APPLICATION OF SPATIAL H_∞ CONTROL TECHNIQUE FOR ACTIVE VIBRATION CONTROL OF A SMART BEAM

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Abstract: This study presents the design and implementation of a spatial H_∞ controller for the active vibration control of a cantilevered smart beam. The smart beam consists of a passive aluminum beam (507x51x2mm) and eight symmetrically surface bonded SensorTech BM500 type PZT (Lead-Zirconate-Titanate) patches (25x20x0.5mm). PZT patches are used as actuators and a laser displacement sensor is used as sensor. The smart beam was analytically modelled by using the assumed-modes method. The model only included the first two flexural vibrational modes and the model correction technique was applied to compensate the possible error due to the higher order modes. The system model was also experimentally identified and both theoretical and experimental models were used together in order to determine the modal damping ratios of the smart beam. A spatial controller was designed for the suppression of the vibrations of the smart beam due to its first two flexural modes. The designed controller was then implemented to experimentally suppress the vibrations. This study also compared the effectiveness of a pointwise controller with the newly developed spatial one.

1 INTRODUCTION

The vibration is an important phenomenon for the lightweight flexible aerospace structures. Those structures may be damaged under any undesired vibrational load. Hence, they require a proper control mechanism to attenuate the vibration levels in order to preserve the structural consistency. The usage of smart materials, as actuators and/or sensors, has become promising research and application area that gives the opportunity to accomplish the reduction of vibration of flexible structures and proves to be an effective active control mechanism.

The smart structure is a structure that can sense external disturbance and respond to that with active control in real time to maintain mission requirements (Çalışkan, 2002). Active vibration control of a smart structure requires an accurate system model of the structure. Smart structures can be modeled by using analytical methods or system identification techniques using the experimental data (Meirovitch, 1986 and Nalbantoğlu, 1998). The system model of a smart structure generally involves a large number of vibrational modes. However, the performance goals are mostly related to the first few vibrational modes since their effect on structural failure is much more prominent. Hence, a reduction of the order of the model is required (Hughes, 1981 and Moheimani, 1997). On the other hand, ignoring the higher modes can affect the system behaviour since directly removing the higher modes from the system model perturbs the zeros of the system. Therefore, in order to minimize the model reduction error, a correction term, including some of the removed modes, should be added to the model (Clark, 1997).

Today, robust stabilizing controllers designed in respect of H_∞ control technique are widely used on active vibration control of smart structures. Yaman et al. (2001 and 2003) showed the effect of H_∞ controller on suppressing the vibrations of a smart beam due its first two flexural modes. Similar work is done for active vibration control of a smart plate,

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and the effective usage of piezoelectric actuators on vibration suppression with H_∞ controller was successfully presented (Yaman, 2002).

Whichever controller design technique is applied, the suppression should be preferred to be achieved over the entire structure rather than at specific points, since the flexible structures are usually those of distributed parameter systems. Moheimani and Fu (1998) and Moheimani et al. (1997) introduced spatial H_2 and H_∞ norm concepts in order to meet the need of spatial vibration control, and simulation-based results of spatial vibration control of a cantilevered beam were presented. Moheimani et al. (1999) studied spatial feedforward and feedback controller design, and presented illustrative results. They also showed that spatial H_∞ controllers could be obtained from standard H_∞ controller design techniques. Halim (2002) studied the implementation of spatial H_∞ controller on active vibration control and presented quite successful results. However his works were limited to a beam with simply supported boundary conditions.

This paper aims to present design and implementation of a spatial H_∞ controller on active vibration control of a cantilevered smart beam.

2 THE SMART BEAM MODEL

The cantilevered smart beam model and its structural properties are given in Figure 1 and Table 1, respectively. The smart beam consists of a passive aluminum beam (507mmx51mmx2mm) with symmetrically surface bonded eight SensorTech BM500 type PZT (Lead-Zirconate-Titanate) patches (25mmx20mmx0.5mm). The beginning and end locations of the PZT patches along the length of the beam are denoted as r_1 and r_2, respectively. The patches are assumed to be optimally placed by considering maximum strain characteristics (Çalışkan, 2002). The parameters L, w, t, ρ, E, A, I, d_{31} denote length, width, thickness, density, Young's modulus, cross-sectional area, second moment of area and piezoelectric charge constant; and the subscripts b and p indicate the beam and PZT patches, respectively. Note that, despite the actual length of the beam is 507mm, the effective length utilized in the study (i.e. the effective span of the beam) reduces to 494mm since it is clamped with a fixture.

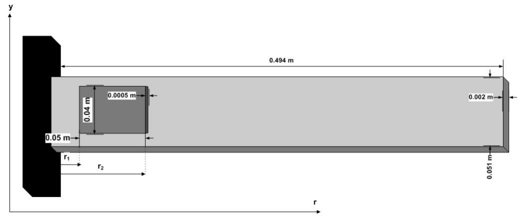


Figure 1: The smart beam model used in the study.

Table 1: The properties of the smart beam.

Aluminum Passive Beam	PZT
$L_b = 0.494m$	$L_p = 0.05m$
$w_b = 0.051m$	$w_p = 0.04m$
$t_b = 0.002m$	$t_p = 0.0005m$
$\rho_b = 2710kg/m^3$	$\rho_p = 7650kg/m^3$
$E_b = 69GPa$	$E_p = 64.52GPa$
$A_b = 1.02x10^{-4}m^2$	$A_p = 0.2x10^{-4}m^2$
$I_b = 3.4x10^{-11}m^4$	$I_p = 6.33x10^{-11}m^4$
-	$d_{31} = -175x10^{-12}m/V$

The assumed-modes model of the smart beam includes large number of resonant modes (Kırcalı, 2005). However, the control design criterion of this study is to suppress only the first two flexural modes of the smart beam. Hence, that higher order model is directly truncated to a lower order one, including only the first two flexural modes. The direct model truncation may cause the zeros of the system to perturb, which consequently affect the closed-loop performance and stability of the system considered (Clark, 1997). For this reason, a general correction term k_i^{opt} is added to the truncated model and the resultant model (Kırcalı, 2005 and 2006) can be expressed as:

$$\bar{G}_C(s,r)=\sum_{i=1}^{2}\frac{\bar{P}_i\phi_i(r)}{s^2+2\xi_i\omega_i s+\omega_i^2}+\sum_{i=3}^{50}\phi_i(r)k_i^{opt} \quad (1)$$

where general correction constant is [18]:

$$k_i^{opt} = \frac{1}{4\omega_c\omega_i}\frac{1}{\sqrt{1-\xi_i^2}}\ln\left\{\frac{\omega_c^2+2\omega_c\omega_i\sqrt{1-\xi_i^2}+\omega_i^2}{\omega_c^2-2\omega_c\omega_i\sqrt{1-\xi_i^2}+\omega_i^2}\right\}\overline{P}_i \quad (2)$$

and

$$\overline{P}_i = \frac{C_p\left[\phi_i'(r_2)-\phi_i'(r_1)\right]}{\rho_b A_b L_b^3 + 2\rho_p A_p L_p^3} \quad (3)$$

The nominal system model of the smart beam is denoted by $\overline{G}_C(s,r)$. The geometric constant $C_p = E_p d_{31} w_p(t_p + t_b)$ is due to bending moment of PZT patches exerted on the beam. The parameter r defines the spatial variation along the longitudinal axis and t is the time. The cut-off frequency of the correction term is denoted by ω_c and the details of all the parameters and the detailed derivation of the equation (1) can be found in reference (Kırcalı, 2006).

Theoretical assumed-modes modeling does not provide any information about the damping of the system. Experimental system identification, on the other hand, when used in collaboration with the analytical model, helps one to obtain more accurate spatial characteristics of the structure. The modal damping ratios and more accurate resonance frequencies were determined by spatial system identification (Kırcalı, 2006) and the results are given in Table-2:

Table 2: The resonance frequencies and modal damping ratios of the smart beam.

ω_1 (Hz)	ω_2 (Hz)	ξ_1	ξ_2
6.742	41.308	0.027	0.008

3 SPATIAL H$_\infty$ CONTROL OF THE SMART BEAM

3.1 Controller Design

Consider the closed loop system of the smart beam shown in Figure 2. The aim of the controller, K, is to reduce the effect of disturbance signal over the entire beam by the help of the PZT actuators.

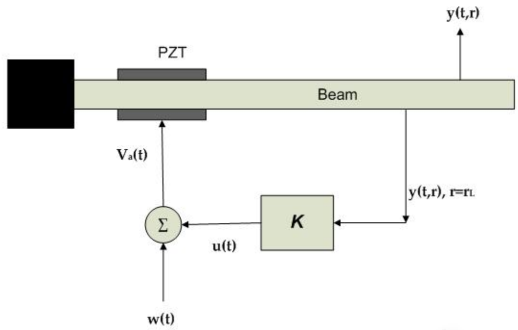


Figure 2: The closed loop system of the smart beam.

The state space representation of the system above can be shown to be (Kırcalı, 2006):

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ y(t,r) &= C_1(r)x(t) + D_1(r)w(t) + D_2(r)u(t) \\ y(t,r_L) &= C_2 x(t) + D_3 w(t) + D_4 u(t) \end{aligned} \quad (4)$$

where x is the state vector, w is the disturbance input, u is the control input, $y(t,r)$ is the performance output, $\tilde{y}(t,r_L)$ is the measured output at location $r_L = 0.99L_b$. The performance output represents the displacement of the smart beam along its entire body, and the measured output represents the displacement of the smart beam at a specific location A is the state matrix, B_1 and B_2 are the input matrices from disturbance and control actuators respectively, Π is the output matrix of error signals, C_2 is the output matrix of sensor signals, Θ_1, Θ_2, D_3 and D_4 are the correction terms from disturbance actuator to error signal, control actuator to error signal, disturbance actuator to feedback sensor and control actuator to feedback sensor respectively. The disturbance $w(t)$ is accepted to enter to the system through the actuator channels, hence, $B_1 = B_2$, $D_1(r) = D_2(r)$ and $D_3 = D_4$.

The state space form of the controller can be represented as:

$$\begin{aligned} \dot{x}_k(t) &= A_k x_k(t) + B_k y(t,r_L) \\ u(t) &= C_k x_k(t) + D_k y(t,r_L) \end{aligned} \quad (5)$$

such that the closed loop system satisfies:

$$\inf_{K\in U} \ \sup_{w\in L_2[0,\infty)} J_\infty < \gamma^2 \quad (6)$$

where U is the set of all stabilizing controllers and γ is a constant.

The spatial cost function to be minimized as the design criterion is:

$$J_\infty = \frac{\int_0^\infty \int_R y(t,r)^T Q(r) y(t,r) dr dt}{\int_0^\infty w(t)^T w(t) dt} \tag{7}$$

where $Q(r)$ is a spatial weighting function that designates the region over which the effect of the disturbance is to be reduced and J_∞ can be considered as the ratio of the spatial energy of the system output to that of the disturbance signal. The control problem is depicted in Figure 3.

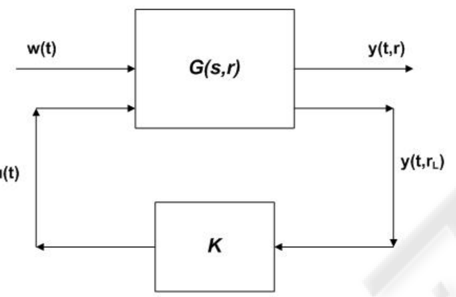


Figure 3: The spatial H_∞ control problem of the smart beam.

The spatial H_∞ control problem can be solved by the equivalent ordinary H_∞ problem (Moheimani et.al, 2003) by taking:

$$\int_0^\infty \int_R y(t,r)^T Q(r) y(t,r) dr dt = \int_0^\infty \tilde{y}(t)^T \tilde{y}(t) dt \tag{8}$$

Hence, following the necessary mathematical manipulations, the adapted state space representation will be:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ \tilde{y}(t) &= \begin{bmatrix} \Pi \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} \Theta_1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} \Theta_2 \\ \kappa \end{bmatrix} u(t) \\ y(t,r_L) &= C_2 x(t) + D_3 w(t) + D_4 u(t) \end{aligned} \tag{9}$$

The derivation of equation (9) and the below state space variables can be found in (Kırcalı, 2006) as:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2 & 0 & -2\xi_1\omega_1 & 0 \\ 0 & -\omega_2^2 & 0 & -2\xi_2\omega_2 \end{bmatrix} \tag{10}$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ \overline{P}_1 \\ \overline{P}_2 \end{bmatrix} \tag{11}$$

$$C_1 = \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \\ 0 \\ 0 \end{bmatrix}^T, C_2 = \begin{bmatrix} \phi_1(r_L) \\ \phi_2(r_L) \\ 0 \\ 0 \end{bmatrix}^T \tag{12}$$

$$\begin{aligned} D_1 = D_2 &= \sum_{i=3}^{50} \phi_i(r) k_i^{opt} \\ D_3 = D_4 &= \sum_{i=3}^{50} \phi_i(r_L) k_i^{opt} \end{aligned} \tag{13}$$

$$\Pi = \begin{bmatrix} diag(L_b^{3/2})_{2x2} & 0_{2x2} \\ 0_{3x2} & 0_{3x2} \end{bmatrix} \tag{14}$$

$$\Theta_1 = \Theta_2 = \begin{bmatrix} 0_{4x1} \\ \left(\sum_{i=3}^{50} L_b^3 \left(k_i^{opt} \right)^2 \right)^{1/2} \end{bmatrix} \tag{15}$$

One should note that, the control weight, κ, is added to the system in order to limit the controller gain and avoid actuator saturation problem. In the absence of the control weight, the major problem of designing an H_∞ controller for the system given in equation (4) is that, such a design will result in a controller with an infinitely large gain (Moheimani et.al, 1999). In order to overcome this problem, an appropriate control weight, which is determined by the designer, should be added to the system. Since the smaller κ will result in higher vibration suppression but larger controller gain, it should be determined optimally such that not only the gain of the controller does not cause implementation difficulties but also the suppressions of the vibration levels are satisfactory. In this study, κ was decided to be taken as 7.87×10^{-7}. The simulation of the effect of the controller is shown in Figure 4 as a Bode plot, and the frequency domain simulation is done by Matlab v6.5.

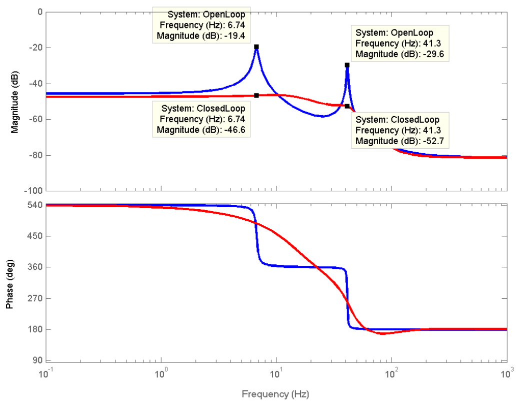


Figure 4: Bode plots of the open and closed loop frequency responses of the smart beam.

The vibration attenuation levels at the first two flexural resonance frequencies were found to be 27.2 dB and 23.1 dB, respectively. The simulated results show that the designed controller is effective on the suppression of excessive vibrational levels.

3.2 Experimental Implementation

The smart beam of this study, shown in Figure 5, consists of the PZT patches that are placed in a collocated manner to have opposite polarity and used as the actuators. A Keyence LB-1201(W) LB-300 laser displacement sensor (LDS) is used as the sensor. The closed loop experimental setup is shown in Figure 6.

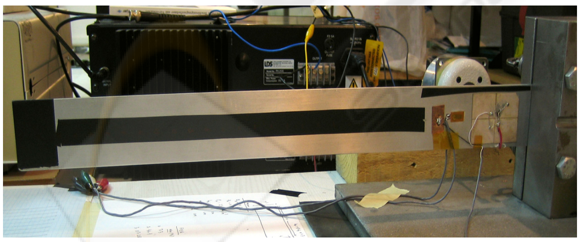


Figure 5: The smart beam used in the study.

The displacement of the smart beam at location $r_L = 0.99 L_b$ was measured by using the LDS and converted to a voltage output that was sent to the SensorTech SS10 controller unit via the connector block. The controller output was converted to the analog signal and amplified 30 times by SensorTech SA10 high voltage power amplifier before applied to the piezoelectric patches. The controller unit is hosted by a Linux machine, on which a shared disk drive is present to store the input/output data and the C programming language based executable code that is used for real-time signal processing.

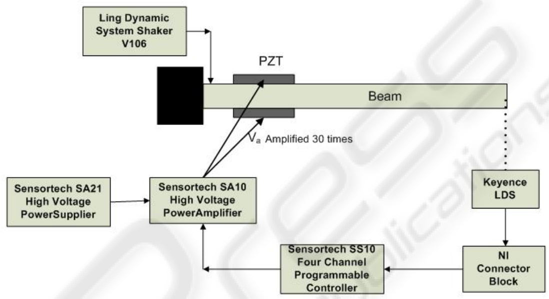


Figure 6: The closed loop experimental setup.

3.2.1 Free Vibration Suppression

For the free vibration control, the smart beam was given an initial 5 cm tip deflection and the open loop and closed loop time responses of the smart beam were measured. The results are presented in Figure 7. Figure 7 shows that the controlled time response of the smart beam settles nearly in 1.7 seconds. Hence, the designed controller proves to be very effective on suppressing the free vibration of the smart beam.

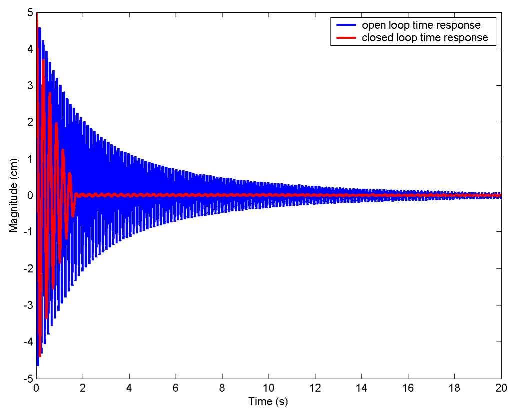
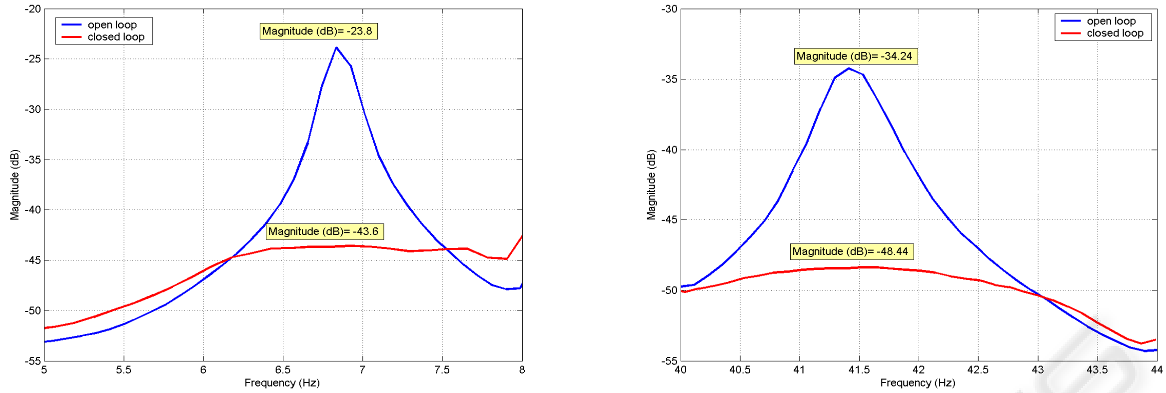
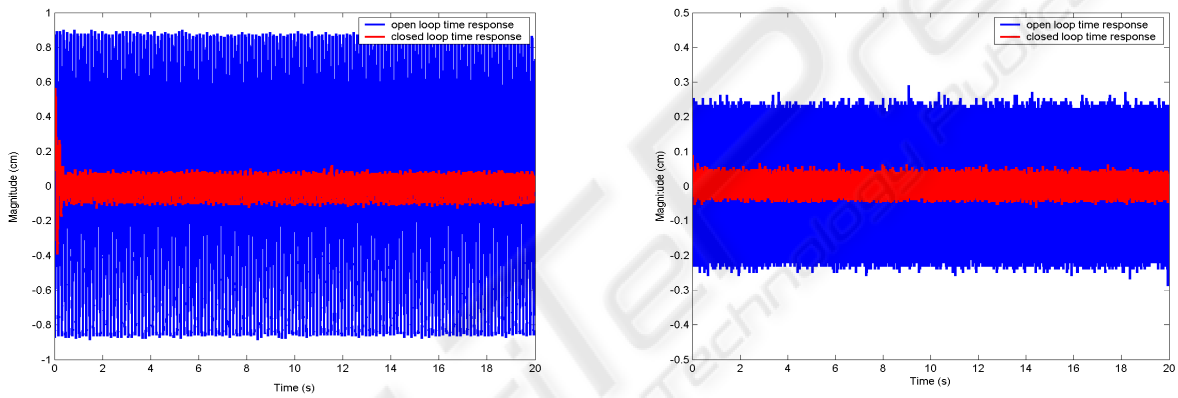


Figure 7: Free vibration suppression of the smart beam.



a) 5-8 Hz b) 40-44 Hz

Figure 8: Bode magnitude plot of the open and closed loop systems.



a) 1st resonance b) 2nd resonance

Figure 9: Open and closed loop time responses of the smart beam under constant excitation at resonance frequencies.

3.2.2 Forced Vibration Suppression

The forced vibration control of the smart beam was analyzed in two different configurations. In the first one, the smart beam was excited for 180 seconds with a shaker located very close to the root of the smart beam, on which a sinusoidal chirp signal of amplitude 4.5V was applied. The excitation bandwidth was taken first 5 to 8 Hz and later 40 to 44 Hz to include the first two flexural resonance frequencies separately. The experimental attenuation of vibration levels were determined from the Bode magnitude plots shown in Figure 8.a-b. The resultant attenuation levels were found as 19.8 dB and 14.2 dB, respectively. In the second configuration, instead of using a sinusoidal chirp signal, a constant excitation was applied for 20 seconds at the resonance frequencies again with a shaker. The ratios of the maximum time responses of the open and closed loop systems, shown in Figure 9.a-b, are considered as absolute attenuation levels. Hence, for this case, the attenuation levels at each resonance frequency were calculated approximately as 10.4 and 4.17, respectively. Consequently, the experimental results show that the controller is effective on suppression of the forced vibration levels of the smart beam.

3.3 Efficiency of the Controller

The efficiency of spatial controller in minimizing the overall vibration over the smart beam was compared by a pointwise controller that is designed to minimize the vibrations only at point $r_L = 0.99L_b$. For a more detailed description of the pointwise controller design, the interested reader may refer to the reference (Kırcalı, 2006). The implementations of the controllers showed that both controllers reduced the vibration levels of the smart beam due to its first two flexural modes in comparable efficiency (Kırcalı, 2006). On the other hand, the simulated H_∞ norms of the smart beam as a function of r, shown in Figure 10, showed that the spatial H_∞ controller has a slight superiority on suppressing the vibration levels over entire beam.

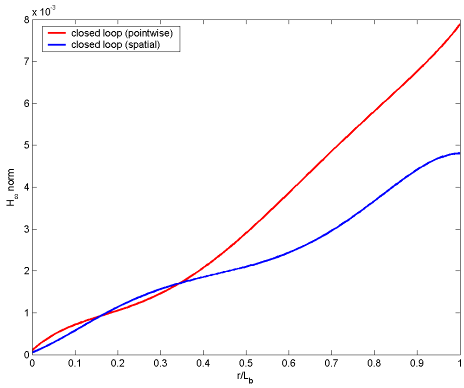


Figure 10: Simulated H_∞ norm plots of closed loop systems under the effect of controllers.

4 CONCLUSION

This study presented the active vibration control of a cantilevered smart beam. A spatial H_∞ controller was designed for suppressing the first two flexural vibrations of the smart beam. The efficiency of the controller was demonstrated both by simulation and experimental implementations. The effectiveness of the spatial controller on suppressing the vibrations of the smart beam over its entire body was also compared with a pointwise controller.

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