A NEW FUZZY LOGIC CONTROLLER FOR TRADING ON THE STOCK MARKET

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Abstract: A common problem that financial operators often meet in their own work is to make, at the right moment, the operational choices on the Stock Market. Once the Market to act on has been chosen, the financial operator has to decide when and how to operate on it, in order to achieve a profit. The problem that we are going to deal with is the planning of an automatic decisional system for the management of long positions on bull market. First, a trading system (TS) will be implemented pointing its features out. Then a fuzzy logic implementation of the TS will be introduced (FTS). The fuzzy system will be optimized by the genetic algorithms. Finally, the two different implementations of the trading system will be compared using some performance indexes.

1 INTRODUCTION

Different ways exist to operate on the Stock Market: following the instinct or smell, reading journals and reports, with the help of experts, or applying more methodic techniques. Among all the available operational techniques, there are the *technical analysis* (Edwards, Magee, 1957) and *fundamental analysis* (Schwager, 1995). Furthermore, in the last years some experimental techniques has been used. These techniques are founded upon the concepts of *soft computing*. Some of them simulate the process of the human reasoning (*expert systems* and *fuzzy systems*), others the biological operation of the brain (*neural networks*) and others the genetic evolution (*genetic algorithms*). All these techniques can leave out the principles of technical analysis and fundamental analysis, but they can use them partially (G.J. Deboeck, 1994).

In Li, Xiong (2005), the authors presents a fuzzy neural network to predict the comprehensive index of Shanghai stock market. In Hiemstra (1994), the author presents a general approach to stock market prediction and introduces an architecture of a fuzzy logic forecasting support system. In Setness, van Drempt, (1999) the authors examine the application of other fuzzy models to the problem of stock market analysis. In H.S. Ng, K.P. Lam a Genetic Fuzzy Expert Trading System (GFETS) was designed to simulate the vague and fuzzy trading rules and give the buy-sell signal. Fuzzy trading rules are optimized and selected using genetic algorithm in GFETS. In H. Dourra, P. Siy (2002) the authors proposed a method to map some technical indicators into new inputs that can be fed into a fuzzy logic system.

In this paper, in the section 2, we shall introduce the classical methodologies of analysis of the Stock Market and the trading systems. In the section 3, the implementation of a trading system on the Mib30 (TS) is introduced. In the section 4 we shall introduce the fuzzy logic, used for the implementation of the fuzzy trading system (FTS). In the section 5, a comparative analysis of the two systems is effected through some performance indexes.

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2 ANALYSIS OF THE STOCK MARKET

2.1 Technical Analysis

The economic phenomena and their reflexes on the Stock Market are very complicated. The most important branch of research is technical analysis. (Malkiel, 1981; Fama, 1989).

Technical analysis mainly founds itself on the observation of the prices (Edwards, Magee, 1957). Technical analysis identifies the direction of a trend and indicates, at the right moment, when the trend direction is changing. In order to do this it uses the graphic and algorithmic tools which are indicators, defined as functions of prices and volumes (Elder, 1993; Sack, 1992).

The most diffused tools of technical analysis are algorithmic (*indicators* and *oscillators*) (Malkiel, 1981; Edwards, Magee, 1957). The indicators and the oscillators offer a different perspective from which to analyze the price action. They are derived by applying a formula to the price data of a security. Price data includes any combination of the open, high, low or close over a period of time. An oscillator is an indicator that fluctuates above and below a centerline or between set levels as its value changes over time (Edwards, Magee, 1957).

Below, we shall describe the features of some principal indicators, which will be used in this research.

2.1.1 MACD (Moving Average Convergence/Divergence)

The MACD is a momentum oscillator, i.e. it measures the strength of the price movement. The MACD is calculated by subtracting the value of a 0.075 (26-period) exponential moving average from a 0.15 (12-period) exponential moving average. A 9-period exponential moving average of the MACD, called *trigger line*, is used to signal buy orders or sell orders.

2.1.2 ADX (Average Directional Movement Index)

The ADX, built by Wilder's smoothing of the DX (Directional Movement Index), measures the strength of a trend and it is useful to individualize the shift from a trend phase to a congestion phase (Hartle, 1991). The DX characterizes the directional movement of the prices and oscillates between 0 and 100 (then also the ADX oscillates between 0 and 100). The values of the ADX, that overcomes a certain threshold (the select values usually go from 20 to 40), point out a strong trend phase, while values that go under the threshold point out a congestion phase.

2.2 Trading Systems

The expression "trading system", as is known, characterizes a rigorous methodology that uses fixed rules (*trading rules*) to decide how to operate on the Stock Market. The aim of the trading system is realizing, through a particular strategy, good profits for the investor (*trader*).

The trading systems are usually implemented to use, at the same time, several tools of technical analysis. The contemporary use of these tools can produce discordant results. This problem can be solved using a computational algorithm that produces buy signals and sell signals when the available data are compatible with all of the established rules.

2.2.1 Evaluation of a Trading System

Once that the trading system has been well defined, there are different criterions with which to appraise the success or the failure of it. The most important tools, predisposed to evaluate the efficiency of a trading system and used in this research, are described below (Elder, 1993).

1) The *Equity Line* is probably the best diagnostic tool for trading system developer. In one graph it shows the sum total of the success or failure of the system being tested, and the resulting effect on your equity. The ideal chart of an equity line should be an increasing curve; if so, there would be constant and increasing profits from time to time.

2) The *Profit* is the aggregate clean profit and it’s achieved supposing to close one’s own positions the last day of the simulation.

3) The *Profit / Loss Index* compares the profit produced by the winning operations to that produced by the lost operations:

$$\text{P/L Index} = \frac{\text{Profit}}{\text{Trade Profit}}$$

where the *Trade Profit* is the profit obtained by the winning operations only.

4) The *Reward/Risk Index* is defined as

$$\text{R/R Index} = \frac{\text{Reward}}{\text{Risk}}$$

where the *Reward* is the aggregate clean profit (*Profit*) and the *Risk* is the lowest point, of equity line. If this index is smaller than +50, we submit the trader to a too elevated stress compared to the profits produced by the trading system.

5) The *Buy & Hold Index* compares the profit obtained by the trading system to that obtained by the strategy Buy & Hold. It consists in opening a long position (buy order) the first day of the simulation and in closing such position (sell order) the last day, without effecting any operations during the select period.

6) The *Win/Lose Index* corresponds to the ratio between winning operations and lost operations

$$\text{W/L Index} = \frac{\text{N° Winning operations}}{\text{N° Lost operations}}$$

3 A TRADING SYSTEM ON MIB30

In this section we shall introduce the design of a trading system (TS), which is the base for the following development of a fuzzy trading system (FTS). Working with FTS, we will be able to apply our algorithmic trading rules using the peculiar properties of fuzzy logic. Before going deep into the development of TS, we have to introduce the time series of prices on which the TS has been applied.

3.1 The Time Series

We have chosen the Italian Stock Market and the security of the Mib30 (*Milan Italy Stock Exchange 30 Index*), based on the 30 leading stocks, that is, the most liquid and most highly capitalized stocks listed on the Italian Stock Exchange. The time series of Mib30 (Figure 1) has been downloaded from Yahoo.

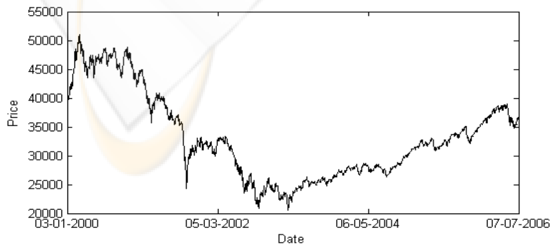

Figure 1: Close prices of MIB30.

Before using the time series, we have integrated some lacking data, using a linear interpolation. The used time series is characterized by a daily frequency, from 03/01/2000 to 07/07/2006, and every sample is made of the open price, of the maximum price, of the minimum price and of the close price of the day (1690 samples).

3.2 Trading Rules

The system has been designed only for the management of long positions in bull market. It produces buy signals in bull market and sell signals in bear market. Furthermore, we didn't take into accounts the criterions for the management of not sustainable losses or of gains higher than fixed profit.

We have decided to individualize three possible market phases: the bull market, bear market and the congested market. Moreover, considering that in the bull market and in the bear market we can apply the same operational tools, we have focused on the identification of only two of the market phases: the *trend market* (whether it is bull market or it is bear market) and the *congested market*. The TS has an initial *filter* which allows it to establish, with a certain degree of approximation, the type of market phases. To detect the type of market phases we have chosen the ADX. The identification between the trend markets and congested market has been made through a threshold. If the ADX is lower than the threshold (congested market), the TS doesn't produce any BUY or SELL signals, but WAIT signal (no signal). If the ADX is higher than the threshold (trend market) instead, the TS uses the MACD oscillator (jointly the trigger line) to produce operational signals according to the following rule: a SELL signal occurs when the MACD falls below its trigger line; a BUY signal occurs when the MACD rises above its trigger line. When the trigger line stays below (or above) the MACD, the TS produces a WAIT signal. The TS is not a very aggressive system but surely it is a solid one.

3.3 Parameters Tuning

Firstly, we must fit the ADX period: a very wide ADX period implicates a slow movement of this indicator; while a narrow ADX period determines a rapid movement of this indicator. In addition, we must fit a second parameter, the ADX threshold (this parameter determines the ADX sensibility).

In relation to the trigger line, we have chosen a standard period of 9 days; therefore, we must tune only the first two parameters.

The most logical method for the choice of values of the ADX period and the ADX threshold is selecting those values which had previously produced the best results. First, we have specified a allowable range for parameters value; then we have simulated all possible trading systems from 03/01/2000 to 26/04/2004 (two-third of the time series); subsequently, we have saved the parameters value that have produced the best results (maximum clean profit). We have finally used the best parameters value to test the TS from 27/04/2004 to 07/07/2006 (the rest one-third of the time series).

The trading system that has produced the best clean profit had the ADX period equal to 7 and the ADX threshold equal to 23.

3.4 System Evaluation

The TS, applied from 03/01/2000 to 07/07/06, have generated 45 buy signals and the same number of sell signals with a clean profit equal to 18399 (unit price). Buy and sell signals are uniformly distributed within six years taken for the simulation. So we can't observe any period of inactivity of the system, even if our system shows the tendency to signal the operations lately. The system is not able to exploit fully the bull market phases and, at the same time, it follows the bear market phases for a too long time. This is due to the characteristics of the instruments of technical analysis we applied. Observing the trend of the system equity line (Figure 2) we can notice that the curve is characterized by growing steps in bull markets and horizontal lines in bear markets. Our system aims to improve profits taking advantage of upwards trends and to limit losses during downwards trends.

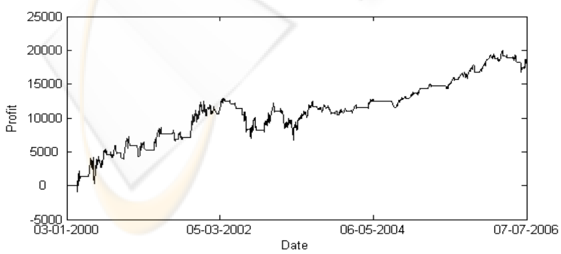

Figure 2: Equity Line of TS.

However we can also notice some periods in which the system has to face some losses because of the decision to maintain the actual position (WAIT) even during a congestion phase. Finally, we have to point out that we have ignored the costs of all the transactions (both for opening and closing operations). That is the costs to open and to close an operation don't affect on the whole profit.

As regards performance indexes, (Table 1), we can notice that:

- the Profit/Loss Index, greater than 50, indicates that, during the six years used for simulation, profits have been greater than losses;
- the Reward/Risk Index, extremely near to 100, allows us to state that the system made profits with a very low risk factor;
- very high value of the Buy & Hold Index assures us that our system is a useful trading system;
- the Win/Lose Index, greater than 1, indicates that we made a number of winning deals greater than the number of losing ones.

Table 1: Performance Indexes.

Profit/Loss Index	60.30
Reward/Risk Index	95.30
Buy & Hold Index	280.04
Win/Lose Index	1.37

4 A FUZZY TRADING SYSTEM ON MIB30

4.1 The Fuzzy Logic and the Stock Market

The studying and forecasting of stock markets often involve vague and inaccurate concepts and reasoning. Fuzzy logic appears as the most natural tool to face this kind of problems, since it has been designed just to represent uncertain knowledge (Zadeh, 1978; Yager, Zadeh, 1992). The application of fuzzy logic in the economic-financial field allows to implement a simple system, whose operations are easy to guess. Furthermore, a good trading system needs the support of a suitable model that allows (Li, Xiong, 2005; Hiemstra, 1994; Setness, Van Drempt, 1999):

- to define and store all the information suitable for the desired forecast;
- to represent the uncertainty and the imperfection that characterize the information

that belongs to the experts of this sector;
- to provide a clear, explanatory and interactive forecast.

Fuzzy logic allows to represent these concepts and to synthesize them in the implementation of a fuzzy controller that can replace a classical trading system.

4.2 The Fuzzy Logic Controller

In this section, we shall introduce the fuzzy controller designed to implement the FTS. The FTS, like the TS, is a system designed only for the management of long positions in bull markets. It produces BUY signals in bull markets and SELL signals in bear markets. Our controller is characterized by three inputs (ADX, perMT1, perMT2) and only one output.

Respective *membership functions* (MF) are associated to all the variables (Zadeh, 1975; Mamdani, Assilian, 1975; King, Mamdani, 1967). For every MF, we have determined their own parameters through Genetic Algorithms (GA) (Davis, 1991; Goldberg, 1989), using as *fitness* the profit obtained applying the systems to the whole historical data series. We have implemented a customized function in order to create our individuals (our initial population was made up of 50 individuals, each of which represents a set of parameters). Also we have implemented two functions to realize, during optimization, the crossover and the mutation process to generate new individuals through evolution. In this way, we have deleted the complex and hard-working phase of manual definition of the parameters (Karr, Gentry, 1993) and we have also easily detected the most correct shapes of the various MF. Moreover we've been able to impose and to respect some ties on the mutual positions of the MF, composing some figures characterized by symmetry characteristics, and individualizing some zones of overlap for the figures themselves. (Karr, Gentry, 1993).

The output of our fuzzy controller, obtained through the well-known mechanism of defuzzification, is a crisp value belonging to the numerical interval [-1;1]. Then we have introduced two numerical thresholds, a positive and a negative one. We have made this in order to identify, in connection with the output value, the corresponding operative signal (BUY, WAIT and SELL). So we have made a selection among all the available output values, choosing just the meaningful ones. In this way, all of the output values, superior to the negative threshold or inferior to the positive threshold, correspond to a WAIT signal. All the output values superior (inferior) to the negative (positive) threshold correspond to a BUY (SELL) signal. We established the two threshold values through an optimization made once more using GA.

4.2.1 ADX

This input corresponds to ADX. As this index can assume values in the range [0 100], the corresponding fuzzy set is the same range. Three different membership functions have been associated to this first input (Figure 3):

- **CO:** a congestion phase, which is a phase in which market is not in a downward trend nor in an upward one;
- **CT**: a not well defined market phase; in this case we can't state if a congestion phase is developing, or if a downward trend or an upward one is going to an end or confirming itself;
- **TR**: a trend, very strengthened or not. This MF has a trapezoidal shape and, in Figure 3, it has been cut out at 70.

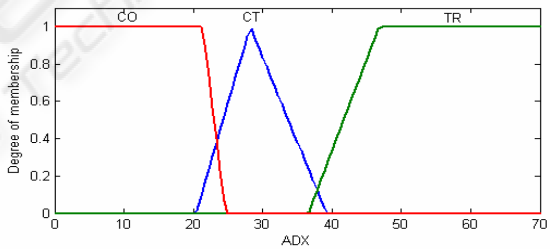

Figure 3: Membership Functions of 'ADX'.

4.2.2 perMT1 And perMT2

In FTS, these two inputs are used to represent the crossing between MACD curve and its trigger line. First of all, we have to point out the crossing between these curves. This crossing can be represented using two helpful situations: yesterday (i.e. at the time k-1), the difference between MACD and trigger line was negative while today (i.e. at the time k) the same difference is positive (MACD's curve crossed trigger line from the bottom upwards, Figure 4-a); yesterday (i.e. at the time k-1), the difference between MACD and Trigger Line was positive while today (i.e. at the time k) the same difference is negative (MACD's curve crossed

Trigger Line from the above downwards, Figure 4-b);

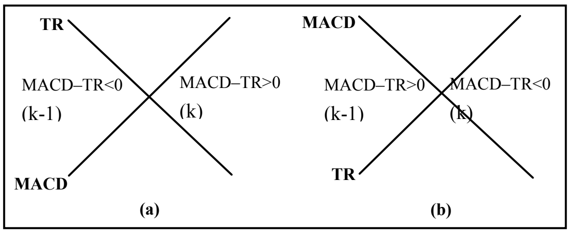

Figure 4: Crossing between curves.

Therefore, our task is to find a suitable formal representation of the algebraic signs of these differences, not of their exact value.

We can define two variables:

$$MT1 = MACD(k-1) - TR(k-1);$$
$$MT2 = MACD(k) - TR(k);$$

Then, we can define the total gap, in the last twenty-four hours, between MACD and its trigger line as

$$MT12 = |MT1| + |MT2|$$

At this point, we can finally introduce our inputs, defined as the percentage variations of the two variables MT1 and MT2 in comparison to the total variation MT12.

This mathematical model makes us sure that the two variable fuzzy sets are finite and that they correspond to the range [-100 100]. Moreover, this model preserves the right signs of the mathematical differences we have considered.

Three different MF have been associated to the variable named perMT1 (Figure 5):

- **N**: negative differences, regarding yesterday measures; this MF, in Figure 5, has been cut out at -60.
- **Z**: differences that are equal to zero, still regarding yesterday measures;
- **P**: positive differences, once more as regards yesterday measures; this MF, in Figure 5, has been cut out at 60.

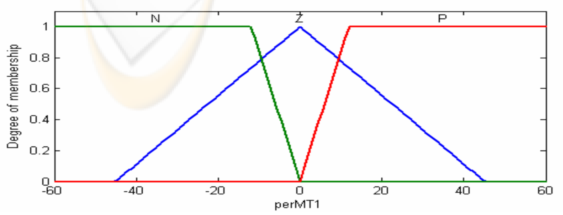

Figure 5: Membership Functions of 'perMT1'.

We used the same MF to represent perMT2.

4.2.3 ACTION

The output variable, named ACTION, represents the real operative signal that comes out from the evaluation of all fuzzy rules made by the fuzzy controller, on the basis of the received inputs. According to fuzzy logic principles, the three different signals we have considered (BUY, SELL and WAIT) have been represented considering some possible vague situations. So five different MF have been associated to our output variable, which can assume values in the range [-1 1] (Figure 6):

- **SELL**: sell orders;
- **ASELL**: sell warnings;
- **WAIT**: wait signals;
- **ABUY**: buy warnings;
- **BUY** : buy orders.

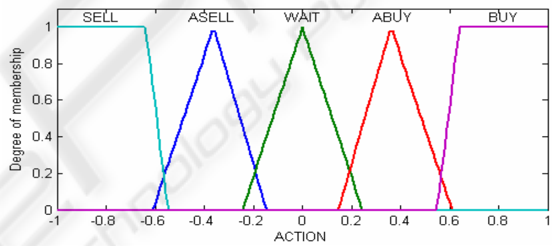

Figure 6: Membership Functions of 'ACTION'.

As we have already said before, output defuzzification is followed by selection of the really meaningful values, through the use of the filter implemented by the two threshold. The optimal values obtained applying GA optimization are:

$$S^{-} = -0{,}18 \qquad S^{+} = 0{,}35$$

4.2.4 Fuzzy Rules

The knowledge base of the inference engine of our fuzzy controller is made up of 27 rules, each of them has a weight equal to 1. As regards the left part of these rules, we have chosen the boolean operator AND as connective. The result of a compound expression is obtained applying the minimum method among all values. Besides, the technique chosen for output defuzzification is the one based on the calculation of the centroid of the area obtained.

Below there are some of the rules we have implemented:

- **IF** ADX **is** TR **AND** perMT1 **is** N **AND** perMT2 **is** N **THEN** ACTION=WAIT;
- **IF** ADX **is** TR **AND** perMT1 **is** N **AND** perMT2 **is** Z **THEN** ACTION=ABUY;
- **IF** ADX **is** TR **AND** perMT1 **is** N **AND** perMT2 **is** P **THEN** ACTION=BUY;

4.3 System Evaluation

We remember that the considerations made for the TS are also valid for the FTS. We have ignored once more the costs of all the transactions made, that is the costs to open and to close an operation don't affect on the whole profit; besides, we haven't applied any criterion to effect the exit from the market. These assumptions are the same for both of the two systems, therefore the base of comparison is valid.

Now we can reassume the results obtained applying the FTS on the whole historical data series. The use of the system from 03/01/2000 to 07/07/2006 has brought the generation of 36 buy orders and of the same number of sell orders, with a net profit of 22894 (unit price). Buy and sell signals are uniformly distributed within the six years taken for the simulation and we can't observe any period of inactivity of the system. The trend of the fuzzy system equity line (Figure 7) supports the acknowledgement that fuzzy rules we have chosen are consistent with the aim of our research, that is managing only long positions in bull markets. In fact, we can notice, in the chart, that the curve is characterized by growing steps in bull market and horizontal lines in bearing market.

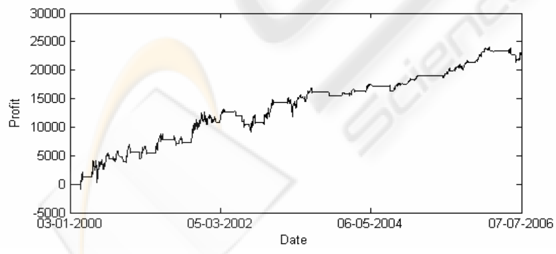

Figure 7: Equity Line of FTS.

We can notice once more some periods when our system has to face reductions as regards net profit. This is due, as for TS, to our choice of maintain our position (WAIT) even in a phase of plain congestion (ADX lower than its threshold).

As regards performance indexes used for FTS evaluation (Table 2), we can notice that:

- the Profit/Loss Index (by far greater than 50) indicates us that on the whole we had more profits that losses;
- the Reward/Risk Index extremely near to 100, allows us to state that the system has made profits with a very low risk factor;
- very high value of the Buy & Hold Index means that our system multiplies by four the profit obtained with a Buy And Hold strategy;
- the Win/Lose Index almost equal to 3 means that the system has made a number of winning deals greater than the number of losing ones.

Table 2: Performance Indexes.

Profit/Loss Index	73.87
Reward/Risk Index	96.23
Buy & Hold Index	324.04
Win/Lose Index	2.27

5 TS VS FTS

In this section we perform a comparative analysis of the two different implementations of a trading system.

The FTS indicates 36 buy signals, and the same numbers of sell signals which are fewer than those produced by TS. In both cases, buy signals (sell signals) are very near to the points in which an upwards (downwards) trend is growing. The FTS shows a better attitude than the TS to take advantages of upwards trends and to point out, at the right time, the downwards trends. This is due to the application of fuzzy logic, which allows to decrease the inaccuracy belonging to technical analysis and to its instruments that we have applied in our research. The reduced number of operations suggested by FTS confirms its attitude to avoid wrong signals. As a matter of fact, fuzzy logic recognizes, better than the TS does, the market phase (initial filter). Fuzzy logic helps the trader avoid some dangerous operations which must be corrected by additional operations. For this reason the TS produces a considerable number of operations but a low profit. Moreover FTS, compared to TS, is able to contain better the amount of losses. This means that FTS can reach a much more higher profit than TS. This profit remains on very high levels during the whole simulation. These last considerations are well evident in the chart (Figure 8) where we have quoted together the equity lines of the two systems:

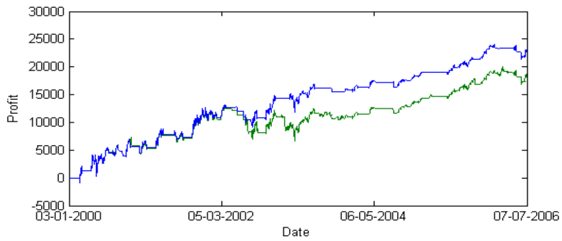

Figure 8: Equity line of FTS and TS.

As far as the performance indexes are concerned, a quick comparison is enough to state that FTS is better than the TS, from every point of view. In fact, the FTS is stronger (it has a better Win/Lose Index and a better Buy & Hold Index) and it is also much more reliable (it has a better Profit/Loss Index and a better Reward/Risk Index) than the TS.

6 CONCLUSIONS

Designing both the non-fuzzy trading system and the fuzzy one haven't any pretension to satisfy real operative aims. The task of our research has been to show that we've been able to improve results of some simple and well-known rules of technical analysis through the application of fuzzy logic principles.

First, we have observed that an automatic decisional system, planned as an application for stock market, has to provide a general model which we have modified and optimized using our own knowledge: fuzzy logic, a well known technique of soft computing. As matter of fact, the "transparent" structure belonging to a fuzzy logic system allows easy interactions with the trader, through an interactive employment, but designing a fuzzy trading system implies some real difficulties to choose the right parameters for the fuzzy logic controller. We have solved this problem using Genetic Algorithms as an optimization technique.

So the task of our research has been the implementation of a fuzzy trading system (FTS) as an alternative to an equivalent non-fuzzy trading system (TS).

Our results have made us state that not only fuzzy logic is a valid alternative to the classical implementation of a trading system, but from every points of view, it also improves its performances.

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