PRESSURE BASED INK DIFFUSION MODEL FOR REAL-TIME SIMULATION OF CHINESE CALLIGRAPHY

Ye Wang and Jon Rokne
Department of Computer Science, University of Calgary, Canada

Keywords: Non-Photorealistic Rendering, Chinese Calligraphy and Ink Painting, Simulation of Ink Diffusion, Pressure Based Modeling.

Abstract: This paper describes a novel approach to simulating Chinese calligraphy for digital image purposes. The approach includes an ink diffusion model, a multi-layer paper model, a brush model, and the simulation of special effects. Special attention is given to the ink diffusion effect which is of importance in Chinese calligraphy. When the ink is deposited onto absorbent paper, it spreads outside the original border of a stroke since the flow of water will transport carbon particles along the capillary tubes found in the paper. The ink flow model is based on a new algorithm simulating dynamic ink diffusion into absorbent paper. In this capillary network based paper model, the pressure at each node can be obtained from Darcy's law applied to the ink used in the calligraphy on each edge and it is proportional to the density of capillaries. The deposition layer of the paper is furthermore used to simulate the deposition of carbon particles into the paper and it is also used to simulate the washing out effects. Ink effects such as irregular edges and back run effect can also be simulated. The system is efficient and can create realistic Chinese calligraphy in real-time.

1 INTRODUCTION

Chinese calligraphy and Chinese painting have been refined over thousands of years into distinctly Oriental artforms. In the past, these artforms could only be practiced using traditional tools and instruments. With the advent of the computer, a powerful tool that can be adapted to a variety of simulation tasks, these artistic expressions can be simulated in a very realistic manner. Since these artforms are artistic expressions of reality they can be considered to be non-photorealistic expressions, although calligraphy might also be considered to be a photorealistic expression since the desired results shown should be as close to the real ones as possible.

In this paper, an efficient novel approach to simulate ink diffusion and percolation based on the physical mechanism of ink diffusion in rice paper is presented. Together with simulation of ink dispersion, paper and ink interaction and Chinese calligraphy brush techniques, this forms a complete system consisting of four main parts: the brush model, the ink and water transfer model, the virtual paper simulation and special effects simulation.

2 RELATED WORK

Simulating Chinese calligraphy and Chinese painting on a computer is an interesting topic which has attracted a number of researchers. One of the earliest attempts is that of Strassmanns (Strassmann, 1986) who simulated the calligraphy brush as a 1D array of bristles. Following this Guo and Kunii (Guo and Kunii, 1991) proposed a model to simulate rice paper which consisted of a mesh with randomly distributed fibers. They divided the simulation into small regions and distributed the fibers so that the local fiber distribution was random and the global distribution uniform. However, the complex mechanisms of ink diffusion and water percolation were not simulated adequately. This meant that the characteristics of the calligraphic brush interaction with the paper through diffusion of the ink could not be simulated sufficiently realistically.

Kunii et al. (Kunii et al., 1995) later proposed a multidimensional diffusion model including three distinct zones (initial zone, black border, and gray zone). Their model simulated ink diffusion, and it can also be applied to black ink rendering. However, the dif-

Wang Y. and Rokne J. (2007).
PRESSURE BASED INK DIFFUSION MODEL FOR REAL-TIME SIMULATION OF CHINESE CALLIGRAPHY.
In *Proceedings of the Second International Conference on Computer Graphics Theory and Applications - GM/R*, pages 271-276
DOI: 10.5220/0002071902710276

fusion rate was a constant in his model, which is not realistic when ink is deposited on the highly absorbent rice paper.

A paper model consisting of three layers was introduced by Curtis et al. (Curtis et al., 1997). They use a cellular automaton to simulate fluid flow and pigment dispersion and the layering to simulate various watercolor effects, such as edge-darking, granulation, back runs, separation of pigments, and glazing. Their approach did not include the permeability effect, which means that the subtle ink patterns of Chinese calligraphy and painting can not be simulated by their model. Lee (Lee, 2001) improved Kunii's (Kunii et al., 1995) model by using a two perpendicular directions model that includes a texture structure and sinusoidal variations to simulate rice paper. In this paper model, the paper is divided into square regions called "papels". They developed the sinusoidal wave schema for representing the flow of ink through a fiber mesh. However, the fiber mesh model used in the paper is quite different from the fiber structure in real paper and the wave algorithm for the diffusion process is different from the real physical diffusion process. This means that the model only simulates processes of this kind which do not conform closely to the physical processes for ink flow in paper. Based on Lee's (Lee, 2001) fiber mesh structure and layer structure for each paper cell, Yu et al. (Yu et al., 2003) presented a local equilibrium model (LEM) which simulated the movement of water and ink on and in paper. Their modified layer model simulated overlapped strokes successfully. However, without the physical process based simulation, the simulation of the ink diffusion phenomenon was not realistic. Xiaofeng (Xiaofeng Mi, 2004) used different droplet models to simulate the tangent area between brush and paper and based on these models they developed a virtual brush model which is inspired by calligraphy and painting experience. Their models were able to create realistic looking results in real time.

Huang et al. (Huang et al., 2003) presented a method which can simulate a variety of tones on different types of paper. Both a regular fiber mesh whose fibers are uniformly aligned and an irregularly distributed fiber mesh consisting of randomly positioned fibers were simulated in this paper. The diffusion of brush strokes can be easily controlled, according to experimental data and users can specify parameters to get the desired effects. Van et al. (T. V. Laerhoven, 2004) developed a layered paper model which divides the paper into a grid of small subpapers. Each layer is implemented as a two-dimensional grid of cells that exchange certain amounts of water with ink. It updates all cell values according to their neighboring cell values in the same layer, and possibly in a layer above and below. They have to distribute the system by breaking up the whole grid in smaller subgrids that are simulated on separate processing units. The results are sent back to the parent application and combined with the results of other subpapers to produce the final image. Tsai et al. (Tsai et al., 2005) discussed diffusion rendering of ink painting and focussed on synthesizing artistic effects of ink-refusal and stroke-trace-reservation. This was the first paper to take the quantity of glue in the paper as one important parameters of the structure of paper in the ink diffusion simulation.

Chu and Tai (Chu and Tai, 2005) presented a fluid flow model based on the lattice Boltzmann equation (LBE) (Succi, 2001) for simulating percolation in absorbent paper. Instead of starting with a macroscopic description of the fluid, the LBE modeled the physics of fluid particles at a macroscopic level. They adapted the basic LBE method to incorporate various features needed for the special case of percolation. The paper thickness patterns obtained by scanning the paper against a dark background were stored aa a texture map.

3 CHINESE INK AND PAPER

Chinese ink is a mixture of soot and glue which is ground together with water to get black liquid ink. The soot is composed of carbon particles which are easily dissolved in water. The dimension of a carbon particle is in the range of 10-150 nm, thus it can seep into the paper easily and produce the extraordinary rendering effects of the Chinese calligraphy.

Rice paper is often used in Chinese calligraphy. It is highly absorbent because of the special materials used and the very thin fiber structures. To reduce its absorbency, the paper can be soaked with alum water. The higher the density of the alum water, the lower the absorbency of the paper will be. So the rendering effect is mainly determined by paper absorbency and fiber structure.

The simulation of Chinese calligraphy includes three parts:

1) **Paper model**: The paper consists of a mesh with randomly distributed fibers. To model this fiber structure, the paper is divided into small square cells. Each cell has four neighbors which are connected with fibers and each cell of the paper is simulated as a network flow model with a node and four edges. Each edge represents a bundle of capillaries and the node acts as a reservoir which stores the water flowing into it. The water will

flow out the cell along the edges only when it is full of water.

2) **Ink flow model**: The two layer model, a diffusion layer and a deposition layer, will be used to simulate the ink flow on the rice paper. The network based flow model will be used in the diffusion layer to model the water flow in the paper and also to determine the final region of the ink marks. The deposition layer can model the carbon particles deposited into the paper. It is also used to simulate the washing out effects when the area is painted later.

3) **Absorption**: As the ink flows in the paper, the carbon particles will be absorbed by the fibers and deposited into the paper. Thus the density of the ink changes as it flows along the fiber network. These effects are considered in our simulation as they affect the final rendering result.

4 PAPER MODEL

To simulate the texture of the paper, we add a uniformly distributed random number for modeling the fiber density of each edge.

If the average fiber density is Den_{ave}, the standard deviation of the uniform distribution is Std_{dev}, then the fiber density of each edge will be:

$$Den_{edge} = Den_{ave} + 2 * Xtd_{dev} * [rand(X_p, X_{p-1} - 0.5] \quad (1)$$

where Y_p and X_p is the size of the fiber mesh of the paper. $rand(M,N)$ is an $M \times N$ matrix with random entries chosen from a uniform distribution on the interval $(0.0, 1.0)$.

After the fiber densities of all edges of each node were generated, the fiber density of each cell, Den_{cell}, could be obtained by averaging the densities of all edges:

$$Den_{cell} = (Den_{edge1} + Den_{edge2} + \cdots)/n.$$

where n is the number of edge connected with this cell.

In our programme, $Den_{ave} = 1 Std_{dev} = 0.2$

- The edge is modeled as a pipe which consists of a bundle of capillaries. The cross section area S and the diffusion coefficient K of each of the four edges is a function of the local fiber density ρ^e. It is assumed that K and S are linear with ρ^e.
- All cells have the same volume and the fiber density $\rho^c_{m,n}$ of the cell (m,n) is random. The fiber density of each cell is obtained by using the average value of the four edges.

5 INK FLOW MODEL

A network based flow model can be used in the diffusion layer.

5.1 Darcy's law

Darcy's law is commonly used to study the flow in porous media in groundwater hydrology. As a macroscopic law, it doesn't tell us about the flow through individual pores. Since paper is a typical porous medium, the water diffusion in it can also be modeled by Darcy's law. As the flow through the individual pores formed by the fibers of the paper is not important, we mainly consider the flow through the capillary bundles from one cell to another. According to Darcy's law, the flow through the pipe composed of capillary bundle is

$$Q = KJS \quad (2)$$

where $Q(m^3/s)$ is the flux of the diffusion, $S(m^2)$ is the area of the cross section of the pipe which is consist of a bundle of capillaries, J is the gradient of the pressure of the flow(the pressure on unit length of the pipe along the flow direction), and $K(m/s)$is the diffusion coefficient of the pipe.

5.2 Flow in Capillary Networks

For two nodes connected with capillaries as shown in Figure 1(a), if the pressure at nodes m and $m+1$ are p_m and p_{m+1}, the length of the edge is L_m, the diffusion coefficient is K_m and the cross-section is S_m, then, the flux will be

$$Q_m = K_m S_m \frac{p_m - p_{m+1}}{L_m} \quad (3)$$

according to Darcy's law, so the flux can be derived if the pressure at each node is known.

For the capillary network with nodes $M \times N$ as shown in Figure 1(c), the pressure at each node can be obtained by applying Darcy's law on each edge. For one node (m,n) in the network, if the pressure at this node is $p_{m,n}$, and the pressures at the four neighboring nodes are $p_{m+1,n}$, $p_{m,n-1}$, $p_{m-1,n}$ and $p_{m,n+1}$, then the flux on the four edges are Q_1,Q_2,Q_3 and Q_4, and

$$Q^i_{m,n} = K^i_{m,n} S^i_{m,n} \frac{\hat{p}_i}{L^i_{m,n}}; \qquad i = 1, \ldots, 4 \quad (4)$$

where $\hat{p}_i$ is the difference of the pressure on the edge,

$$\hat{p_1} = p_{m+1,n} - p_{m,n} \quad (5)$$

$$\hat{p_2} = p_{m,n+1} - p_{m,n} \quad (6)$$

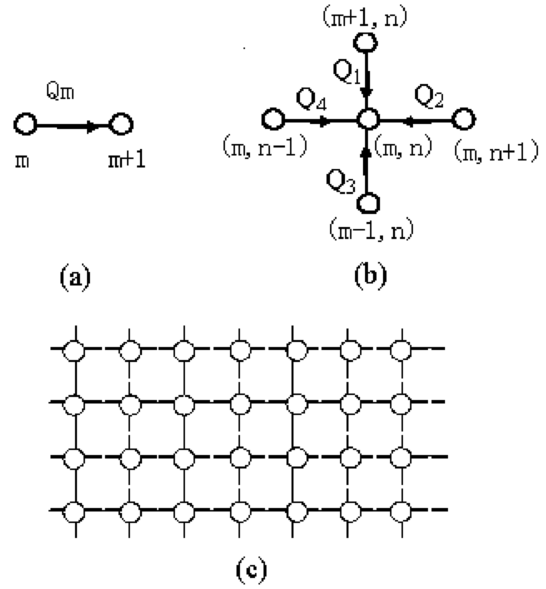

Figure 1: Nodes and edges in the networks.

$$\hat{p_3} = p_{m-1,n} - p_{m,n} \tag{7}$$

$$\hat{p_4} = p_{m,n-1} - p_{m,n} \tag{8}$$

Since the flux is conservative, the flux flow into node (m,n) is zero, that is

$$\sum_i Q^i_{m,n} = 0 \tag{9}$$

Let

$$c^i_{m,n} = \frac{K^i_{m,n} S^i_{m,n}}{L^i_{m,n}}; \qquad i = 1,\ldots,4 \tag{10}$$

Then

$$c^1_{m,n} p_{m+1,n} + c^2_{m,n} p_{m,n+1} + c^3_{m,n} p_{m-1,n} + c^4_{m,n} p_{m,n-1} - \sum_i c^i_{m,n} p_{m,n} = 0 \tag{11}$$

The same equation can be derived for each node, and the following equation can be obtained

$$A_{MN\times MN} P_{MN\times 1} = 0 \tag{12}$$

where $P_{MN\times 1}$ is the pressure vector

$$\mathbf{P_{MN\times 1}} = \begin{pmatrix} p_{1,1} \\ \vdots \\ p_{M,1} \\ \vdots \\ p_{1,N} \\ \vdots \\ p_{M,N} \end{pmatrix} \tag{13}$$

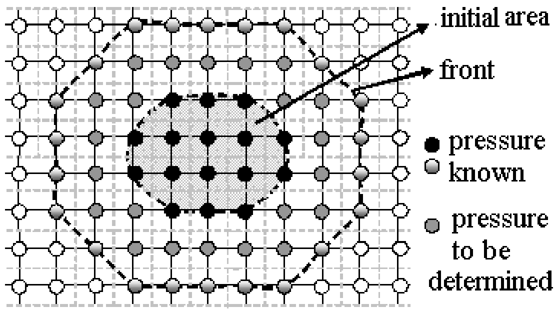

Figure 2: Diagram of the diffusion.

and $A_{MN\times MN}$ is a coefficient matrix which is determined only by the properties of the paper.

The initial region of the ink is simulated as a reservoir, and for those nodes within this region, the pressures can be assumed to be zero. Since the flow is caused by the capillary attraction, it is also assumed that the flow is driven by pressures on the front (the boundary of the diffused area) nodes. The pressure at the nodes on the front can be assumed to be a constant p_0.

When the ink is diffusing, the front is moving outwards. Therefore, at each step of the diffusion, only pressures of the nodes in the diffused region have to be calculated because pressures of the nodes in the initial region and at the front are already given as shown in Figure 2. Furthermore, pressures of the nodes outside the diffusion region are not needed as there is no ink there, and the matrix A can be simplified when the corresponding components in P are deleted from the equation. Now by replacing the components corresponding to the pressures at the nodes in the initial region and at the front in P with their values, eq. (12) can be written as

$$\hat{A}\hat{P} = B \tag{14}$$

where $\hat{P}$ is the pressures of the nodes within the diffused region.

Since the pressure at nodes within the diffusion region is varying as the ink is diffusing, $\hat{A}$ and $\hat{P}$ are varying with time. However, at each time step, the pressure $\hat{P}$ can be obtained by eq. (14). With the pressure at each node, the flux can be then derived with eq. (4) .

In this model, fiber texture is denoted by parameters $K_{m,n}$ and $S_{m,n}$, which will affect the percolation effect of the final rendering result. If the average values of $K_{m,n}$ and $S_{m,n}$ are K_0 and S_0. So the three factors K_0, S_0 and the pressure p_0 at the front will determine the diffusion speed. So in order to simulate the physical process of the ink diffusion in a paper, K_0, S_0 and p_0 are determined by experiments.

5.3 Algorithm

- Step 1: find the source region of the ink. The pressure of the nodes in this region can be assumed to be zero or a function of the amount of ink remaining in this area.
- Step 2: find the front nodes. The pressure of the node on the front is a function of the fiber density of the cell. For simplicity, a constant pressure of the node in the front can also be assumed.
- Step 3: find the mid-nodes of the diffusion network and form the coefficient matrix A to obtain pressures of these nodes.
- Step 4: solve the linear equation $\hat{A}\hat{P} = B$ to get the pressures.
- Step 5: calculate the flux of each edge. The evaporation of water can be considered in this step by subtracting it from the flux of each edge.
- Step 6: calculate the time when a front cell is full of ink and the amount of ink flows into each cell on the front.
- Step 7: calculate the amount of the carbon particles deposited in the deposition layer.
- Step 8: update the amount of ink in the source region. If there is no ink left then stop the diffusion, else go to step 2.

6 ABSORPTION

The size of the carbon particles in the ink is random with a probability distribution. However, in order to simplify the calculation, it is assumed that the carbon particles all have the same size and all have the mean dimension value. Thus the density of the ink is only determined by the number of the particles in it.

The rendering effect can be calculated directly from the carbon particles deposited in the paper. It is assumed that the absorption rate is independent of the velocity of the flow and only related to the local fiber density of the paper and the diameters of the particles. As the ink flows in the paper, the carbon particles will be absorbed by the fibers and deposited into the paper. Thus the density of the ink changes as it flows along the fiber network. At the end of the diffusion, all particles in the ink remaining in each cell will deposit on the paper after the water is evaporated. Furthermore, following assumptions are made in our simulaition:

- The absorption rate of the paper is proportional to the local fiber density of each cell.
- The color intensity of the image is a proportional to the carbon density at each cell.

Since the source region is modeled as a reservoir, the ink is uniformly distributed in this region and flow out from its boundary. As the ink flows out and the carbon particles deposit in the paper, there will be a deep dark edge at the initial boundary if the absorbency of the paper is high. This is because the ink flow in the initial region is ignored. In order to avoid this unrealistic phenomena, the carbon particles deposited in the diffusion region is re-distributed by subtracting parts of the particles proportionally from each cell and averagely putting them into the source region to make the density of the deposited particles continuous in the initial boundary.

If the density of the deposited carbon particles in the diffusion region is $d_{m,n}$ and the average value at the initial boundary is d_0, the number of cells and the average density in the source region are N_s and d_s. Then

$$d_0\beta = d_s + (1-\beta)\frac{1}{N_s}\sum_{m,n} d_{m,n} \tag{15}$$

where

$$\beta = \frac{d_s + \frac{1}{N_s}\sum_{m,n} d_{m,n}}{d_0 + \frac{1}{N_s}\sum_{m,n} d_{m,n}} \tag{16}$$

is the coefficient to balance the dark edge,

Therefore, the carbon density at each cell in the diffusion region can be balanced by multiplying β to correct the dark edge problems.

7 EXPERIMENTS

7.1 Rendering of a Circular Region

For a drop of ink diffusing in a paper, the image is a circle. The result is shown in Figure 3.

```
Paper size:        100 by 100 cells
Paper texture:     Uniform distribution on
                   the interval (0.8,1.2)
Original region:   a circle at the center,
                   diameter 40
Amount of ink:     1200
Ink density:       1
Brightness of the ink:    0.8(Maximum:1)
Average absorption rate:  0.2
```

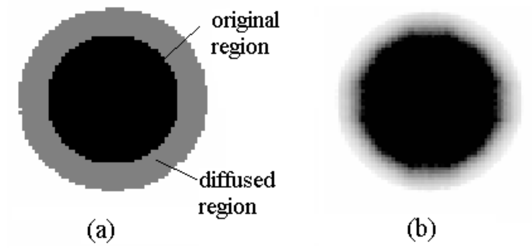

Figure 3: Diffusion from a circular region (a) region diffused; (b) rendering effect.

7.2 Rendering of a Chinese Character

Using the Chinese character ”ZHONG” as a sample to show the rendering effect of our method. The result is shown in Figure 4.

```
Paper size:       150 by 150 cells
Paper texture:    Uniform distribution
                 interval (0.8,1.2)
Original region:  Chinese character
                  "ZHONG"
Amount of ink:    2000
Ink density:      1
Brightness of the ink:    0.8(Max:1)
Average absorption rate:  0.2
```

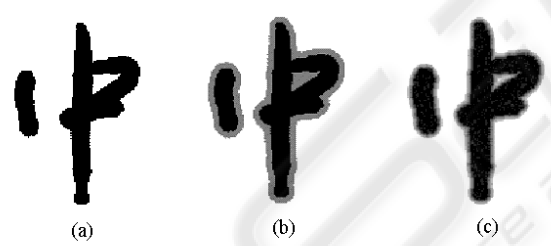

Figure 4: Diffusion of a Chinese character ”Zhong” (a) original character; (b) region diffused; (c) rendering effect.

The results of the two case studies showed that our method can simulate the ink diffusion in the rice paper very well. The effect of the paper texture on the diffusion can be considered by randomized fiber density. The percolation effect can also be modeled in our model by the randomized fiber density and absorption rate of the paper.

8 CONCLUSION

Our ink flow model is based on the Darcy’s law. By calculating the pressure of each node on the paper we can simulate the physical process of ink diffusion and percolation unlike many other methods which can only simulate the effect of the diffusion but can not simulate the physical process. The experiments show that our new method can give very satisfied result including to simulate the irregular edge and fiber structure in a realistic way. In addition, compared to the fluid model, there is no need to solve the partial differential equations. This means that the calculations are fast. There are some other effects like wash out, ink draw back which will be added into our model later.

REFERENCES

Chu, N. S.-H. and Tai, C.-L. (2005). MoXi: Real-Time Ink Dispersion in Absorbent Paper. *SIGGRAPH 2005*, 24(3):504–511.

Curtis, C. J., Anderson, S. E., Seims, J. E., Fleischer, K. W., and Salesin, D. H. (1997). Computer-Generated Watercolor. In Whitted, T., editor, *SIGGRAPH97*, CG-PACS, pages 421–430, New York.

Guo, Q. and Kunii, T. L. (1991). Modeling the diffuse painting of sumie. *IFIP Modeling in Computer Graphics*, pages 329–338.

Huang, S., Way, D., and Shih, Z. C. (2003). Physical-Based Model of Ink Diffusion in Chinese Ink Paintings. In *Proceedings of WSCG 2003*, volume 10, pages 520–527.

Kunii, T. L., Nosovskij, G. V., and Hayashi, T. (1995). A diffusion model for computer animation of diffuse ink painting. pages 98–102, Geneva, Switzerland.

Lee, J. (2001). Diffusion Rendering of Black Ink Paintings Using New Paper and Ink Models. *j-CAG*, 25(2):295–308.

Strassmann, S. (1986). Hairy brush. *Proc. of SIGGRAPH ’86*, pages 225–232.

Succi, S. (2001). *Lattice Boltzmann equation for fluid dynamics and beyond*. Oxford University Press.

T. V. Laerhoven, J.Liesenborgs, F. R. (2004). Real-time watercolor painting on a distributed paper model. pages 640–643, Crete, Greece.

Tsai, C., Liaw, C., Huang, C., and Lee, J. (2005). Synthesizing the artistic effects of ink painting. *Optical engineering*, pages 359–368.

Xiaofeng Mi, Min Tang, J. D. (2004). Droplet: A virtual brush model to simulate chinese calligraphy and painting. *Journal of Computer Science and Technology*, 19(3):393–404.

Yu, Y. J., Lee, D. H., Lee, Y. B., and Cho, H. G. (2003). Interactive Rendering Technique for Realistic Oriental Painting. *Journal of WSCG*, 11:538–545.