APPLICATION OF CAUCHY INTEGRAL TO APPROXIMATE THE FIELD COMPONENTS AND CURRENT IN METAL & DIELECTRICAL POLYGONS

Method of the Field (Current) Restoration Inside and Outside Flat Closed Contour based on Its Known Values at the Contour

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Abstract: The Cauchy integral was proposed as a means to approximate the field components and current in metal and dielectrical polygons. The report illustrates that such a technique can significantly scale down electromagnetic issues solved in spatial- frequency domain. The technique also makes it possible to evaluate the strength of electromagnetic field not only inside the considered closed contour, but outside it even in case when there is no a prior information about object configuration and its physical characteristics.

1 INTRODUCTION

The purpose of this work is to illustrate applicability of Cauchy integrals to description of electromagnetic (EM) field or current in some electromagnetic objects whose surface allows a piecewise planar approximation. By example of dielectrical and metal cubes, it was confirmed that such a description could help restore the field (current) values within an object surface (or object section) for a wide frequency band. These simple electromagnetic objects were selected because their main properties had been studied very thoroughly by a lot of scientists - both through numerical computation of electrodynamics boundary values and through real experiments as in (Mittra, 1977). The Cauchy integral was then proposed as a means to scale down the systems of linear algebraic equations derived from electromagnetic vector boundary equations and drawn in terms of space and frequency. The report illustrates that such a technique makes it possible to evaluate the strength of electromagnetic field not only inside the considered closed contour, but outside it even in case when there is no a prior information about object configuration and its physical characteristics. It is proved that proposed technique can be used for reducing systematic errors in measurement of emitter's angular coordinates by means of mobile direction finders and increasing DF resolution and accuracy.

There are a lot of highly effective methods for electromagnetic field approximation (both for entire electrodynamic objects and for their components, e.g. in their finite elements) including the cases when the object dimensions exceed the free-space wavelength. Thus, to describe the field behavior via modified finite elements method, we suggest using Lagrange interpolation polynomials in this work (Milan, Branislav, 2006). The finite elements approach proposed by the authors (Milan, Branislav, 2006) allows to significantly reduce the number of computations to describe the field, as compared with regular polynomials. However, if you do not have a priori information about the object geometry and materials, it cannot restore the structure of the EM field inside the object and outside of it. The authors of this report suppose that the above problem can be solved if the field/current component is taken as an analytical complex variable function with a Cauchy integral and this work is just an approach to that solution.

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2 THEORY

For practical purposes, it is advisable to treat the analyzed surface S as a total of plane polygons of different size. Suppose that there is a plane polygon over a closed contour L.

Then, the Cauchy integral can be evaluated through the known discrete values of $U(\xi_k)$ ($k = 1,2,...,N$) using L (Privalov, 1984) integration contour:

$$U(z) = \frac{1}{2\cdot\pi\cdot i}\cdot\oint_L \frac{U(\xi)d\xi}{\xi - z} \approx \frac{1}{2\cdot\pi\cdot i}\cdot\sum_{k=1}^{N}\frac{U(\xi_k)\cdot\Delta\xi_k}{\xi_k - z},$$

is an increment of the integration complex variable, where the $\Delta\xi_k = \Delta x_k + i\cdot\Delta y_k$ point is center of the intervals $l_k = \sqrt{\Delta x_k^{\ 2} + \Delta y_k^{\ 2}}$ whose length is ξ_k.

Below are computational modeling results illustrating applicability of Cauchy integration for approximation of field/current components in the simplest electrodynamic objects - dielectrical or steel cube.

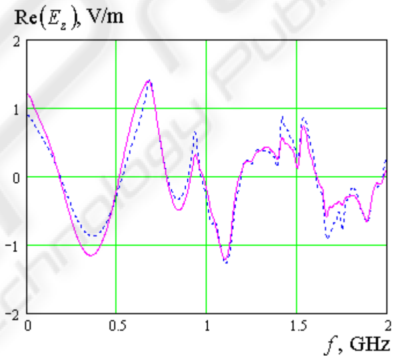
The dash lines in Figs. 1, 2 reflect the actual (derived through rigorous numerical solutions of diffraction problems) frequency profiles of the real and imaginary part of E_z-component (total EM field) and J_z-component (current density on the metal surface). The solid lines show the restored profiles derived through Cauchy integrals.

Let's consider diffraction of a plane polarized EM-wave on a 110x110x110 mm dielectrical cube where ($\varepsilon_r = 10$, $tg\delta_э = 0.003$) - Figure 1. The coordinates of the normalized wave vector $\vec{k}_{0_n} = \vec{k}_0 / \left|\vec{k}_0\right|$ were taken as $(-0.577;\ -0.577;\ -0.577)$. Projection of the normalized vector $\vec{E}_n^{nad.} = \vec{E}^{nad.} / \left|\vec{E}^{nad.}\right|$ onto the coordinate axis $(x; y; z)$ were (-0.408; -0.408; 0.816) respectively. The sight point coordinates were taken as (45; 45; -55) mm (near the bottom cube face angle).

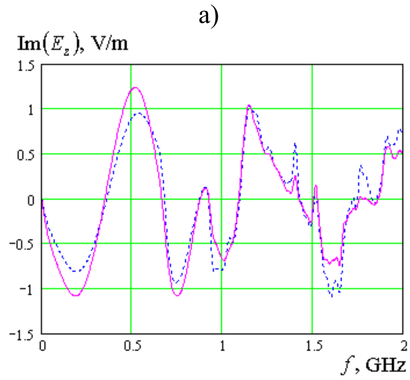
The results of numerical experiments to restore the current density on the "shady" surface of the metal cube are shown in Figure 2. For the above profiles, the values of the normalized wave vector coordinates $\vec{k}_{0_n} = \vec{k}_0 / \left|\vec{k}_0\right|$ amounted to (-0.667; -0.667; -0.333); and projections of that vector $\vec{E}_n^{nad.} = \vec{E}^{nad.} / \left|\vec{E}^{nad.}\right|$ to $(x; y; z)$ axis were taken as $(-0.236;\ -0.236;\ 0.943)$. The surface current density was to be restored in the middle of a "shady" face of the cube ($y = -55$ mm); and the sight point coordinates were taken as (0; -55; 0) mm.

With this, we would like to draw attention to the actual and imaginary part of the E_z- component (total EM field, Figure 1). There, the Cauchy integral can "scan" almost all the frequency band in question and "trace" even the slightest resonances around such frequencies as 1.4, 1.5 and 1.85 GHz.

The available errors between the actual and estimated profiles are, most probably, caused by numerical integration errors and by the nature of the function describing the field structure (current) – it is not purely analytical.

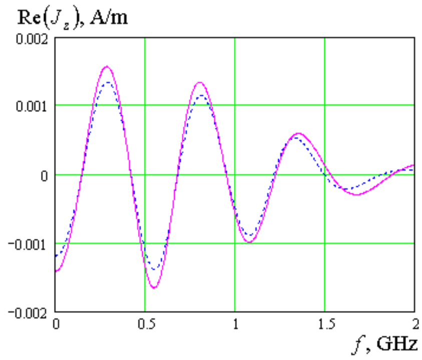


a)



b)

Figure 1: Restoration of the Field at the Bottom of a Dielectric Cube, section $z = -55$ mm).



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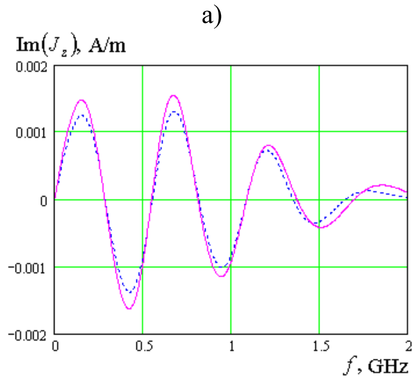

b)

Figure 2: Restoration of the Current Surface Density on the "Shady" Side of a Metal Cube ($z = -55$ mm).

Now let us consider a method for restoration of the EM field outside a closed contour and try to reduce the angular coordinates systematic error in emitters' position finding by means of a mobile antenna array - see Figure 3.

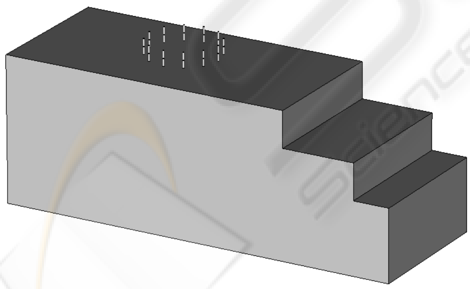

Figure 3: Electromagnetic object: carrier enclosure + antenna array.

Suppose that an EM wave is falling onto an array, then the general vector of the voltage amplitudes, which emerged on the antenna resistors, is $\vec{U} = [U_1, U_2, U_3, \ldots, U_N]^T$. In this case, the values of the function describing the field at the circle with R radius comprising the elements of the array and, consequently, N electrically short symmetrical vibrators will be as follows:

$$U(z = R \cdot \exp[i \cdot \xi]) = \sum_{n=1}^{N+1} B_{n-1} \cdot \exp[i \cdot (n-1) \cdot \xi / (N+1)].$$

The values of B_n complex factors are determined through solution of the following algebraic equations with several complex unknowns:

$$U_k = B_1 + B_2 \exp\left[ik \frac{2\pi/(N+1)}{N}\right] + \ldots + B_{N+1} \cdot \exp\left[ik \frac{2\pi/(N+1)}{N} N\right],$$

$$k = 1, 2, \ldots, N$$

The values of the R function measured and approximated at the outer circle with U radius shall conform to Cauchy or Poisson integral formula:

$$U(R) = \sum_{n=1}^{N+1} B_{n-1} \exp[i(n-1)\varphi/(N+1)] =$$
$$= \frac{1}{2\pi} \int_0^{2\cdot\pi} U_{внешнее}(r \exp[i\psi]) \frac{r^2 - R^2}{r^2 - 2rR\cos(\psi - \varphi) + R^2} d\psi =$$
$$= \frac{1}{2\pi} \int_0^{2\cdot\pi} \left\{ \sum_{k=1}^{K+1} X_{k-1} \exp[i(k-1)\psi/(K+1)] \right\} \cdot$$
$$\cdot \frac{r^2 - R^2}{r^2 - 2rR\cos(\psi - \varphi) + R^2} d\psi.$$

So we obtain an a first kind Fredholm integral equation related to unknown $U_{внешнее}(r \cdot \exp[i \cdot \psi])$ function, describing the scalar field on r-radius:

$$U_{внешнее}(r \exp[i\psi]) = \sum_{k=1}^{K+1} X_{k-1} \exp[i(k-1)\psi/(K+1)],$$

And this function is fully determined by X-factors.

To solve this integral equation is an incorrect mathematical problem and therefore we had to use Tikhonov regularization (Bakhvalov, Zhidkov, Kobelkov; 1987). We have also applied a collocation method (Bakhvalov, Zhidkov, Kobelkov; 1987) and thus reduced the initial integral equation to a system of linear algebraic equations with complex variables.

According to the numerical analysis of the bearing frequency dependence (provided that the bearing was measured with an array with Radius $R = 0.5$ m and that the real azimuth angle was $\varphi_{EMW\_fall} = 45^0$ for the falling wave), the maximum systematic errors was 16^0 at 90 MHz: the measured bearing value was 61^0. The developed technique can significantly reduce the amount of DF systematic errors caused by scattered waves from the carrier. Thus, when restoring the P&A structure of the scalar field on the circle with $r = 1.5$ m, the error will be reduced from 16^0 to 1^0 regardless of the carrier's geometry and its location relative to the array (Figure 4). Moreover, since the radius of a virtual array is increased by three times and since the number of its elements rises from 12 to 36, the scanning resolution of the azimuth angular coordinates will also rise.

Figure 4: Application of the proposed method for reduction of direction finding systematic errors and increase of azimuth resolution on a mobile carrier (the full line shows the array pattern as in Figure 3 and the dash line shows the array pattern for the array with the triple diameter).

3 CONCLUSION

The EM field and current surface density components in metal-dielectric structures can be approximated by means of Cauchy integrals treated by the method of average and based on the finite aggregate of scalar field values taken through the integration contour. With this, it can also be possible to restore the phase and frequency characteristics of the field or current within the integration contour. Besides, these characteristics will be more exact than those of amplitude and frequency and will cover a wider frequency band.

In most of the reviewed cases, the field (current) approximation accuracy will decrease as the sight point approaches to the integration contour (if the grid pitch is fixed). To increase such approximation around the integration contour, a tapered grid can be used. Its pitch will decrease as the sight point approaches the contour.

Approximation of the field (or current) through Cauchy integral can be applied to scale down electromagnetic issues solved in terms of space and frequencies and to allow for diffusers affecting antenna systems and their directional properties.

It is proved that proposed technique can be used for reducing systematic errors in measurement of emitter's angular coordinates by means of mobile direction finders and increasing DF resolution and accuracy.

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