SHAPE COMPARISON OF FLEXIBLE OBJECTS
Similarity of Palm Silhouettes

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Abstract: We consider the problem of shape comparison for elastic objects presented by binary bitmaps. Our approach to similarity measuring of such objects is based on the conception of a flexible object. A flexible object is defined as a planar graph with a family of circles centered on graph edges. A set of admissible deformations is connected with each flexible object. These deformations are described as a group of planar graph vertices transforms. We define the flexible objects similarity through matching and alignment within the group of admissible deformations. The regular method for approximation of the binary bitmap shape by the flexible object is presented. The flexible object is designed as a subgraph of continuous skeleton of the binary bitmap. The proposed approach is applied to a problem of palm shape recognition for personal biometrical identification.

1 INTRODUCTION

The problem of binary bitmap shapes comparison arises in many applications. In particular, the problem of the palm shape recognition for biometrical identification is reduced to it. The shape of human hand (palm geometry) is an important feature used for personal identification in access control systems.

The color or the grayscale image of a palm received by means of any device (for example, of the web camera or the scanner) may be transformed to the monochrome bitmap in which an object (a palm) is presented by black pixels, and a background – by white. But a human palm is a difficult object to classify. The person can't repeat the same position of a palm even if he wants to. Two photos of the same palm and two photos of two different palms can have differences of the same range (Figure 1).

Known approaches to shape comparison of the objects based on alignment of their outline contours (Sederberg and Greenwood, 1992) are unsuitable for solving these problems. These methods don't preserve important invariants of the palm shape – finger's width and curvature.

Another approach to compare general structure features of object shapes is based on their skeletons (Sebastian and Kimia, 2001). A skeleton is more detailed description of topological structure of a shape. However, it isn't enough for comparison of palms. The topology of skeletons of palms anyway is almost identical. There are only 5 topologically different skeletons of palm. But skeletons don't allow comparing such important features as palm's outlines.

The proposed approach solves this problem. We propose a measure for human palms comparison (and other similar elastic objects) using both this

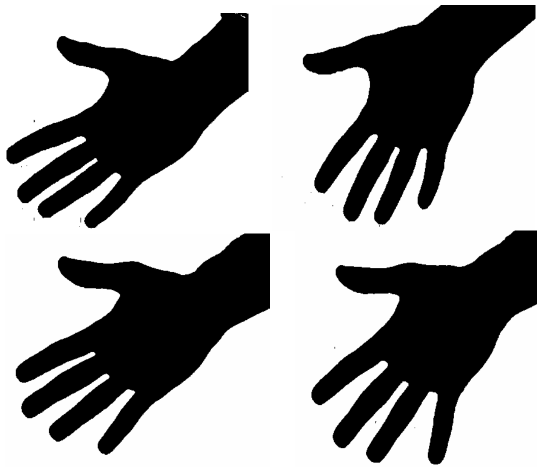

Figure 1: The silhouettes of palms (the first row – the same person, the second row – different persons).

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features. Alignment of skeletons allows comparing general structure of objects and deforming of their outlines. Then measure of difference between objects is obtained by matching of outlines.

The basis of this approach is the model of the so-called flexible object. Such object has a shape which can change within certain limits. The limits of these changes are described by special group of transforms. The transforms are organized in such a way that some elements of the shape are fixed and constant, and others can vary. With reference to a palm constant elements are metacarpus and phalanges of fingers, but changeable elements are joints. The problem of flexible object comparing is reduced to selection of such admissible transforms of these objects, in which their shapes will be the closest to one another. The difference of their shapes in this (the closest) position is accepted as a measure of the distinction of objects.

2 A FLEXIBLE OBJECT

Let's consider a set of points T on the Euclidean plane R^2, which is presented by a planar graph with tree type structure. This graph has a finite set of vertices, and its edges are continuous lines.

We bind with each point $t \in T$ in graph T some circle c_t with the center in this point. Let's call this family of circles $C = \{c_t, t \in T\}$ *a circular tree*. A graph T we call *an axial graph* of the circular tree. And a union $S = \bigcup_{t \in T} c_t$ of all circles of family C (as point sets) we call *a silhouette* of the circular tree. The silhouette of the circular tree represents a closed connected set of the Euclidean plane $S \subset R^2$. The outline of this set is an envelope of the whole family of circles C. There is an example of the circular tree on Figure 2. The set C is infinite, bat we use its finite subset for visualization on Figure 2 and other Figures.

Among all vertices of a circular tree we extract some subset of the points $P \subset T$ called bend points. We connect a range of angles between each couple of edges incident to bend point. We forbid the changing a relative position of edges for the rest vertices of the graph.

A change of angles in bend points in an admissible range is called *a deformation* of a circular tree. Such deformation implies not only the change of an axial graph, but also moving of a family of circles and a respective alteration of a circular silhouette of a tree.

Let V be a group of deformations of a circular tree C. The couple of circular tree and its group of deformations we call *a flexible object* $G = (C, V)$.

If some deformations of two circular trees make their silhouettes coincide then these trees are called *equivalent*. A shape of the flexible object is described by a set of silhouettes of all its equivalent circular trees.

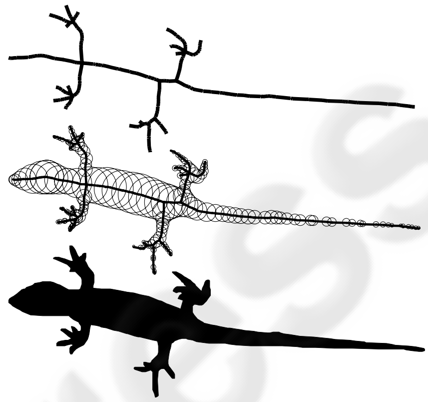

Figure 2: An axial graph, a family of circles and a silhouette of the circular tree.

3 THE COMPARISON OF FLEXIBLE OBJECTS

We introduce the metrics on the set of flexible objects in the following way. We define distance $\rho(G_1, G_2)$ between two flexible objects $G_1 = (C_1, V_1)$ and $G_2 = (C_2, V_2)$ as the minimal distance between their circular trees on the whole set of admissible deformations, i.e.

$$\rho(G_1, G_2) = \inf_{\substack{v_1 \in V_1 \\ v_2 \in V_2}} \mu[v_1(C_1), v_2(C_2)].$$

Here $v_1(C_1)$ and $v_2(C_2)$ are circular trees C_1 and C_2 transformed by means of deformations v_1 and v_2, and is a measure of distance of circular trees.

$$\mu(C_1, C_2) = Area(S_1 \setminus S_2 \cup S_2 \setminus S_1)$$

Here S_1 and S_2 are silhouettes of circular trees C_1 and C_2. And $\mu(C_1, C_2)$ is equal to the area of a symmetric difference of S_1 and S_2 minimized for all possible variants of matching.

Thus, the problem of an estimation of similarity of flexible objects consists in their matching on each other and a choice of such deformation of these objects and such matching at which the value of the distinctions of their shapes will be minimal. With reference to human palms it means, that it is necessary to apply palms to each other and move fingers in such a way that silhouettes of palms have coincided at the greatest degree.

4 THE CONSTRUCTION OF A FLEXIBLE OBJECT

Let it be a binary bitmap (Figure 1). A construction of a flexible objects which approximating this bitmap includes the following steps:

1. An approximation of the binary bitmap outline by the minimal perimeter polygon. The polygon is a closed path of the minimal length separating black and white pixels on the binary bitmap (Figure 3a).

2. A construction of the continuous skeleton of the polygon (Mestetskiy, 1998, 2006). The skeleton of the polygon is a locus of the centers of its inscribed maximal circles (Figure 3a). The skeleton of the polygon with its circles forms a circular tree, and the polygon itself is a silhouette of this tree.

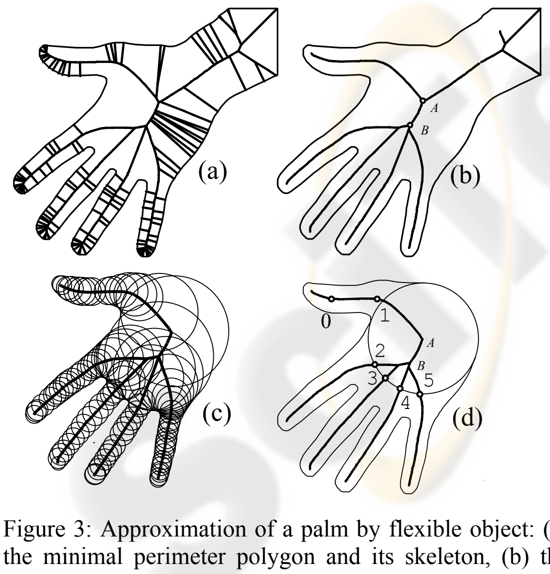

Figure 3: Approximation of a palm by flexible object: (a) the minimal perimeter polygon and its skeleton, (b) the skeletal base, (c) the circular tree, (d) banding points.

3. A pruning of a skeleton to get the so-called skeletal base of the polygon (Figure 3b).

Let M be a polygon, and S - a silhouette of circular tree of a connected subgraph of its skeleton. We will call the minimal subgraph of the skeleton, at which a silhouette of a circular tree differs from a polygon in the Hausdorf metrics no more than on the given value ε, *a skeletal base* of the polygon M, i.e. $H(M,S) \le \varepsilon$.

A skeletal base has a much more simple structure, than the skeleton of a polygon (Figure 3b) and is more stable to noise distortions connected with the source binary bitmap.

5 THE COMPARISON OF SILHOUETTES OF PALMS

Let's choose a third degree vertex of a skeletal base graph which is incidence with the branch of the thumb (the vertex A on Figure 3b). The branches of the thumb and a wrist are crossing in this vertex. The image of a wrist is extraneous information for palm shape description. Therefore we delete the branch of a wrist in the skeletal base. The obtained graph is an axial graph of a circular tree of a flexible object (Figure 3c). The vertex A is a root of the circular tree. Its circle is called "root" circle. The next third degree vertex B we call "center" of a palm (Figure 3c) and its circle is called "middle" circle.

The analysis of a real skeleton of a human palm shows, that it is enough to consider six bend points: two points of the thumb and one point of each of the rest four fingers (Figure 3d). We choose two bend points (0 and 1) of maximal curvature at thumb branch and four bend points (2-5) as crossing of finger branches and the "root" circle.

We can estimate the distance $\rho(G_1,G_2)$ between two palms by more simple measure:

$$\rho(G_1,G_2) = \inf_{\substack{v_1\in V_{1_1} \\ v_2\in V_2}} \mu[v_1(C_1),v_2(C_2)] \le$$

$$\le \inf_{v_2\in V_2} \mu[C_1,v_2(C_2)] = \hat{\rho}(G_1,G_2)$$

The estimation $\hat{\rho}(G_1,G_2)$ represents a measure of distance of flexible objects G_1 and G_2, received by a static position of G_1 and a deformation of G_2.

Alignment of two palms is carried out by the following steps.

1. Coincidence of centers of "middle" circles (vertices B in Figure 3d).

2. Coincidence of directions from the centers of "middle" circles to the centers of "root" circles (vectors $\overline{BA}$ in Figure 3d).

3. Deformation of the axial graph of the second palm for coincidence with the axial graph of the first palm (Figure 4). For that we “rotate fingers” of the second palm (branches of axial graph) around bending points. The Hausdorf metrics can be used as a measure of coincidence of these branches.

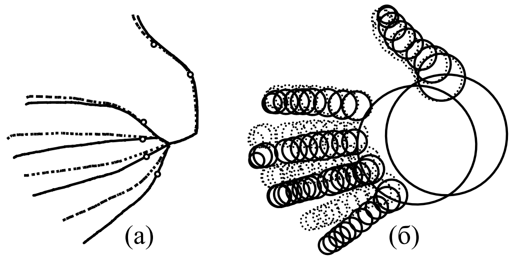

Figure 4: Deformation of the circular tree: (a) rotation of branches, (b) moving of circles.

4. Construction of circular tree silhouettes as envelopes of a family of circles.

5. Comparing of silhouettes (Figure 5). The effective algorithm for computation of the areas of a symmetric difference is designed with the help of methods of computational geometry.

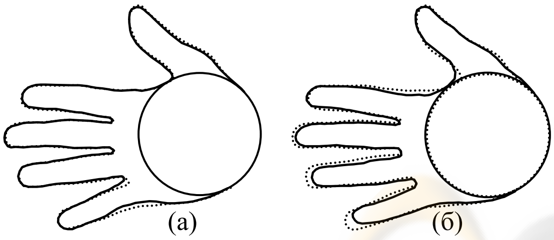

Figure 5: Comparison of silhouettes: (a) images of the same palm, (b) palms of different persons.

The computing experiment was carried out for testing of proposed method. The data base of 1662 bitmaps of 320 palms (4-6 images per person) has been used in this experiment. All images 640×480 were obtained for the same conditions (camera, distance, brightness). The approximating flexible objects have been constructed for each of these bitmaps. The measure of distance between silhouettes S_1 and S_2 was computed as

$$\rho(S_1, S_2) = \frac{Area(S_1 \setminus S_2 \cup S_2 \setminus S_1)}{Area(S_1)} \cdot 1000 .$$

The left diagram on Figure 6 shows the distribution of distances to the nearest sample of the same person (left curve), and of different people (right curve). Such distance enables to construct a classification rule by the nearest neighbor. The diagram of classification errors for different values of the threshold is shown on Figure 6 (right).

The running time for binary bitmap approximation of one bitmap by the flexible object is 15 msec, and for two palms comparison is 0.5 msec using Intel processor 1.3 GHertz.

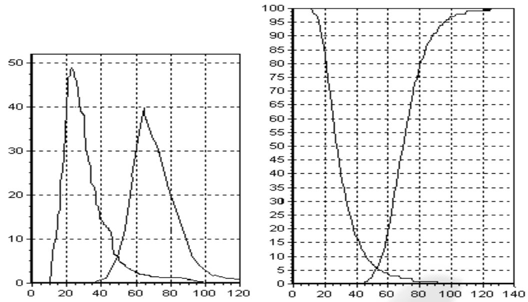

Figure 6: Left – distance distribution to the nearest palm, right – classification errors (FRR – left curve, FAR – right curve).

6 CONCLUSIONS

The combination of two constructions – an outline and a skeleton – opens up opportunities for the comparison of objects which don’t have strictly fixed shapes using a matching method. The proposed method is well adjusted with common sense, is easily visualized and allows efficient implementation.

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