

Linear least squares predictor of Y_f with given W_p and U_f can be written as Frobenius norm minimization

$$\min_{L_w, L_u} \left\| Y_f - \begin{bmatrix} L_w & L_u \end{bmatrix} \begin{bmatrix} W_p \\ U_f \end{bmatrix} \right\|_F^2$$

Table 1: Four different techniques of control design from experimental data.

	With Plant Model	Without Plant Model
Online	Indirect Adaptive	Direct Adaptive
Offline	Model Based Design	Direct Control Design

From subspace orthogonal project, L_w and L_u is calculated as

$$\begin{bmatrix} L_w & L_u \end{bmatrix} = Y_f \begin{bmatrix} W_p \\ U_f \end{bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} W_p \\ U_f \end{bmatrix} \begin{bmatrix} W_p \\ U_f \end{bmatrix}^{\mathrm{T}} \right)^{\dagger} \quad (2)$$

where † denotes pseudo-inverse. Future outputs can be predicted from past inputs, outputs, and future inputs.

$$\begin{bmatrix} \hat{y}_k \\ \vdots \\ \hat{y}_{k+i-1} \end{bmatrix} = L_w \begin{bmatrix} u_{k-i} \\ \vdots \\ u_{k-1} \\ y_{k-i} \\ \vdots \\ y_{k-1} \end{bmatrix} + L_u \begin{bmatrix} u_k \\ \vdots \\ u_{k+i-1} \end{bmatrix} \quad (3)$$

Pseudo-inverse is normally calculated through Singular Value Decomposition (SVD) but Woodley et al. presented another way by using Cholesky factorization, which is computationally faster and consumes less memory (Woodley et al. (2001b)). It has already been used for guidance and control of unmanned vehicle (Kelbley (2006)).

4.2 Advantages of Subspace Identification Methods

Subspace Identification Methods (SIM) have many advantages over classical system identification techniques (Overschee and Moor (1996)). Notables are:

- From plant's input and output data, a predictor is found same as Kalman filter states, which makes it a simple least square problem. The whole architecture is streamlined and user-friendly.
- When implemented in direct adaptive control, plant model is not needed to be simplified, which can omit useful information from plant, as in SIM, all the plant information is stored in a compact form of subspace predictor.
- Output of subspace identification methods is in state space form which makes it easy to implement in computer but it's architecture has been exploited in different model free implementations as well (Woodley et al. (2001b); Favoreel et al. (1999b,a)).

Wernholt used SIM to solved system identification problem for ABB IRB 6600 robot (Wernholt (2004)). Hsu et al. used N4SID in style translation for human motion. These are some of the examples that how SIMs are being used.

4.3 Reported Problems in Subspace Identification Methods

There are a few problems in subspace identification methods. Many of these problems have been discussed in recent literature and partial remedies have been suggested (Chou and Verhaegen (1997); Lin et al. (2004); Wang and Qin (2004); Chiuso and Picci (2005)). Some of these problems are:

- Biased estimate for closed loop data.
- Errors-in-variables situation due to a projection performed in the algorithm.
- Assumption of noise-free input.

It is expected that in direct adaptive system, which calculates plant's model and designs controller in realtime, this problem will not faced but final answer to this can only be given after its implementation.

4.4 Types of Subspace Identification Methods

There are many implementations of subspace identification methods. Notables are:

- Canonical variate analysis (CVA) (Larimore (1990)).
- Multivariable output-error state space (MOESP) (Verhaegen and Dewilde (1992)).
- Numerical algorithms for subspace state space system identification (N4SID) (Overschee and Moor (1994)).
- Eigensystem realization analysis (ERA) (Juang (1994)).
- Subspace fitting (Jansson and Wahlberg (1996)).
- Stochastic subspace identification method using principal component analysis (SIMPCA) (Wang and Qin (2004)).

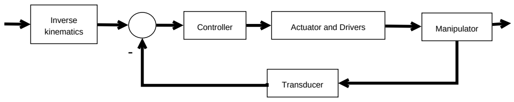

Figure 1: Joint space control.

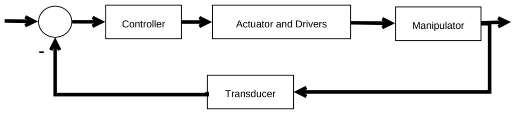

Figure 2: Operational space control.

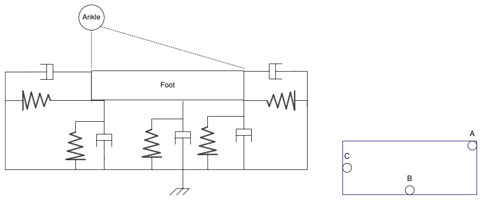

Figure 3: Foot ground interaction. On the left is the side view and on the right is the top view of foot model where points A, B, and C are connected to three dampers and springs. Dampers and springs connected on sides are responsible for friction with the ground.

5 PROPOSED IMPLEMENTATION

Joint space control is consisted of two subproblems. First, manipulator inverse kinematics is performed and then joint space control scheme is devised which allows the end effector to follow the reference input. The main computational burden in this scheme is because of inverse kinematics, which is normally performed by using different optimization techniques, as in a redundant system, there can be infinite solutions for a given task (Lope et al. (2003); Gupta et al. (1993); Kim et al. (2003)). Many implementations of joint space control can be found in the literature (Laib (2000); Kelly (1997); Arimoto (1995); Kelly (1993); Wen et al. (1992); Tomei (1991); Takegaki and Arimoto (1981); Zhang et al. (2000)).

In many applications, desired path of end effector is specified in operational space. Operational space control, on the other hand, is used for constrained manipulator motions (Sciavicco and Siciliano (2000); Sapio and Khatib (2005)). These constraints can be because of gravity or kinematically imposed. It can be seen in figure 2 that inverse kinematics is embedded in the closed-loop control law but not explicitly performed as shown in figure 1 (Sciavicco and Siciliano (2000)). Operational space control and task space control sometimes allude to the same concept (Khalil and Dombre (2004); Xie (2003); Sciavicco and Siciliano (2000)). Sapio and Khatib has simulated operational control schemes in physiological model of a human body under constrained conditions (Sapio and Khatib (2005)).

6 EXPERIMENT

MATLAB®and Simulink®by MathWorks Inc. have been employed to simulate a bipedal leg with torso. Under the action of normal gravity and exogenous force signals at each joint, the leg falls down and trajectory of torso is recorded. Using Subspace Identification, a predictor is found. This predictor is then applied on input joint signals. First, predicted trajectories are presented and then trajectories are predicted by updating previous trajectory from actual outputs after every prediction.

Following algorithm gives error between actual and predicted trajectories:

1. Prediction horizon, i is chosen and experiment is performed with given input and resultant torso trajectory is noted
2. From noted trajectory, a predictor is calculated using subspace projection algorithm
3. Outputs are calculated using subspace predictor and given inputs at joints
4. Difference between calculated values and actual values are plotted for each axis
5. Prediction horizon is changed and the whole process is repeated

One of the challenges in simulations was to simulate foot-ground interaction. Many implementations can be found in the literature (Hsu et al. (2005); Ogihara and Yamazaki (2001); Wang (2005); Wolkotte (2003)). Model with three contact points was devised after inspiration from human foot. This is shown in figure 3.

6.1 Assumptions

It is assumed that there are only three points where foot can touch the ground as shown in figure 3 and there is no air friction.

6.2 Results

For prediction horizon i less than a certain value, the system simply fails to predict the future outputs. Some suggest that the value of i should be 2 to 3 times

Table 2: Supposed values of different parameters for simulation.

	Length or radius [m]	Width [m]	Height [m]	Mass [kg]
Torso	0.1	0.4	0.5	20
Thigh	0.05		0.4	10
Calf	0.05		0.4	5
Foot	0.3	0.07	0.3	2
	Shape	I_1 [kg m^2]	I_2 [kg m^2]	I_3 [kg m^2]
Torso	Parallelepiped	0.688	0.4333	0.2833
Thigh	Cylinder	0.2333	0.2333	0.05
Calf	Cylinder	0.0698	0.0698	0.0063
Foot	Parallelepiped	9.6667e-4	0.0151	0.0158

the expected order of the system for stable and accurate results (Woodley (2001)), however, there is no hard and fast rule. In our experiments, the prediction horizon more than 10 did not improve the accuracy of the prediction. Increasing the value of i can also be computationally expensive as even with Cholesky/SVD factorization technique, the complexity of finding a subspace predictor is $O(ij+i^3)$, where j is number of prediction problems (Golub and Loan (1996)). It can be seen in the simulation and graphs that for movements of more than 1 meter, the error is in the order of micrometers. These results are very encouraging especially when there are multiple rigid bodies which are coupled together with rotatory joints and ground-foot interaction is present with given friction.

7 FUTURE WORK

To find subspace predictor, Hanekel matrix structure can be exploited for a better real-time operation. This work can be extended to a complete implementation of a model-free control system such as the one suggested by Woodley et al.. One of the challenges in the actual implementation is determination of uncertainty block Δ for the given system using techniques such as model unfalsification but without excessive overload of high computations (Woodley et al. (1998), Paul B. Brugarolas (2004)).

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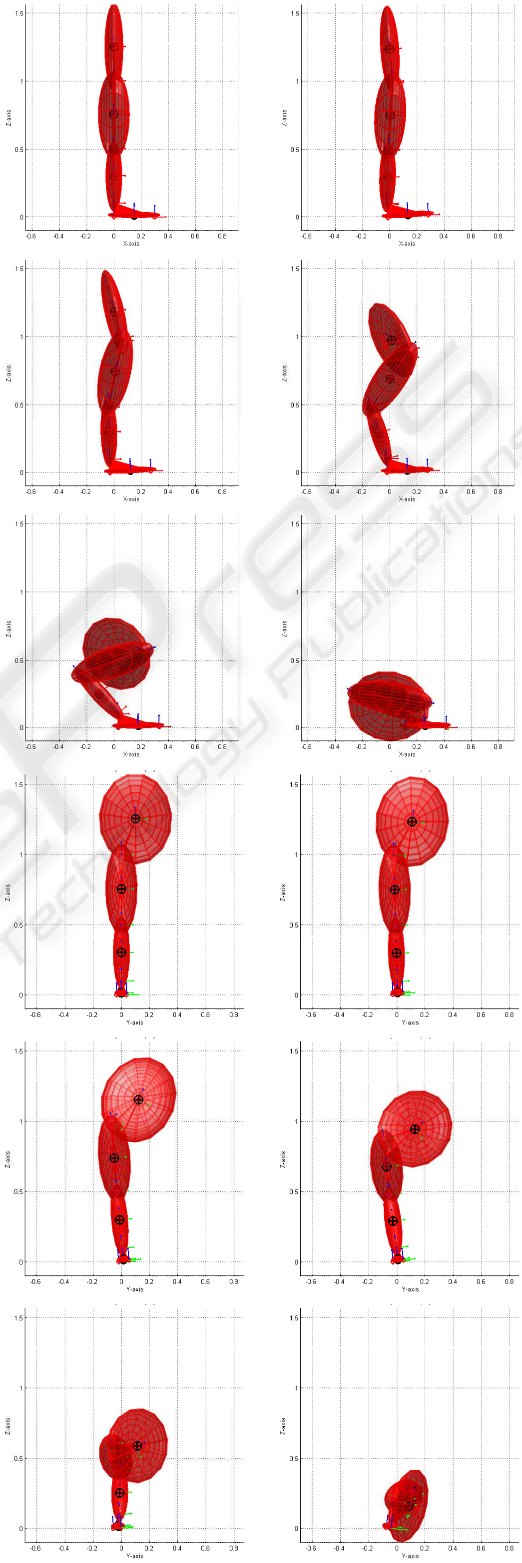

Figure 4: Free fall of a biped leg with exogenous force signals at its joints. Top six and bottoms ones are shots taken from the same simulation but from different angles after every 0.1 seconds.

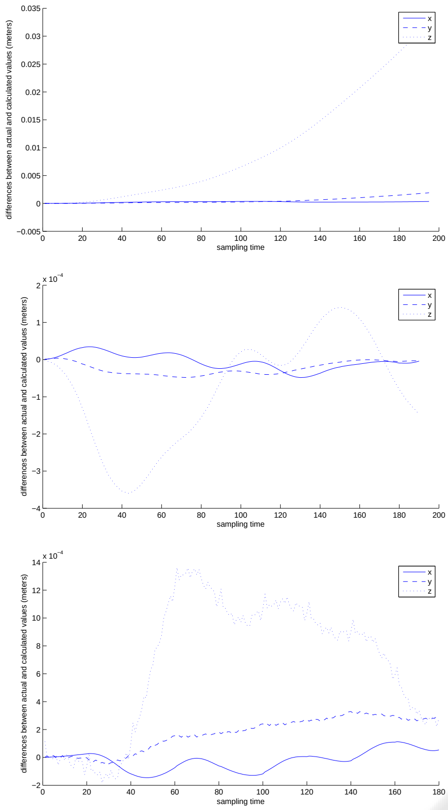

Figure 5: Error in the calculation of torso position. Above graphs are with $i = 5, i = 10$, and $i = 20$ respectively. Note that the largest movement of torso is in the z-direction, the error is also the most in this direction.

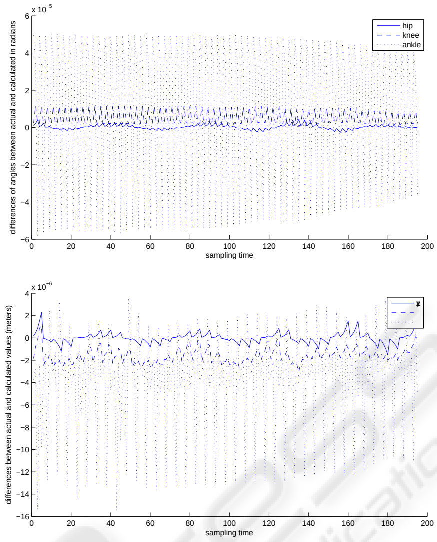

Figure 6: Error in the calculation of torso position when data is updated from actual torso position after every prediction. Above graphs are with $i = 5$ and $i = 20$ respectively. Note that even for very small prediction horizon *i.e.* $i = 5$, the error is very small.