LOCAL MULTIRESOLUTION OF A MESH BASED ON $\sqrt{3}$ SUBDIVISION AND SURFACE DISCONTINUITIES

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Abstract: We build a local multiresolution of meshes when the connectivity is resulting from an enhanced $\sqrt{3}$ subdivision of a coarse mesh template. We use the concept of biorthogonality and lifting to develop a set of filters for local analysis and local synthesis. The enhanced $\sqrt{3}$ subdivision, we developed, takes into account natural surface discontinuities during the subdivision process. The multiresolution based on our enhanced $\sqrt{3}$ subdivision permits to obtain a great compression ratio.

1 INTRODUCTION

A project of surface meshing for still and animated images and associated software has been developed at L3i for several years. Our goal is to build a multiresolution analysis to compress mesh information in order to permit fast transmission of shapes on networks and to allow fast visualization via levels of details. We use wavelet theory. The wavelet functions are deduced from scaling functions based on subdivisions.

There are two main categories of subdivision schemes: the subdivision inserting vertices on edges and the subdivision inserting vertices on faces. Each one of these can use an approximation method or an interpolation method. Among all the approximation methods for subdivisions inserting vertices on edges, we can cite (Doo et al. 1978), (Catmull and Clark 1978) and (Loop 1987). Among interpolation methods for subdivisions inserting vertices on edges the method of (Halstead et al. 1993) is a modified Catmull-Clark subdivision. The most famous one is the "butterfly" method (Dyn et al. 1990), which gives a G1 continuity of the limit surface, with a minimal number of neighbors and whatever the connectivity of the vertices. A modified butterfly method which ensures a better continuity has been proposed by (Dyn et al. 1993) and (Zorin et al. 1996). Among approximation methods for subdivisions inserting vertices on faces, we can cite the $\sqrt{3}$ subdivision of Kobbelt (Kobbelt 2000). Among interpolation methods, we can cite the $\sqrt{3}$ Subdivision of Labsik and Greiner (Labsik and Greiner 2000). All these subdivision methods do not take into account the natural discontinuities of the surface.

Subdivision of meshes with natural surface discontinuities has been studied by (Hoppe et al. 1994). The study is focused on subdivision inserting vertices on edges and does not include a multiresolution analysis.

The basic principles of multiresolution analysis and wavelets were given by (Meyer 1986), (Meyer 1988) for mathematical aspects, by (Mallat 1989) for signals and images and by (Lounsbery et al. 1997) and (Schröder and Sweldens 1995) for surfaces. Schröder and Lounsbery worked on subdivision inserting vertices on edges for meshes without natural surface discontinuities.

A subdivision allows synthesizing a shape. It increases the resolution of a coarse mesh, called "control mesh", and converges on a limit surface. A multiresolution analysis permits to decrease the resolution of a fine mesh without lost of information. In the case of a subdivided mesh, the synthesis is simply the subdivision of the coarse mesh and the analysis consists in rebuilding the coarse mesh from

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the subdivided mesh. In the case of a complex mesh that is not the result of a subdivision but has the connectivity of a subdivided mesh, the analysis provides a low-resolution approximation mesh. There exists remeshing methods to obtain this kind of mesh (Eck et al. 1995), (Lee et al. 1998). In order to rebuild the original mesh by the synthesis of the low-resolution mesh, the analysis computes and store errors, called details, at every level.

The multiresolution theory assures us that the analyzed mesh is the best approximation of the original mesh at this level for the chosen dot product.

Because the subdivision tends to a smooth limit surface, the analysis of a mesh representing a smooth limit surface generates many null details. That's why the analysed version of a mesh and the non null details can be stored more efficiently than the original mesh.

In this paper we introduce a multiresolution analysis of meshes when the connectivity is resulting from an enhanced $\sqrt{3}$ subdivision taking into account discontinuities. We choose a subdivision inserting vertices on faces because the meshes are growing slower than methods by insertion of vertices on edges. We enhanced the original $\sqrt{3}$ subdivision of Labsik and Greiner to take into account the natural discontinuities of a surface, such as darts, creases and boundaries (Guillot and Gourret 2006a) (Guillot and Gourret 2006b). Having this subdivision scheme, we build a multiresolution analysis, which handles discontinuities. Moreover, due to the high amount of data in recent meshes, we develop a local analysis, i.e. the calculation should include only a part of the data at a time.

The multiresolution analysis developed in this paper works with every subdivision scheme. An analogous approach is done by (Olsen et al. 2005). Their calculation starts from a very small neighbourhood which is recursively enhanced by a method similar to the lifting scheme. So, the size of filter is growing until an optimization of the magnitude of details is reached. We build our wavelets with only one lifting scheme. Then a recursive approach constrains the number of null details. Moreover our method uses natural discontinuities to minimize magnitude of details.

In section 2, we present the principles of a global multiresolution and some definitions. In section 3, we explain how to process the local calculus of scaling and wavelet functions using biorthogonality and lifting scheme. In Section 4 we describe how to perform locally a synthesis and in section 5 we describe how to perform locally an analysis. Section 6 is dedicated to results, and section 7 is dedicated to conclusion and future works.

2 MULTIRESOLUTION ANALYSIS BASED ON SUBDIVISION

2.1 Our Enhanced $\sqrt{3}$ Subdivision

The enhanced $\sqrt{3}$ subdivision is based on the insertion of a new vertex in each triangular face. We start from a control mesh at subdivision level j=0. A vertex is inserted in each face. Then new faces are created joining the new vertices to the initial vertices and to the new vertices in immediate neighbourhood.
Doing this, a new vertex always shares 6 faces.
The mesh resulting from one subdivision of level j is called the level j+1.
A mesh M^j of level j has n_j vertices and f_j faces. We call Y^j the set of vertices in the mesh of level j and K^j its connectivity, so $M^j = (K^j, Y^j)$ represent the level j.
A direct property is that
$n_{j+1} - n_j = f_j = 3.f_{j-1} = \ldots = 3^j.f_0$
Far of natural surface discontinuities and far of extraordinary vertices (non 6 connected vertices), our enhanced $\sqrt{3}$ subdivision uses the method of Labsik-Greiner or of Kobbelt (for accuracy we introduce the name Labsik-Greiner formula and Kobbelt formula in this paper). Otherwise we developed a formulation explained in (Guillot and Gourret 2006b).

2.2 Multiresolution Analysis

Generally a mesh M is obtained from a cloud of points acquired by a 3D scanner. So the number of vertices and the connectivity are not the result of recursive subdivisions ($K \neq K^j$). Our multiresolution analysis method needs that the connectivity of the starting high level mesh is the result of a recursive subdivision ($K = K^j$). We suppose that $K = K^j$ in what follows. It means for example that most of the vertices are 6-connected and that extraordinary vertices, of connectivity different of 6, are sufficiently spaced. Note that as said in section 2.1, a new vertex always shares 6 faces: it is always of connectivity 6, so the extraordinary vertices are not introduced by the subdivision process. They are defined in the control mesh M^0 and they remain through the recursive subdivisions.

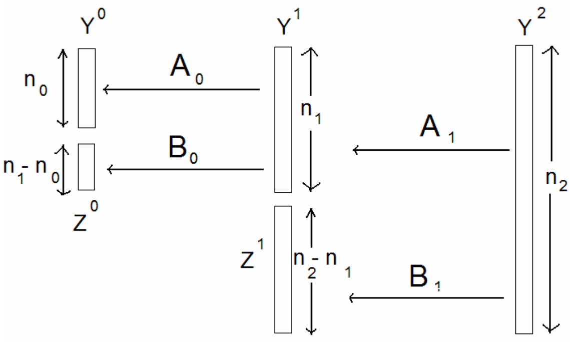

Figure 1: Analysis from level 2 to level 0.

The analysis of a level j+2, builds a level j+1 and a set of details called Z^{j+1}. Recursively the level j+1 can be analysed as a level j and a set of details Z^j.
Let's call A^j the matrix that transform Y^{j+1} into Y^j and B^j the matrix that transform Y^{j+1} into Z^j.
The detail Z^{j+1} has exactly three times more vertices than Z^j. We show in Figure 1 the analysis from level 2 to level 0.
Let's call $Y^j(n)$ the n^{th} vertex of Y^j. The details Z^j are considered as a list of virtual points (points not connected via K^j).
In what follows V_i is the i^{th} vertex of Y^j, V'_i is the i^{th} vertex of Y^{j+1} and W_i is the $(i - n_j)^{th}$ vertex of Z^j.

2.3 Multiresolution Synthesis

The synthesis is the reconstruction of the level j+1 from the level j and the details Z^j. A second synthesis rebuilds the level j+2 from the level j+1 rebuilt and from the details Z^{j+1}. Thus the level j+2 can be obtained from the level j and the details Z^j and Z^{j+1}.

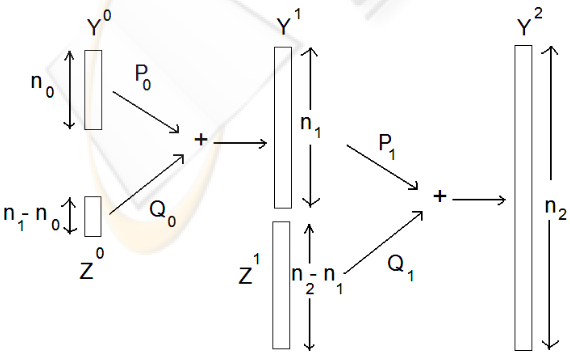

Figure 2: Synthesis from level 0 to level 2.

Let's call P^j the matrix that subdivide Y^j and call Q^j the matrix that uses the details Z^j. The action of Q^j on Z^j gives the difference between Y^{j+1} and the subdivision of Y^j. We show in Figure 2 the synthesis from level 0 to level 2.

3 LOCAL COMPUTATION OF SCALING AND WAVELET FUNCTIONS

In order to build a multiresolution, we need a scaling function φ and a wavelet function ψ. They define the global matrices P^j and Q^j as shown in Figure 3. We never compute the whole P^j or Q^j matrices, only rows or columns of these matrices.

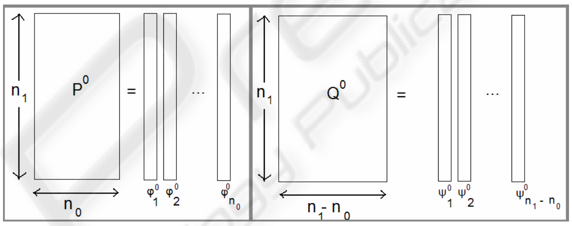

Figure 3: Definition of the global matrices P^0 and Q^0.

3.1 Local Computation of φ^j

Note that "local computation of φ^j " means that the computation itself is local, not the function φ^j. Let's V'_i be a vertex in Y^{j+1} and D'_i be the d-disk in Y^{j+1} centred on V'_i. The d-disk centered on V'_i is the set of vertices connected to V'_i by less than d edges.
In what follows, we will suppose that every calculation is local around a vertex, which means that to synthesize a vertex V'_i in Y^{j+1}, we need to consider some vertices in Y^j and some points in Z^j.
The vertices of Y^j and the points of Z^j are in
$D_i = \{V_i ; V'_i \in D'_i\} \cup \{ W_i ; W'_i \in D'_i \}$.

If i is in $[1,n_j]$, φ^j_i represents the influence of the vertex V_i in the computation of the new vertices in a one level subdivision, it is the scaling function associated with V_i. The influence of the vertex V_i is local, so we can compute it locally, in fact V_i only influences the vertices of D'_i.
The scaling function φ^j_i depends on the connectivity of the vertices around D'_i.
For example, when we use the Labsik-Greiner interpolation formula with twelve neighbours (Figure 4) the disk D'_i is shown in Figure 5. Note that because the Labsik-Greiner formula is an interpolation method only black vertices are

influenced by V_i, white vertices are not influenced by V_i.

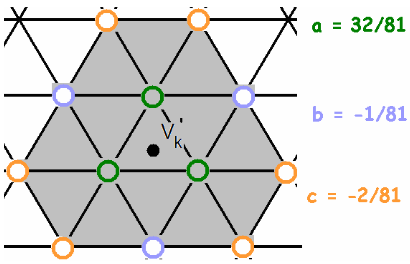

Figure 4: Stencil of Labsik-Greiner formula (the twelve circled neighbours are weighted a, b or c to calculate the black vertex V'_k).

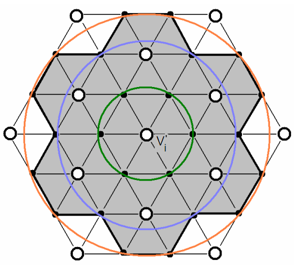

Figure 5: 3-Disk D'_i for Labsik-Greiner formula (V_i influences black vertices, with a, b, c weights).

For example, when we use Kobbelt formula (Figure 6), D'_i is the 2-disk shown in Figure 7. Note that because Kobbelt is an approximation method, black and white vertices are influenced by V_i.

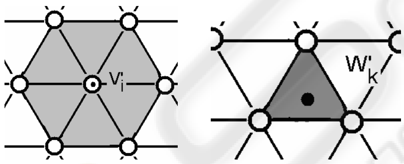

Figure 6: Stencils for kobbelt formula.

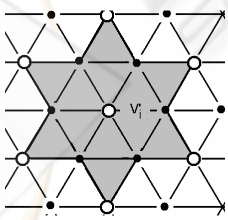

Figure 7: D'_i for the Kobbelt formula.

D'_i depends on the method of subdivision and on the proximity or not of natural discontinuities or extraordinary vertices. See (Guillot and Gourret 2006b) for more explanations on the enhanced $\sqrt{3}$ subdivision. Calculations are now implemented in our software MEFP3C (Khamlichi and Gourret 2004). Note that taking discontinuities into account enlarges the size of D'_i.

For every V'_k in Y^{j+1}, we note $\varphi^j_i(k)$ the influence of V_i on the computation of V'_k. If V'_k is not in D'_i, V_i has no influence, so $\varphi^j_i(k) = 0$. If V'_k is in D'_i, $\varphi^j_i(k)$ can be obtained by simulating the calculation of V'_k in the subdivision algorithm, we obtain the weight of V_i in the stencil around V'_k.

3.2 Local Computation of ψ^j

Note that "local computation of ψ^j " means that the computation itself is local, not the function ψ^j. For k in $[1 , n_{j+1}- n_j]$ ψ^j_k is the wavelet function associated with W_k ($W_k = Z^j (k - n_j)$).
Knowing the φ^j function, we should build the ψ^j function using the global orthogonal condition between φ^j and ψ^j :
For all i in $[1,n_j]$, for all k in $[1 , n_{j+1}- n_j]$
$< \varphi^j_i , \psi^j_k > = 0$.
But a global computation is too expensive. So we release this constraint to something local.

We use the concept of biorthogonality with lazy wavelet (Sweldens 1996). For $\psi_{lazy\ k}$ we choose the dirac δk. Let's V_a, V_b and V_c be the vertices of the face of Y^j in which we insert W'_k. Our lifting operation to enhance the orthogonality of the lazy wavelets consists in writing :
$\psi^j_k = \psi^j_{lazy\ k} + \alpha.\ \varphi^j_a + \beta.\ \varphi^j_b + \gamma.\ \varphi^j_c$, where α, β, γ are real numbers that we compute for every k and every j writing the system :

$< \varphi^j_a , \psi^j_k > = 0$
$< \varphi^j_b , \psi^j_k > = 0$
$< \varphi^j_c , \psi^j_k > = 0$

It is a 3x3 system, easily solved, done for every k.
φ^j_a is known by its values on every vertex of Y^{j+1} so it can be seen as a vector of R^{nj+1}. Let's consider ψ^j_k as a vector of R^{nj+1}. To calculate $< \varphi^j_a , \psi^j_k >$ we use the Euclidean inner product of φ^j_a and ψ^j_k as vectors of R^{nj+1}. We do not use the usual inner product first defined by Lounsbery.
Our wavelet function is the sum of three scaling functions. Thus we can compute locally ψ^j_k . An example for the regular case of the Labsik-Greiner formula is shown in Figure 8.

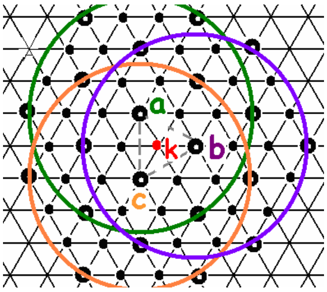

Figure 8: 4-Disk D'_k of the wavelet function ψ^j_k associated with the Labsik-Greiner formula.

4 LOCAL SYNTHESIS

The synthesis is the sum of the result of the P^j and Q^j matrices applied to Y^j. The synthesis is computed with the matrices P^j and Q^j as show in Figure 9.

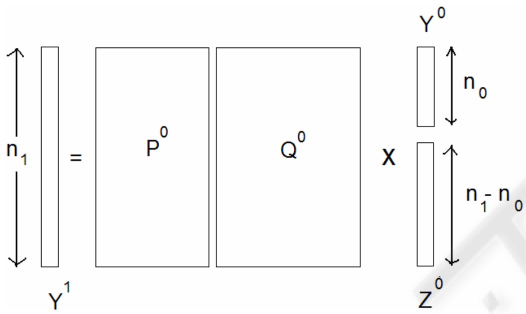

Figure 9: Synthesis from level 0 to level 1.

The action of P^j on Y^j is exactly the result of the subdivision which is a local computation, i.e. the computation of Y_i^{j+1} is local around Y_i^j.
To compute the action of Q^j on Z^j, we need the i^{th} row of Q^j. We have already assumed that this row has only non null factors in the columns representing the vertices of D'_i. Let's W_k be a point of Z^j. We know how to compute ψ_k^j where only the i^{th} coefficient interests us.
The calculation of the i^{th} row of Q^j needs just the calculation of ψ_k^j for every W_k in Z^j.

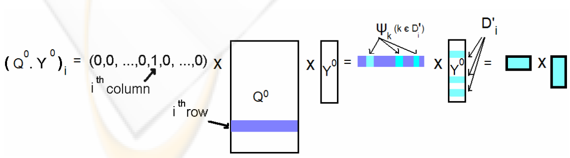

Figure 10: Computation scheme of $Y^0_i = Q^0_i.Y^0$.

5 LOCAL ANALYSIS

The analysis is globally computed with the matrices A^j and B^j as shown in Figure 11.

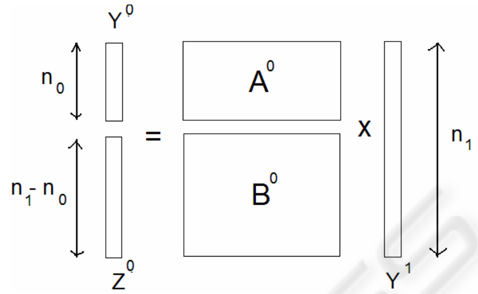

Figure 11: Analysis from level 1 to level 0.

A^j and B^j are usually computed as the inverse of the global matrix $[PQ]^j$ whose properties are shown in Figure 12.

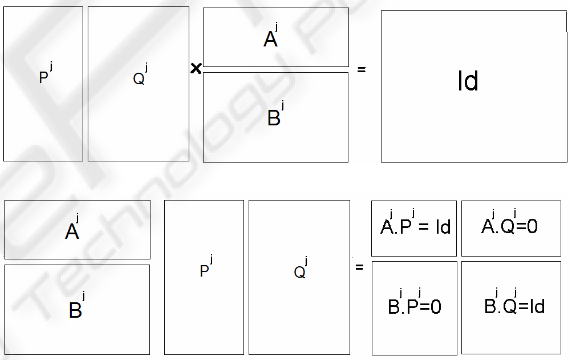

Figure 12: Global matrices.

5.1 Local Analysis: the Built of Y^j

Let's V_i be in Y^j. The i^{th} row of A^j applied on Y^{j+1} gives us V_i.
We assume that the action of A^j around V_i can be computed locally on D'_i. If A^j_{ik} is the term of A^j on the i^{th} row and the k^{th} column, we can assume that $A^j_{ik} = 0$ if $Y^{j+1}(k)$ is not in D'_i. To compute the result of the i^{th} row of A^j, we need the $(A^j)_{ik}$ for k verifying $Y^{j+1}(k)$ in D'_i. In section 3 we saw that we can compute φ_k and ψ_k for every k.
Let's A^j_i be the i^{th} row of A^j with only the coefficients corresponding to a column h that verify $Y^{j+1}(h)$ in D'_i.
Let's call $(P^j_k)_{k \in D'i}$ the matrix containing the P^j_k where k is in D'_i. It is also the matrix containing the φ_k^j when $Y^{j+1}(k)$ in D'_i, with just the rows of number h verifying $Y^{j+1}(h)$ in D'_i.

From the equation $A^j.P^j = Id$ we have just kept some rows and some columns so we can write the following equation:
$A^j_i .(P^j_k)_{k\epsilon D'i} = (0, 0, 0, \ldots, 1, 0, 0, \ldots, 0)$
The one is placed as i in D'_i.
In the same way we build $(Q^j_k)_{k\epsilon D'i}$ as the matrix containing the ψ^j_k when $Y^{j+1}(k)$ is in D'_i with just the rows of number h when $Y^{j+1}(h)$ is in D'_i. The equation $A^j.Q^j = 0$ gives
$A^j_i .(Q^j_k)_{k\epsilon D'i} = (0, \ldots, 0)$.
The matrix $[(P^j_k)_{k\epsilon D'i}, (Q^j_k)_{k\epsilon D'i}]$ is a square matrix (of size given by the number of vertices in D'_i). We compute A^j_i with the inverse of $[(P^j_k)_{k\epsilon D'i}, (Q^j_k)_{k\epsilon D'i}]$.
Eventually, $V_i = A^j_i \,.\, Y^{j+1}$.

5.2 Local Analysis: the Built of Z^j

The i[th] row of B^j applied on Y^{j+1} gives us W_{i+nj}.
Let's use again the matrices B^j_i, $(P^j_k)_{k\epsilon D'i}$ and $(Q^j_k)_{k\epsilon D'i}$, from the equations:
$B^j.P^j = 0$ and $B^j.Q^j = Id$
we can write:
$B^j_i.\ (P^j_k)_{k\epsilon D'i} = 0$ and $B^j_i.\ (Q^j_k)_{k\epsilon D'i} = (0, 0, \ldots, 1, 0, \ldots, 0)$ with the one placed as i in D'_i.
By just inverting $[(P^j_k)_{k\epsilon D'i}, (Q^j_k)_{k\epsilon D'i}]$ we obtain B^j_i and so W_i.

6 RESULTS

Examples presented are not realistic ones. There are only given to prove working of our method. We show in Figure 13, two examples. In order to get meshes with the connectivity of a subdivided mesh that are not just the result of a previous subdivision, we created meshes with discontinuities and we subdivided them 4 times with our approximation enhanced $\sqrt{3}$ subdivision scheme. Then we have implemented our multiresolution analysis from our interpolating enhanced $\sqrt{3}$ subdivision scheme.
The meshes to analyse have the connectivity of meshes subdivided four times, so we can analyse them 4 times.
We show in Figure 14 the result of four analyses.
Note that because our enhanced $\sqrt{3}$ subdivision is a subdivision inserting vertices on faces, the discontinuities do not belong to edges of the meshes of odd level 1 and 3 (Guillot and Gourret 2006b), only the even level produced by the analysis (level 0 and 2) should be visualized as shown in Figure 15.

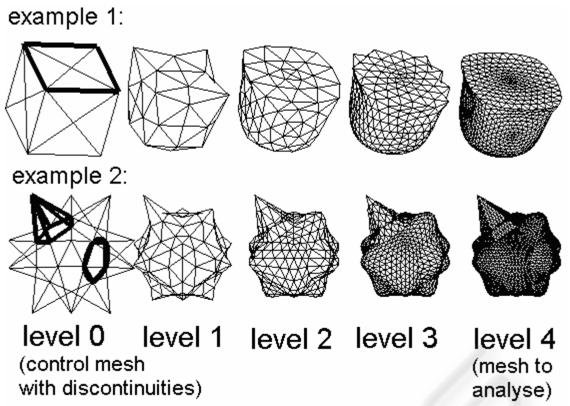

Figure 13: The construction of the meshes to analyse (construction with our enhanced $\sqrt{3}$ approximation method).

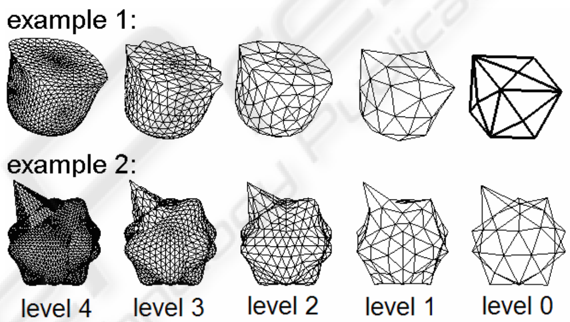

Figure 14: The result of four analyses with our enhanced $\sqrt{3}$ interpolation method.

6.1 Compression

With the mesh of level 0 and the four details Z^0, Z^1, Z^2 and Z^3, we can rebuild exactly the original mesh. The size of (Y^4) is the same as the size of $(Y^0, Z^0, Z^1, Z^2, Z^3)$. Because the subdivision tends to a smooth limit surface, the analysis of a mesh representing a smooth limit surface generates many approximately null details. Considering some of this details as null do not generate a great difference between the rebuilt mesh and the original mesh. That is why the analysed version of a mesh and the non null details can be stored more efficiently than the original mesh. We developed a constraint on the details (not explained in this paper) in order to ensure that the rebuilt mesh will not differ from the original mesh by more than a given tolerance epsilon. It means that for every vertex i, $|Y_i^4{}_{original} - Y_i^4{}_{rebuilt}| \leq$ epsilon * diameter of the bounding sphere. We show in Table 1, the percentage of details kept because they cannot be considered as null factors for

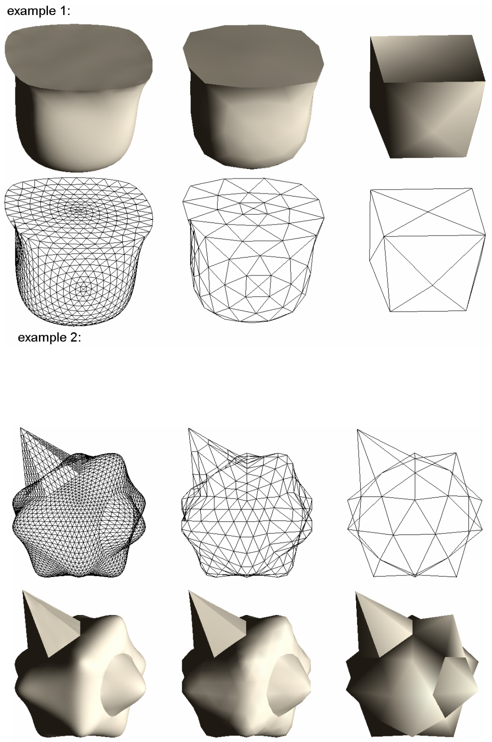

Figure 15: The meshes to analyse (level 4) and the level 2 and 0 obtained from analysis with discontinuities.

the 2 examples. The third column when the analysis takes into account natural surface discontinuities.

Our multiresolution analysis method based on the standard Labsik-Greiner formula, modified around extraordinary vertices, and without natural surface discontinuities processing permits to obtain a good compression ratio as shown in the second column of table 1.

Our multiresolution analysis method based on our enhanced $\sqrt{3}$ subdivision, with natural surface discontinuities, permits us to obtain an even better compression ratio.

We show in Figure 16 a third example, which is the second example at level 4, deformed by MEFP3C.

Table 1: Compression ratio in order to archive a precision of ε.

	ϵ	without discontinuities	with discontinuities
Example 1	1%	81%	87%
Example 2	1%	81%	88%
Example 2	1‰	59%	63%
Example 3	1%	65%	65%

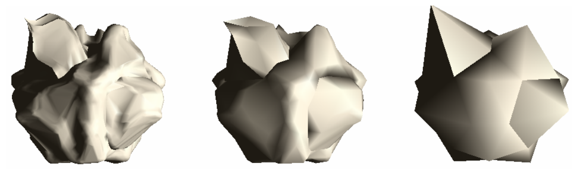

Figure 16: The mesh to analyse (level 4 of example 2 deformed) and the level 2 and 0 obtained from analysis with discontinuities.

7 CONCLUSION AND FUTURE WORK

The local multiresolution analysis of meshes presented in this paper uses our enhanced $\sqrt{3}$ subdivision. Because the calculations are local, the algorithm could be parallelized on multiprocessor computers.

Without discontinuities, the disk D'_i to synthesize a vertex V'_i is a 3-disk for the scaling function and a 4-disk for the wavelet function. Because our software deduces ψ from φ, it is only necessary to impose the disk size for φ. Without discontinuities, the disk D'_i to analyze a vertex V'_i is a 4-disk. This result is an experimental result.

The multiresolution analysis developed in this paper works with every subdivision scheme.

Our multiresolution analysis method based on our enhanced $\sqrt{3}$ subdivision which takes into account natural surface discontinuities permits us to obtain a great compression ratio.

We are presently working on boundaries that are handled by our enhanced subdivision but not yet implemented in our multiresolution analysis and we are also working on remeshing algorithms. Then we will be able to process realistic shapes such as faces and bodies.

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