FUSION PREDICTORS FOR DISCRETE-TIME LINEAR SYSTEMS WITH MULTISENSOR ENVIRONMENT

Ha Ryong Song and Vladimir Shin
Department of Mechatronics, Gwangju Institute of Science and Technology
1 Oryong-Dong, Buk-Gu, Gwangju, 500-712, South Korea

Keywords: Discrete-time linear system, Kalman predictor, fusion formula, multisensor.

Abstract: New fusion predictors for linear dynamic systems with different types of observations are proposed. The fusion predictors are formed by summing of the local Kalman filters/predictors with matrix weights depending only on time instants. The relationship between them and the optimal predictor is discussed. High accuracy and computational efficiency of the fusion predictors are demonstrated on the first-order Markov process and the damper harmonic oscillator motion with multisensor environment.

1 INTRODUCTION

The integration and fusion of information from a combination of different types of observed instruments (sensors) are often used in the design of high-accuracy control systems. Typical applications that can benefit, the use of multiple sensors, are industrial tasks, military command, mobile robot navigation, multi-target tracking, and aircraft navigation (see Hall, 1992; Bar-Shalom and Li, 1995). If it is decided that all local sensors observe the same target, then the next problem is how to combine the correspondence local estimates.

Several distributed fusion architectures were discussed in Bar-Shalom (1990) and Bar-Shalom and Campo (1986) and Li et al. (2004) and algorithms for distributed estimation fusion have been developed in Bar-Shalom and Campo (1986) and Shin et al. (2004, 2006) and Zhou et al. (2006). The Bar-Shalom and Campo fusion formula (FF) for two-sensors systems has been generalized for an arbitrary number of sensors in Shin et al. (2004, 2006). FF represents an optimal mean-square linear combination of the local estimates with the matrix weights satisfying the linear algebraic equations. The explicit expression for the matrix weights has been derived in Zhou et al. (2006). Application of FF to some estimation and filtering problems was proposed in Bar-Shalom and Campo (1986), Li et al. (2004), and Shin et al. (2004, 2006). The main purpose of this paper is development of fusion predictors to forecast the future state of the linear multisensor systems.

This paper is organized as follows. In Section 2, we present the statement of the prediction problem with multisensor environment and give its optimal solution. In Section 3, we propose two fusion predictors, which are derived by using the FF. In Section 4, the fusion predictors are tested and compared. Finally, Section 5 is the conclusion.

2 STATEMENT OF PROBLEM KALMAN PREDICTOR

Consider a discrete-time linear dynamic system with additive white Gaussian noise,

$$x_{k+1} = F_k x_k + G_k v_k, \quad k = 0,1,\dots , \tag{1}$$

where $x_k \in \Re^n$ is state vector, and $v_k \in \Re^r$ is white Gaussian noise, $v_k \sim N(0, Q_k)$.

Suppose that *overall* observation vector $Y_k \in \Re^m$ is composed of N different types of observation subvectors (local sensors) $y_k^{(1)}, \dots, y_k^{(N)}$,

$$Y_k = \left[y_k^{(1)^T} \ \cdots \ y_k^{(N)^T} \right]^T , \tag{2}$$

where $y_k^{(i)}$ are determined by the equations

$$\begin{aligned} y_k^{(1)} &= H_k^{(1)} x_k + w_k^{(1)}, \quad y_k^{(1)} \in \Re^{m_1}, \\ &\vdots \\ y_k^{(N)} &= H_k^{(N)} x_k + w_k^{(N)}, \quad y_k^{(N)} \in \Re^{m_N}, \end{aligned} \tag{3}$$

Ryong Song H. and Shin V. (2007).
FUSION PREDICTORS FOR DISCRETE-TIME LINEAR SYSTEMS WITH MULTISENSOR ENVIRONMENT.
In *Proceedings of the Second International Conference on Signal Processing and Multimedia Applications*, pages 119-124
DOI: 10.5220/0002131201190124

where $w_k^{(1)},\ldots,w_k^{(N)}$ are white Gaussian noises, $w_k^{(i)} \sim N(0, R_k^{(i)})$, $m_1+\cdots+m_N=m$. The initial state is modeled as a Gaussian random vector, $x_0 \sim N(\bar{x}_0, P_0)$. The system and observation noises v_k and $w_k^{(i)}$, $i=1,\ldots,N$, and the initial state x_0 are mutually uncorrelated.

Prediction (or ***fixed-lead prediction***) is the estimation of the state at future time $k+s$, $s \geq 0$ beyond the observation interval, that is, based on data up to an earlier time k,

$$\hat{x}_{k+s|k} = E(x_{k+s}|Y_{[0,k]}), \quad Y_{[0,k]} = \{Y_i, \ i=0,\ldots,k\}. \tag{4}$$

The Kalman predictor (KP). The optimal predictor $\hat{x}_{k+s|k} \equiv \hat{x}_{k+s|k}^{KP}$ and its error covariance $P_{k+s|k} \equiv P_{k+s|k}^{KP}$ are given by the Kalman predictor equations:

$$\begin{aligned} &\hat{x}_{k+s|k}^{KP} = F_{k+s-1}\hat{x}_{k+s-1|k}^{KP}, \\ &P_{k+s|k}^{KP} = F_{k+s-1}P_{k+s-1|k}^{KP}F_{k+s-1}^T + \tilde{Q}_{k+s-1}, \\ &\tilde{Q}_{k+s-1} = G_{k+s-1}Q_{k+s-1}G_{k+s-1}^T, \quad s=1,2,\ldots, \end{aligned} \tag{5}$$

with initial conditions $\hat{x}_{k|k}^{KP}, P_{k|k}^{KP}$ determined by the standard Kalman filter (KF) equations(Bar-Shalom et al. 2001, Lewis 1981). Note that the optimal predictor $\hat{x}_{k+s|k}^{KP}$ represents the centralized predictor, which processing the *overall* observations $Y_{[0,k]}$ simultaneously.

Many advanced systems now make use of a large number of sensors in practical applications ranging from aerospace and defense, robotics automation systems, to the monitoring and control of process generation plants. Recent developments in integrated sensor network systems have further motivated the search for decentralized signal processing algorithms. An important practical problem in the above systems is to find a fusion estimate to combine the information from various local estimates to produce a global (fusion) estimate.

In next Section, we propose two new fusion predictors for multisensor discrete-time dynamic systems (1), (3).

3 TWO FUSION PREDICTORS

The derivation of the fusion predictors is based on the assumption that the *overall* observation vector Y_k combines the local (individual) sensors $y_k^{(1)},\ldots,y_k^{(N)}$, which can be processed separately. According to (1) and (3), we have N unconnected dynamic subsystems ($i=1,\ldots,N$) with the state x_k and local sensor $y_k^{(i)}$:

$$\begin{aligned} &x_{k+1} = F_k x_k + G_k v_k, \\ &y_k^{(i)} = H_k^{(i)} x_k + w_k^{(i)}, \end{aligned} \tag{6}$$

where i is the fixed-number of subsystem. Then the optimal mean-square local filtering $\hat{x}_{k|k}^{(i)} = E(x_k|y_{[0,k]}^{(i)})$ and prediction $\hat{x}_{k+s|k}^{(i)} = E(x_{k+s}|y_{[0,k]}^{(i)})$ estimates are determined by the recursive Kalman filtering equations, $y_{[0,k]}^{(i)} = \{y_j^{(i)}, \ j=0,\ldots,k\}$ (Bar-Shalom et al. 2001). We have

$$\begin{aligned} &\hat{x}_{k+s|k}^{(i)} = F_{k+s-1}\hat{x}_{k+s-1|k}^{(i)}, \quad s=1,2,\ldots, \\ &P_{k+s|k}^{(ii)} = F_{k+s-1}P_{k+s-1|k}^{(ii)}F_{k+s-1}^T + \tilde{Q}_{k+s-1}, \end{aligned} \tag{7}$$

$$\begin{aligned} &\hat{x}_{k+1|k} = F_k\hat{x}_{k|k}, \quad \hat{x}_{0|0} = \bar{x}_0, \\ &\hat{x}_{k+1|k+1}^{(i)} = \hat{x}_{k+1|k}^{(i)} + L_{k+1}^{(i)}\left[y_k^{(i)} - H_k^{(i)}\hat{x}_{k+1|k}^{(i)}\right], \\ &P_{k+1|k}^{(ii)} = F_k P_{k|k}^{(ii)} F_k^T + \tilde{Q}_k, \quad P_{0|0}^{(ii)} = P_0, \\ &L_{k+1}^{(i)} = P_{k+1|k}^{(ii)} H_{k+1}^{(i)^T}\left[H_{k+1}^{(i)^T} P_{k+1|k}^{(ii)} H_{k+1}^{(i)} + R_{k+1}^{(i)}\right]^{-1}, \\ &P_{k+1|k+1}^{(ii)} = \left[I_n - L_{k+1}^{(i)} H_{k+1}^{(i)}\right] P_{k+1|k}^{(ii)}, \end{aligned} \tag{8}$$

where I_n is an $n \times n$ identity matrix, and $P_{k|k}^{(ii)}$ and $P_{k+s|k}^{(ii)}$ are the filtering and prediction local error covariances, respectively, i.e.,

$$\begin{aligned} &P_{k+j|k}^{(ii)} = \mathrm{cov}(\tilde{x}_{k+j|k}^{(i)}, \tilde{x}_{k+j|k}^{(i)}), \\ &\tilde{x}_{k+j|k}^{(i)} = x_{k+j} - \hat{x}_{k+j|k}^{(i)}, \quad j=0,1,2,\ldots . \end{aligned} \tag{9}$$

Thus we have N local Kalman estimates $\hat{x}_{k|k}^{(i)}$, $\hat{x}_{k+s|k}^{(i)}$, and the corresponding error covariances $P_{k|k}^{(i)}$, $P_{k+s|k}^{(i)}$ for $i=1,\ldots,N$. Using these local estimates and covariances we propose two fusion prediction algorithms.

3.1 The Fusion of Local Predictors (FLP Algorithm)

The fusion predictor $\hat{x}_{k+s|k}^{FLP}$ of the state x_{k+s} based on the overall sensors (2) is constructed from the local predictors $\hat{x}_{k+s|k}^{(i)}$ by using FF (Shin et al. 2004, 2006 and Zhou et al. 2006):

$$\hat{x}_{k+s|k}^{FLP} = \sum_{i=1}^{N} a_{k,s}^{(i)}\hat{x}_{k+s|k}^{(i)}, \qquad \sum_{i=1}^{N} a_{k,s}^{(i)} = I_n, \tag{10}$$

where $a_{k,s}^{(1)},\ldots,a_{k,s}^{(N)}$ are $n \times n$ time-varying matrix weights determined from the mean-square criterion,

$$J^{FLP}_{k+s|k} = E\left(\left\| x_{k+s} - \sum_{i=1}^{N} a^{(i)}_{k,s}\hat{x}^{(i)}_{k+s|k}\right\|^2\right) \to \min_{a^{(i)}_{k,s}}. \tag{11}$$

The following Theorem and Corollary completely define the fusion predictor $\hat{x}^{FLP}_{k+s|k}$ and its error covariance, $P^{FLP}_{k+s|k} = \mathrm{cov}\left(\tilde{x}^{FLP}_{k+s|k}, \tilde{x}^{FLP}_{k+s|k}\right)$.

Theorem: *Let $\hat{x}^{(1)}_{k+s|k},\ldots,\hat{x}^{(N)}_{k+s|k}$ are the local Kalman predictors of an unknown state x_{k+s}. Then*

(a) The weights $a^{(1)}_{k,s},\ldots,a^{(N)}_{k,s}$ satisfy the linear algebraic equations

$$\sum_{i=1}^{N} a^{(i)}_{k,s}\left[P^{(ij)}_{k+s|k} - P^{(iN)}_{k+s|k}\right] = 0, \quad \sum_{i=1}^{N} a^{(i)}_{k,s} = I_n, \tag{12}$$

and they can be explicitly written out in the following form

$$a^{(i)}_{k,s} = \sum_{j=1}^{N} P^{(ij)\,(-1)}_{k+s|k}\left(\sum_{\ell,h=1}^{N}\left(P^{(\ell h)\,(-1)}_{k+s|k}\right)\right)^{-1}, \quad i=1,\ldots,N; \tag{13}$$

(b) The local cross-covariances

$$P^{(ij)}_{k+s|k} = \mathrm{cov}\left(\tilde{x}^{(i)}_{k+s|k}, \tilde{x}^{(j)}_{k+s|k}\right), \quad i,j=1,\ldots,N; \quad i \neq j \tag{14}$$

satisfy the following recursions:

$$P^{(ij)}_{k+s|k} = F_{k+s-1}P^{(ij)}_{k+s-1|k}F^{T}_{k+s-1} + \tilde{Q}_{k+s-1}, \quad s=1,2,\ldots, \tag{15}$$

$$\begin{aligned} P^{(ij)}_{k|k} &= \left(I_n - L^{(i)}_k H^{(i)}_k\right)\cdot\left(F_{k-1}P^{(ij)}_{k-1|k-1}F^{T}_{k-1} + \tilde{Q}_{k-1}\right) \\ &\times\left(I_n - L^{(j)}_k H^{(j)}_k\right)^{T}, \quad P^{(ij)}_{0|0} = P_0 \end{aligned} \tag{16}$$

with the gains $L^{(i)}_k$ determined by (8).

(c) The fusion error covariance $P^{FLP}_{k+s|k}$ is given by

$$P^{FLP}_{k+s|k} = \sum_{i,j=1}^{N} a^{(i)}_{k,s}P^{(ij)}_{k+s|k}a^{(j)^{T}}_{k,s}. \tag{17}$$

Corollary: *If $\hat{x}^{(1)}_{k+s|k},\ldots,\hat{x}^{(N)}_{k+s|k}$ are unbiased local Kalman estimates then the fusion predictor $\hat{x}^{FLP}_{k+s|k}$ in (10) is unbiased.*

The proofs of Theorem and Corollary are given in Appendix.

Thus the local Kalman filtering estimates (8), and the recursive fusion equations (10)-(17) completely define FLP algorithm.

In particular case at $N=2$, FF (10)-(13) reduces to the Bar-Shalom and Campo formula:

$$\begin{aligned} \hat{x}^{FLP}_{k+s|k} &= a^{(1)}_{k,s}\hat{x}^{(1)}_{k+s|k} + a^{(2)}_{k,s}\hat{x}^{(2)}_{k+s|k}, \\ a^{(1)}_{k,s} &= \left(P^{(22)}_{k+s|k} - P^{(21)}_{k+s|k}\right)D^{-1}_{k,s}, \\ a^{(2)}_{k,s} &= \left(P^{(11)}_{k+s|k} - P^{(12)}_{k+s|k}\right)D^{-1}_{k,s}, \\ D_{k,s} &= P^{(11)}_{k+s|k} + P^{(22)}_{k+s|k} - P^{(12)}_{k+s|k} - P^{(21)}_{k+s|k}. \end{aligned} \tag{18}$$

Further, in parallel with FLP we offer the other algorithm for fusion prediction.

3.2 The Prediction of Fusion Filter (PFF Algorithm)

This algorithm consists of two parts. The first part fuses the local Kalman filtering estimates $\hat{x}^{(1)}_{k|k},\ldots,\hat{x}^{(N)}_{k|k}$ using FF. We have

$$\hat{x}^{FF}_{k|k} = \sum_{i=1}^{N} b^{(i)}_k\hat{x}^{(i)}_{k|k}, \quad \sum_{i=1}^{N} b^{(i)}_k = I_n, \tag{19}$$

where the weights $b^{(1)}_k,\ldots,b^{(N)}_k$ satisfy the linear algebraic equations (Shin et al. 2006)

$$\sum_{i=1}^{N} b^{(i)}_k\left[P^{(ij)}_{k|k} - P^{(iN)}_{k|k}\right] = 0, \quad \sum_{i=1}^{N} b^{(i)}_k = I_n, \tag{20}$$

or explicitly expressed from the formula (Zhou et al. 2006):

$$b^{(i)}_k = \sum_{j=1}^{N} P^{(ij)(-1)}_{k|k}\left(\sum_{\ell,h=1}^{N}\left(P^{(\ell h)(-1)}_{k|k}\right)\right)^{-1}, \quad i=1,\ldots,N; \tag{21}$$

where the local cross-covariances $P^{(ij)}_{k|k}$ are determined by (8) and (16).

In the second part we predict the fusion filtering estimate $\hat{x}^{FF}_{k|k}$ using one-step prediction:

$$\begin{aligned} \hat{x}^{PFF}_{k+s|k} &= F_{k+s-1}\hat{x}^{PFF}_{k+s-1|k} = F_{k+s-1}F_{k+s-2}\hat{x}^{PFF}_{k+s-2|k} = \cdots \\ &= A_{k,s}\hat{x}^{FF}_{k|k}, \quad A_{k,s} = \prod_{j=0}^{s-1} F_{k+s-1-j}, \quad \hat{x}^{PFF}_{k|k} \equiv \hat{x}^{FF}_{k|k}. \end{aligned} \tag{22}$$

Remark 1 *(Estimation accuracy)*: Experimentally, FLP and PFF have very close accuracy, as in Section 4. Unfortunately, now we do not have a rigorous proof or disproof of this result.

Remark 2 *(Computational complexity)*: In general, the both results, namely, linear equations (12), (20) and expressions (13),(21) are equivalent, being the implicit and explicit forms of the solution, respectively. However, from the computational point of view, when the number of sensors N is large or the local cross-covariance matrices $P^{(ij)}_{k+s|k}$ are ill-conditioned, the linear equations may be more preferable than the explicit expressions.

To predict the state x_{k+s} using the FLP we need to compute the matrix weights $a^{(1)}_{k,j},\ldots,a^{(N)}_{k,j}$ for each lead $j=1,2,...,s$ in contrast to the PFF, wherein the weights $b_k^{(1)},...,b_k^{(N)}$ are computed only once, since they do not depend on the leads $s \geq 1$. Therefore the FLP is more complex than the PFF, especially for the large leads $s >> 1$.

Remark 3 *(Real-time implementation)*: We may note, that the local filter gains $L_k^{(i)}$, the error cross-covariances $P_{k|k}^{(ij)}$, $P_{k+s|k}^{(ij)}$, and the weights $a_{k,s}^{(i)}, b_k^{(i)}$ may be pre-computed, since they do not depend on the current observations Y_k. But only on the noises statistics Q_k and $R_k^{(i)}$, and the system matrices $F_k, G_k, H_k^{(i)}$, which are part of the system model (1), (3). Thus, once the observation schedule has been settled, the real-time implementation of the fusion predictors FLP and PFF requires only the computation of the local Kalman estimates $\hat{x}_{k|k}^{(1)},\ldots,\hat{x}_{k|k}^{(N)}$ and final fusion predictors $\hat{x}_{k+s|k}^{FLP}$ and $\hat{x}_{k+s|k}^{PFF}$.

Remark 4 *(Parallel implementation)*: The local Kalman estimates $\hat{x}_{k|k}^{(1)},\ldots,\hat{x}_{k|k}^{(N)}$ are separated for different sensors. Therefore, they can be implemented in parallel for various types of observations $y_k^{(i)}$, $i=1,...,N$.

4 EXAMPLES

4.1 Prediction for a Scalar Multi-sensor System

Consider a scalar system described by

$$x_{k+1} = ax_k + v_k, \quad k = 0,1,\ldots,k_T, \tag{23}$$

$$y_k^{(i)} = x_k + w_k^{(i)}, \quad i = 1,2,\ldots,N, \tag{24}$$

where $v_k \sim N(0,q)$, $w_k^{(i)} \sim N(0,r_i)$, $x_0 \sim N(\bar{x}_0,\sigma_0^2)$. This represents the model which takes N sensor modes. The parameters are subject to $a=0.9$, $q=0.2$, $k_T=20$, $\bar{x}_0=0.5$, $\sigma_0^2=1$, $N=4$. The optimal Kalman predictor, and two fusion predictors FLP and PFF were used to estimate x_{k+s}, $s \geq 1$. The noise statistics were taken as follows: $r_1=2.0$, $r_2=1.8$, $r_3=1.5$, $r_4=0.5$.

Table 1: Comparison of MSEs at N=3,4.

N=3	Type of Fusion Predictors		
k	KP	FLP	PFF
0	1.00000	1.00000	1.00000
1	1.04623	1.04602	1.04623
2	0.96947	0.98315	0.98314
3	0.95727	0.96658	0.96657
4	0.95417	0.96131	0.96131
5	0.95330	0.95962	0.95962
⋮	⋮	⋮	⋮
10	0.95295	0.95918	0.95918
N=4	Type of Fusion Predictors		
k	KP	FLP	PFF
0	1.00000	1.00000	1.00000
1	1.04623	1.04602	1.04623
2	0.95045	0.96037	0.96050
3	0.94361	0.94967	0.94966
4	0.94257	0.94753	0.94753
5	0.94239	0.94718	0.94718
⋮	⋮	⋮	⋮
10	0.94235	0.94735	0.94735

Table 1 illustrates the mean-square errors (MSEs) $P_{k+s|k}^{KP}$, $P_{k+s|k}^{FLP}$, and $P_{k+s|k}^{PFF}$ at the lead $s=10$ and $N=3,4$. Note that the MSEs $P_{k+s|k}^{FLP}$ and $P_{k+s|k}^{PFF}$ are very close and reduced from $N=3$ to $N=4$. Moreover, the differences between the optimal KP and fusion predictors are negligible, especially for steady-state regime at $k \geq 10$. The numerical simulations were performed using a computer with the following specification: Intel® Pentium® 4 CPU 3.0GHz 1G RAM. The CPU time for evaluation of the estimate $\hat{x}_{k+s|k}^{PFF}$ is 4.9 times less than for $\hat{x}_{k+s|k}^{FLP}$. This is due to the fact that the PFF's weights $b_k^{(i)}$ do not depend on leads $s=1,...,10$ in contrast to the FLP's weights $a_{k,s}^{(i)}$.

4.2 The Damper Harmonic Oscillator Motion

System model of the harmonic oscillator is considered in Lewis (1986):

$$\dot{x}_t = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\alpha \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t, \quad 0 \leq t \leq 4, \tag{25}$$

where $x_t = [x_{1,t} \quad x_{2,t}]^T$, and $x_{1,t}$ is position, and $x_{2,t}$ is velocity, v_t is zero-mean white Gaussian noise with intensity q, $E(v_t v_s) = q\delta(t-s)$, $x_0 \sim N(\bar{x}_0, P_0)$.

Assume that the observation system contains two sensors which are observing the position $x_{1,t}$. Then we have

$$y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} x_t + \begin{bmatrix} w_t^{(1)} \\ w_t^{(2)} \end{bmatrix}, \tag{26}$$

where $w_t^{(1)}$ and $w_t^{(2)}$ are uncorrelated white Gaussian noises with zero-mean and intensities r_1 and r_2, respectively.

After discretization of the system and observation models (25), (26) three predictors were applied: KP, FLP and PFF. The performance of the fusion predictors was expressed in the terms of computation load (CPU time) and loss in estimation accuracy (MSE) with respect to the optimal KP. The model parameters, noises statistics, initial conditions, and lead were taken to

$$\omega_n^2 = 0.64, \quad \alpha = 0.16, \quad q = 1, \quad r_1 = 2, \quad r_2 = 1,$$
$$\bar{x}_0 = [0.0 \quad 0.0]^T, \quad P_0 = \mathrm{diag}[0.2 \quad 0.1], \quad s = 10.$$

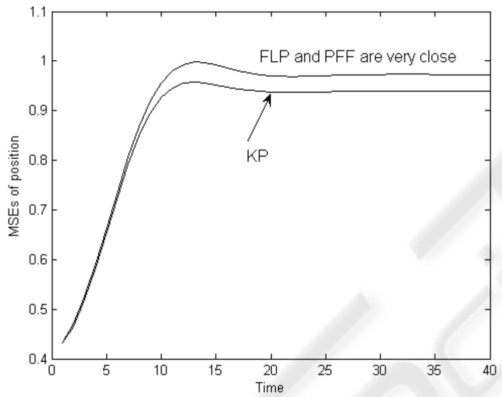

Figure 1: KP, FLP and PFF MSEs for position.

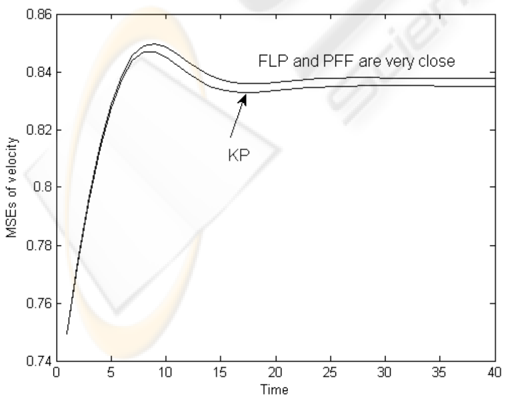

Figure 2: KP, FLP and PFF MSEs for velocity.

In Figs.1 and 2 we show the MSEs for position (x_1), $P_1^{KP} = P_{1,k+s|k}^{KP}$, $P_1^{FLP} = P_{1,k+s|k}^{FLP}$, $P_1^{PFF} = P_{1,k+s|k}^{PFF}$, and analogously for velocity (x_2) P_2^{KP}, P_2^{FLP}, P_2^{PFF}, respectively.

The analysis of results in Figs. 1 and 2 shows that the fusion predictors FLP and PFF have very close accuracy, i.e., $P_i^{FLP} \approx P_i^{PFF}$, $i = 1,2$. Moreover, the differences between both fusion MSEs P_i^{FLP}, P_i^{PFF}, and optimal one P_i^{KP} are negligible, especially for steady-state regime. The CPU times for KP, FLP, and FPP are equaled to

$$T^{KP} = 0.015(s), \quad T^{FLP} = 0.094(s), \quad T^{PFF} = 0.016(s),$$

respectively. Thus these combined effects provide the best balance between the computational efficiency and desired prediction accuracy for the PFF.

5 CONCLUSION

In this paper, we present two fusion predictors (FLP and PFF) for discrete-time linear systems with multisensor environment. Both of these predictors represent the optimal linear combination of an arbitrary number of local Kalman filters or predictors. Each local filter (predictor) is fused by the MMSE criterion. Experimentally the FLP and PFF algorithms have very close accuracy. In view of the computational complexity, however, the PFF more efficient than the FLP. The examples demonstrate the efficiency and high-accuracy of the proposed predictors.

REFERENCES

Bar-Shalom, Y., 1990. *Multitarget-Multisensor Tracking: Advanced Applications*, Artech House, Norwood, MA.

Bar-Shalom, Y., Campo, L., 1986. The effect of the common process noise on the two-sensor used track covariance, *IEEE Trans. Aerospace and Electronic Systems*, Vol. 22, No. 11, pp. 803-805.

Bar-Shalom, Y., Li, X.R., Kirubarajan, T., 2001. *Estimation with Applications to Tracking and Navigation*, John Wiley & Sons, New York.

Bar-Shalom, Y., Li, X. Rong, 1995. *Multitarget-Multisensor Tracking: Principles and Techniques*, YBS Publishing.

Hall, D. L., 1992. *Mathematical Techniques in Multisensor Data Fusion,* Artech House, London.

Lewis, F.L., 1986. *Optimal Estimation with an Introduction to Stochastic Control Theory*, John Wiley & Sons, New York.
Li, X.R., Zhu, Y.M., Wang, J., Han, C.Z., 2004. Optimal linear estimation fusion – Part I: Unified fusion rules, *IEEE Trans. Inf. Theory*, vol. 49, no. 9, pp. 2192-2208.
Shin, V.I., Lee, Y., Choi, T., 2004. Suboptimal Linear Filtering and Generalized Millman's Formula, *Proc. IASTED Inter. Conf. Signal and Image Process*, Honolulu, Hawaii, USA, pp. 369-374.
Shin, V., Lee, Y., Choi, T.-S., 2006. Generalized Millman's formula and its applications for estimation problems, *Signal Processing*, vol.86, No.2, pp. 257-266.
Zhou, J., Zhu, Y., You, Z., Song, E., 2006. An efficient algorithm for optimal linear estimation fusion in distributed multisensor systems, *IEEE Trans. Syst., Man, Cybern.*, vol.36, no.5, pp. 1000-1009.

APPENDIX

Proof of Theorem and Corollary:

(a) Equations (12) and expression (13) immediately follow as a result of application of the general FF (Shin et al. 2006 and Zhou et al. 2006) to the optimization problem (11).

(b) Equation for the local error takes the form

$$\tilde{x}_{k+s|k}^{(i)} = x_{k+s} - \hat{x}_{k+s|k}^{(i)} = F_{k+s-1}\tilde{x}_{k+s-1|k}^{(i)} + G_{k+s-1}v_{k+s-1}.$$

Then equation for the cross-covariance (15) associated with the $\tilde{x}_{k+s|k}^{(i)}$ and $\tilde{x}_{k+s|k}^{(j)}$ follows from the standard propagation equation for $P_{k+s|k}^{(ij)} = E\left(\tilde{x}_{k+s|k}^{(i)}\tilde{x}_{k+s|k}^{(j)^T}\right)$.

Equation (16) was given in Shin et al. (2006).

(c) Using (10) the fusion error covariance can be rewritten as

$$\begin{aligned} P_{k+s|k}^{FLP} &= E\left((x_{k+s} - \hat{x}_{k+s|k}^{FLP})\cdot(x_{k+s} - \hat{x}_{k+s|k}^{FLP})^T\right) \\ &= E\left\{\left[x_{k+s} - \sum_{i=1}^{N} a_{k,s}^{(i)}\hat{x}_{k+s|k}^{(i)}\right]\cdot\left[x_{k+s} - \sum_{j=1}^{N} a_{k,s}^{(j)}\hat{x}_{k+s|k}^{(j)}\right]^T\right\} \\ &= E\left\{\sum_{i=1}^{N} a_{k,s}^{(i)}\tilde{x}_{k+s|k}^{(i)}\sum_{j=1}^{N}\left(a_{k,s}^{(j)}\tilde{x}_{k+s|k}^{(j)}\right)^T\right\} = \sum_{i,j=1}^{N} a_{k,s}^{(i)}P_{k+s|k}^{(ij)}a_{k,s}^{(j)^T}. \end{aligned}$$

This completes the proof of Theorem.

If the local predictors $\hat{x}_{k+s|k}^{(i)}$ are unbiased, i.e., $E\left(\hat{x}_{k+s|k}^{(i)}\right) = E(x_{k+s})$, then we have

$$E\left(\hat{x}_{k+s|k}^{FLP}\right) = \sum_{i=1}^{N} a_{k,s}^{(i)}E\left(\hat{x}_{k+s|k}^{(i)}\right) = \left[\sum_{i=1}^{N} a_{k,s}^{(i)}\right]E(x_{k+s}) = E(x_{k+s}).$$

Corollary is proved.