SURFACE MODELING OF MULTI-POINT, MULTI-FLUTE CUTTING TOOLS

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Abstract: Cutting tools are usually represented by two-dimensional representation schema(s). The two-dimensional nomenclatures have their inherent limitations. This paper outlines a detailed surface based modeling paradigm for a variety of multi-point, multi-flute cutting tools. The work presents the generic biparametric surface based models of slab mills, end mills and drills. The flutes are modeled as helicoidal surfaces. The relations to map proposed three-dimensional (3D) rotational angles that generate 3D geometric models to conventional angles (forward mapping) and their reverse relations (inverse mapping) are also developed. The new paradigm offers immense technological advantages through numerous downstream applications.

1 INTRODUCTION

Traditionally, the geometry of cutting tools has been defined using the principles of projective geometry. Such definitions are two-dimensional (2D) in nature. The developments in the field of Computer Aided Geometric Design (CAGD) now provide a designer to specify the cutting tool geometry in terms of biparametric surface patches. Such an approach provides the comprehensive three-dimensional (3D) definitions of the cutting tools. The surface model of a cutting tool can be converted into a solid model and then may be used for down stream applications. The existing 2D representation schemes are unable to directly provide the necessary data for such applications. The 2D modeling data need to be converted into 3D models before they can be used. The primary goal of this work is to outline surface models of multi-point, multi-fluted cutters.

A wide range of cutters used in practice are multi-point and multi-fluted in geometry (Drodza, 1983). Considerable work has been done in the area of geometric modeling of the drill (Armarego, 1998, Hsieh, 2002, Wang, 2001), helical milling cutters (Sheth, 1990) and end mills (Chen, 2001); however, the works are not in the direction of development of unified representation schemes. Tandon et al. have proposed the unified modeling schemes for slab mills (Tandon, 2004) and end mills (Tandon, 2005).

In the present work, mathematical models of the complex geometry of the fluted cutters are formulated as a combination of surface patches. The orientation of the surface patches is defined by 3D angles, termed as rotational angles. Relations to calculate the conventional two-dimensional (2D) tool angles and the 3D rotational angles from one to other are developed. Finally, graphics output in the form of rendered image of the cutter is shown for verification of the methodology.

2 SLAB MILLING CUTTERS

Slab mills produces flat surfaces parallel to the axis of the spindle. The cutting edges can be straight or helical and are on the circumference (Drodza, 1983).

2.1 Modeling the Surface of Slab Mill

The flutes of the slab mill is made up of five surface patches and they are face (Σ_1), land (Σ_2), flank (Σ_3), tooth back (Σ_4) and fillet (Σ_5). The tooth geometry is

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completed by left and right hand planar surfaces (Σ_6/Σ_7). The flute is modeled as a sweep surface. For this, a generic composite curve perpendicular to the axis and a sweeping rule is required. When the sweeping is linear, the straight tooth cutter is formed and when the sweep is a combination of rotational and parallel sweep, then the resultant surface is helicoidal surface and the cutter is helical slab mill.

The composite curve in XY plane is composed of vertices $V_1 \ldots V_7$ (Figure 1). Let D, D_R and l_l be cutter diameter, root circle diameter and width of primary land respectively and α_i, β_i, γ_i be the angles of rotation of surface patch 'i' about X, Y, Z axis respectively.

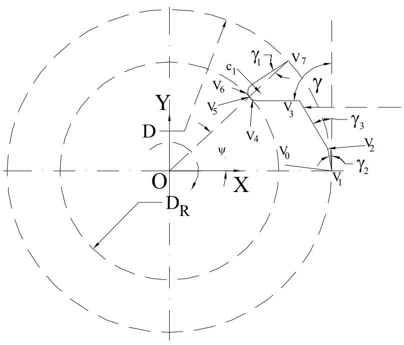

Figure 1: Sectional Curve of a Slab Mill.

The cross-section curve V_1V_7 is composed of five parametric curve segments defined in terms of parameter 's'. The curve segments $\mathbf{p}_1(s)$, $\mathbf{p}_2(s)$, $\mathbf{p}_3(s)$ and $\mathbf{p}_5(s)$ are straight lines between vertices V_1V_2, V_2V_3, V_3V_4 and V_6V_7 respectively, while the curve $\mathbf{p}_4(s)$ is a circular arc of radius R. The sweep surfaces Σ_1 to Σ_5 are formed as $\mathbf{p}_i(s, \phi) = \mathbf{p}_i(s).\mathbf{T}_s$. For helical mill's tooth, if ϕ is the rotational angle and P the pitch of the helical cutter, then the transformation matrix $\mathbf{T}_s$ is

$$\mathbf{T}_S = \begin{bmatrix} \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-P\phi}{2\pi} & 0 \end{bmatrix}; \ \frac{-\pi L}{P} \le \phi \le \frac{\pi L}{P} \qquad (1)$$

The body of the slab mill has left end surface (Σ_{50}), right end surface (Σ_{51}), bore surface (Σ_{52}) and the keyway. The body of slab mill has eight transitional surfaces in the form of chamfers. The chamfer between surface patch Σ_i and Σ_j is denoted as $\sigma_{i,j}$, where i = 50, 51 and j = 52, 53, 54, 55.

2.2 Mapping Relations for Slab Mills

The process of conversion of tool angles given in one nomenclature to other is known as mapping. When 3D rotational angles developed in this work are converted to conventional 2D nomenclatures, the mapping is called the forward mapping. On the contrary, when angles defined as per existing standards are mapped to proposed rotational angles, the process is called the inverse mapping. The angles for a slab mill are shown in Figure 2.

Radial Rake Angle (γ_R) is formed by face Σ_1 with ZX plane and viewed on projection to XY plane. To find γ_R, the normal to Σ_1 is projected on XY plane and dot product of the projected normal with the unit vector along Y axis is taken. This gives $\cos\gamma_R = \cos(\gamma_1+\phi)$. For straight tooth cutter, angle ϕ is zero, while for helical cutters, γ_R is evaluated on z=0 plane. This gives $\gamma_R = \gamma_1$

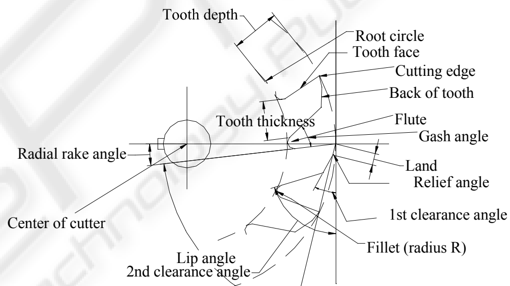

Figure 2: Conventional Tool Geometry of Slab Mill.

In the same fashion, relief angle (α_P) is formed by land Σ_2 about YZ plane when projected on XY plane and is given as $\alpha_P = \gamma_2$. Surface patch Σ_3 (Flank) of tooth forms first clearance angle (α_{1P}) with YZ plane on projection to the XY plane and the angle α_{1P} is expressed by the following relation:

$$\alpha_{1P} = \cos^{-1}\left[\frac{\left(V_{3x} - \frac{D}{2} + l_1\sin\gamma_2\right)\sin\phi + \left(V_{3y} - l_1\cos\gamma_2\right)\cos\phi}{\sqrt{\left(V_{3x} - \frac{D}{2} + l_1\sin\gamma_2\right)^2 + \left(V_{3y} - l_1\cos\gamma_2\right)^2}}\right]$$

Second Clearance Angle (α_{2P}) is formed by patch Σ_4 and can be expressed as $\alpha_{2P} = \gamma_4$. Gash angle (δ) is the angle of fillet and is $908+\gamma_1-\gamma_4+(2\pi/N)$, where N is the number of teeth of the slab mill. Lip angle is the angle formed by the tooth solid side at the cutting edge. In terms of rotational angles the lip angle is evaluated as $\upsilon_{la} = 908+\gamma_1-\gamma_2$.

3 MODELING OF END MILL

End mills are multi-point cutters with cutting edges both on the end face and the circumference (Drodza, 1983). Figure 3 shows the projected geometry of a flat end mill. The geometry of an end mill consists of geometry of fluted shank and end geometry.

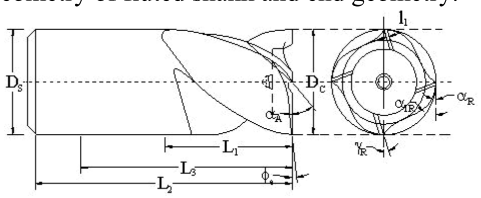

Figure 3: Two-Dimensional Geometry of End Mill.

3.1 Geometry of Fluted Shank

A single tooth of the end mill is modeled with nine surface patches, labeled Σ_1 to Σ_9 (Table 1). Surfaces Σ_1 to Σ_6 are the surfaces on the fluted shank. These surfaces are formed as helicoidal surfaces. The composite sectional curve ($P_1 \ldots P_8$) is composed of six segments (Figure 4). Three segments of the composite curve are straight lines and correspond to the three land widths, namely peripheral land, heel and face. While the other three segments are circular arcs of radii r_3, r_2 and R and correspond to fillet, back of tooth and blending surface.

Table 1: Surface Patches of End Mill.

Symbol	Surface Patch Name	Symbol	Surface Patch Name
Σ_1	Face	Σ_6	Fillet
Σ_2	Peripheral Land	Σ_7	Face Land
Σ_3	Heel	Σ_8	Minor Flank
Σ_4	Blending Surface	Σ_9	Rake Face Extension
Σ_5	Back of Tooth		

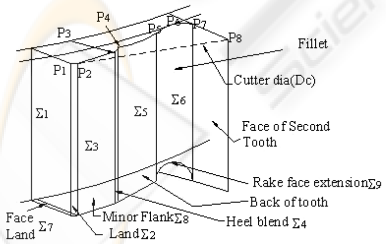

Figure 4: Modeling of an End Mill Tooth.

The helicoidal surface for fluted shank is described as $\mathbf{p}(s,\phi) = \mathbf{p}(s).\mathbf{T}_S$, where

$$\mathbf{T}_S = \begin{bmatrix} \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{P\phi}{2\pi} & 1 \end{bmatrix} \quad \text{for} \quad 0 \le \phi \le \frac{2\pi L}{P} \tag{2}$$

In the above equation, L is the length of fluted shank. The length may be equal to L_1 for flat end mills and (L_1-D_c/2) for ball end mills. Three different sweeping rules can be formulated for the fluted shank and the end profile of the cutter. These rules are for cylindrical, conical and hemispherical helical path.

3.2 Mapping Relations for End Mill

Mapping guide table (Table 2) shows the planes that forms the conventional angles. The forward mapping relations are summarized in Table 3.

Table 2: Mapping Guide Table for End Mill.

Conventional Angles	Formed by	About the Plane	Plane of Projection
γ_R	Σ_1	ZX	XY
α_R	Σ_2	YZ	XY
α_{1R}	Σ_3	YZ	XY
ϕ_e	Σ_7	XY	ZX
α_A	Σ_7	XY	YZ

Table 3: Forward Mapping Relations for End Mill.

Conventional Angles		Rotational Angles
Radial Rake Angle, $\pm\gamma_R$	=	$\mp\gamma_1$
Radial Relief Angle, α_R	=	γ_2
Radial Clearance Angle, α_{1R}	=	γ_3
Axial Relief Angle, α_A=		$\cos^{-1}\left[\frac{\cos\alpha_7}{\sqrt{\cos^2\gamma_1\sin^2\alpha_7+\cos^2\alpha_7}}\right]$
End Cutting Edge , ϕ_e =		$\cos^{-1}\left[\frac{\cos\alpha_7}{\sqrt{\sin^2\gamma_1\sin^2\alpha_7+\cos^2\alpha_7}}\right]$

4 MODELING OF TWIST DRILL

Drills are rotary cutting tools used for the generation of holes (Drodza, 1983). In this paper, modeling of a two-flute, right-cut, straight shank type of solid twist drill is presented. This is the most commonly used drill for originating holes.

Geometrically a drill is made of (i) drill body and (ii) shank. Drill body is the portion responsible for material removal and the part by which drill is held and driven in a drilling machine is shank. The drill body may be segmented into (i) flute and (ii) end geometry. The flute is the cutting portion of the drill. The end of the drill is the portion that facilitates entry of the drill into the workpiece. The conventional two-dimensional projected geometry of a twist drill is shown in Figure 5.

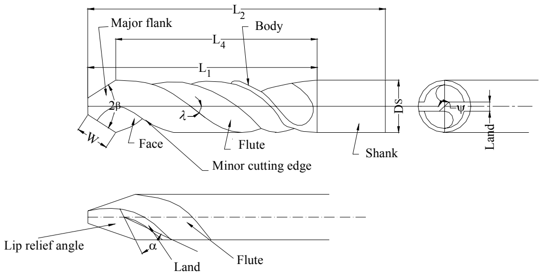

Figure 5: 2D Projected Geometry of a Twist Drill.

The sectional geometry of the fluted shank has three segments, out of which one is a straight line and forms land (Σ_2). The other two segments are circular in geometry and on sweeping form flank (Σ_3) and face (Σ_1) respectively. The drill end is made of as many surface patches as the number of flutes. For a two-flute drill, two surface patches form the drill end. They are labeled as Σ_4 and Σ_5 and known as lip relief surfaces. The lip relief surfaces can be planar, cylindrical, conical and helicoidal. For a drill, the forward mapping relations are:
Half Point Angle,

$$\beta = \cos^{-1}\left[\frac{\sin\beta_4\cos\gamma_4}{\sqrt{\sin^2\beta_4\cos^2\gamma_4+\cos^2\beta_4}}\right] \tag{3}$$

Chisel Edge Angle, $\psi = 90^\circ - \gamma_4$. (4)
Relief Angle,

$$\alpha = \cos^{-1}\left[\frac{\cos\beta_4}{\sqrt{\sin^2\beta_4\sin^2\gamma_4+\cos^2\beta_4}}\right] \tag{5}$$

Helix Angle, $\lambda = \tan^{-1}\left(\frac{P}{\pi D_c}\right)$ (6)

Peripheral Relief Angle, $\alpha_p = \gamma_2$ (7)

5 VALIDATION

This section presents an example on 3D modeling of an end mill. The parameters used to construct the model of end mill are referred in *ANSI/ASME B94.19-1985* standards. The resultant cutter is rendered (Figure 6) in OGL environment.

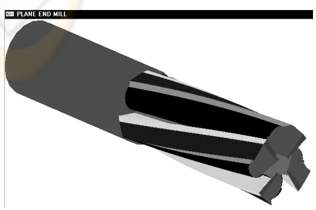

Figure 6: Rendering of an End Mill.

6 CONLUSIONS

The present work has covered the 3D modeling of the multi-point fluted cutters (slab mills, end mill and drills) by mathematically expressing the geometry of the cutting tools in terms of various biparametric surface patches. Four rotational angles (γ_1, γ_2, γ_3, γ_4) are used to define the geometry of a slab mill along with other dimensional parameters. Similarly, four rotational angles (γ_1, γ_2, γ_3, α_7) and three rotational angles (γ_2, β_4, γ_4) are defined to model an end mill and drill respectively. The mathematical definitions of the surfaces have been used to obtain the standard 2D tool angles from these proposed rotational angles. The inverse relationships to obtain the rotational angles from the conventional angles are also obtained. The entire exercise attempts to recast the method of defining a cutting tool in terms of 3D geometric models.

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