SOLVING THE MULTI-OBJECTIVE MIXED MODEL ASSEMBLY LINE PROBLEM USING A FUZZY MULTI-OBJECTIVE LINEAR PROGRAM

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Abstract: This paper develops a fuzzy multi-objective linear program (FMOLP) model for solving the multi-objective mixed model assembly line problem. In practice, vagueness and imprecision of the goals, constraints and parameters in this problem make the decision-making complicated. The proposed model attempts to simultaneously minimize total utility work cost, total production rate variation cost, and total setup cost. In this paper, an asymmetric fuzzy-decision making technique is applied to enable the decision-maker to assign different weights to various criteria in a real industrial environment. The model is explained by an illustrative example.

1 INTRODUCTION

Mixed model assembly lines are a type of production line where a variety of product models similar in product characteristics are assembled. The effective utilization of a mixed-model assembly line requires solving two problems in a sequential manner as follows: 1) line design and balancing and 2) determination of the production sequence for different models. In this paper, we assume that the line has already been balanced and sequencing problem is only considered.

Korkmazel and Meral (2001) consider two major goals in the mixed model sequencing problems: (1) smoothing the workload on each workstation on the assembly line, and (2) keeping a constant rate of usage of all parts used on the assembly line. In their study, first, some well-known solution approaches with goal (2) are analyzed through minimizing the sum-of-deviations of actual production from the desired amount. The approaches that are found to be performing better than the others are extended for the bi-criteria problem considering both goals, simultaneously.

Ponnambalam et al. (2003) investigate the performance of genetic algorithms for sequencing problems in mixed model assembly lines.

Mansouri (2005) presents a Multi-Objective Genetic Algorithm (MOGA) approach to a Just-In-Time (JIT) sequencing problem where variation of production rates and number of setups are to be optimized simultaneously.

Ding et al. (2006) compares two weighted approaches in sequencing mixed model assembly lines for a joint objective of multiple objectives. Minimizing the weighted sum of percentage differences from the best solution values of the respective objectives is considered as the joint objective.

Mixed model assembly line is a multi-objective decision-making problem, in which criteria should have different weightes. Vagueness of the information in this problem, make the decision-making complicated. In this paper, a fuzzy multi-objective model developed to assign different weights to the various criteria.

2 THE MULTI-OBJECTIVE MIXED MODEL ASSEMBLY LINE (MMAL) MODEL

2.1 Mixed-model Assembly Line

The design of the MMAL involves several issues such as determining operator schedules, product mix, and launch intervals. Two types of operator schedules early start schedule and late start schedule, are found in Bard et al. (1994). An early start

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schedule is more common in practice and is used in this paper (Chul et al. (1998)). Second, the master production schedule (MPS) production, which this strategy is widely accepted in mixed model assembly lines, is also used in this paper. MPS is a vector representing a product mix, such that $(d_1,\ldots,d_M)=(D_1/h,\ldots,D_M/h)$; where M is the total number of models, D_m is the number of products of model type m which needs to be assembled during the entire planning horizon and h is the greatest common divisor or highest common factor of $D_1,D_2,\ldots,D_M$. This strategy operates in a cyclical manner. The number of products produced in one cycle is given by $I=\sum_{i=1}^{M} d_i$. Obviously, h times the repetition of producing the MPS products can meet the total demand in the planning horizon. Third, the launch interval (γ) is set to $T/(I\times J)$, in which T is the total operation time required to produce one cycle of MPS products (Chul et al. (1998)).

2.2 Objective Function

2.2.1 Minimizing Total Utility Work Cost

The utility work is typically handled by the use of utility workers assisting the regular workers during the work overload. Let L_j be the fixed line length of station j and U_{ij} be the amount of the utility work required for product i in a sequence at station j. The following model is presented by Chul et al. (1998).

$$\text{Minimize} \sum_{j=1}^{J} \left(\sum_{i=1}^{I} U_{ij} + Z_{(i+1)j} / v_c\right) \quad (1)$$

s.t.

$$\sum_{m=1}^{M} x_{im} = 1 \qquad \forall i \quad (2)$$

$$\sum_{i=1}^{I} x_{im} = d_m \qquad \forall m \quad (3)$$

$$Z_{(i+1)j} = \max \quad (4)$$

$$[0, \min(Z_{ij} + v_c \sum_{m=1}^{M} x_{im} t_{jm} - (\gamma\times v_c), L_j - (\gamma\times v_c))]$$

$$\forall i, j$$

$$U_{ij} = \max$$

$$[0, (Z_{ij} + v_c \sum_{m=1}^{M} x_{im} t_{jm} - L_j)/v_c] \ \forall i, j \quad (5)$$

$$x_{im} = 0 \text{ or } 1 \quad \forall i, m \quad (6)$$

$$Z_{1j} = 0, \quad Z_{ij} \ge 0 \forall i, j \quad (7)$$

$$U_{ij} \ge 0 \forall i, j \quad (8)$$

Z_{ij} is the starting position of the work on product i in a sequence at station j, and x_{im} is 1 if product i in a sequence is the m^{th} model; otherwise x_{im} is 0. The second term in the objective function takes into account for the utility work that may be required at the end of a cycle. Eq. (2) ensures that exactly one product is assigned to each position in a sequence. Eq. (3) guarantees that demand for each model is satisfied. Eq. (4) indicates the starting position of the worker at each station j on product i+1 in a sequence. Utility work U_{ij} for product i in a sequence at station j is determined by Eq. (5).

2.2.2 Minimizing Total Production Rate Variation Cost

One basic requirement of JIT systems is continual and stable part supply. Since this can be realized when the demand rate of parts is constant over time, the objective is important to a successful operation of the system. Thus, the objective can be achieved by matching demand with actual production. The following model is suggested by Miltenberg (1989).

$$\text{Minimize} \sum_{i=1}^{I}\sum_{m=1}^{M}\left(\left|\sum_{l=1}^{i}\frac{x_{lm}}{i} - \frac{d_m}{I}\right|\right) \quad (9)$$

s.t.

Constraints (2), (3), and (6).

The first term in the objective function is the production ratio of model m until product i is produced. The second term is the demand ratio of model m.

2.2.3 Minimizing Total Setup Cost

In many industries, sequence-dependent setups are considered as an important item in assembly operations. The model considering sequence-dependent setups developed by Chul et al. (1998) is considered in this paper.

$$\text{Minimize} \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{r=1}^{M} x_{imr} c_{jmr} \quad (10)$$

s.t.

$$\sum_{m=1}^{M}\sum_{r=1}^{M} x_{imr} = 1 \quad \forall i \quad (11)$$

$$\sum_{m=1}^{M} x_{imr} = \sum_{p=1}^{M} x_{(i+1)rp} \quad i = 1,\ldots,I-1, \forall r \quad (12)$$

$$\sum_{m=1}^{M} x_{\mathrm{Im}r} = \sum_{p=1}^{M} x_{1rp} \qquad \forall r \quad (13)$$

$$\sum_{i=1}^{I}\sum_{r=1}^{M} x_{imr} = d_m \qquad \forall m \quad (14)$$

$$x_{imr} = 0 \text{ or } 1 \qquad \forall i, m, r \tag{15}$$

where c_{jmr} is the setup cost required when the model type is changed from m to r at station j. x_{imr} is 1 if model type m and r are assigned respectively at position i and i+1 in a sequence; otherwise x_{imr} is 0. Eq. (11) is a set of position constraints indicating that every position in a sequence is occupied by exactly one product. Eqs. (12) and (13) ensure that the sequence of products is maintained while repeating the cyclic production. Eq. (14) imposes the restriction that all the demands should be satisfied in terms of MPS.

In a real case, DMs do not have exact and complete information related to decision criteria and constraints. For mixed model assembly line problems the collected data does not behave crisply and they are typically fuzzy in nature.

2.3 The Fuzzy Mixed Model Assembly Line Model

In this section, first the general multi-objective model for mixed model assembly line is presented and then appropriate operators for this decision-making problem are discussed.

A general linear multi-objective model can be presented as:

Find a vector x written in the transformed $x^T = [x_1, x_2, \ldots, x_n]$ which minimizes objective function Z_k with

$$Z_k = \sum_{i=1}^{n} c_{ki} x_i \qquad k = 1,2,\ldots,p. \tag{16}$$

and constraints:

$$x \in X_d, \qquad X_d = \left\{ x \,/\, g(x) = \sum_{i=1}^{n} a_{ri} x_i \le b_r, \quad r = 1,2,\ldots,m, \; x \ge 0 \right\} \tag{17}$$

where c_{ki}, a_{ri} and b_r are crisp or fuzzy values.

Zimmermann (1978) has solved problems (16-17) by using fuzzy linear programming. He formulated the fuzzy linear program by separating every objective function Z_j into its maximum Z_j^+ and minimum Z_j^- value by solving:

$$Z_k^+ = \max Z_k, \quad x \in X_a, \qquad Z_k^- = \min Z_k, \quad x \in X_d \tag{18}$$

Z_k^- is obtained through solving the multi-objective problem as a single objective using, each time, only one objective and $x \in X_d$ means that solutions must satisfy constraints.

Since for every objective function Z_j, its value changes linearly from Z_j^- to Z_j^+, it may be considered as a fuzzy number with the linear membership function $\mu_{Z_j}(x)$ as shown in Fig.1.

Assuming that membership function, based on preference or satisfaction is the linear membership for minimization goals (Z_k) is given as follows:

$$\mu_{Z_k}(x) = \begin{cases} 1 & \text{for } Z_k \le Z_k^-, \\ \dfrac{\left(Z_k^+ - Z_k(x)\right)}{\left(Z_k^+ - Z_k^-\right)} & \text{for } Z_k^- \le Z_k(x) \le Z_k^+, \\ 0 & \text{for } Z_k \ge Z_k^+. \end{cases}$$

$$\text{for } k = 1, 2, \ldots, p, \tag{19}$$

The linear membership function for the fuzzy constraints is given as

$$\mu_{g_r}(x) = \begin{cases} \dfrac{g_r(x) - b_r^-}{b_r - b_r^-} & \text{for } b_r^- < g_r(x) < b_r, \\ \dfrac{b_r^+ - g_r(x)}{b_r} & \text{for } b_r < g_r(x) < b_r^+, \\ 0 & \text{for } g_r(x) \le b_r^- \text{ and } g_r(x) \ge b_r^+. \end{cases} \tag{20}$$

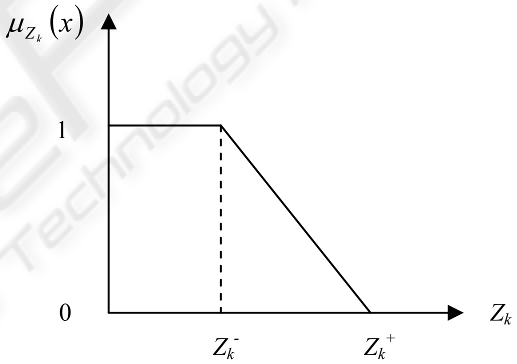

Figure 1: Objective function as fuzzy number for minimizing objective function. Z_k.

b_r^-, b_r^+ are the subjectively chosen constants expressing the limit of the admissible violation of the rth inequalities constraints.

In order to find optimal solution (x^*) in the above fuzzy model, it is equivalent to solve the following crisp model (Zimmermann, 1978):

$$\text{Maximize } \alpha \tag{21}$$

s.t.

$$\alpha \le \mu_{Z_j}(x) \qquad j = 1, 2, \ldots, p$$

(for all objective functions), (22)

$$\alpha \le \mu_{g_r}(x) \qquad r = 1, 2, \ldots, h$$

(for fuzzy constraints), (23)

$$g_p(x) \le b_p \qquad p = h+1, \ldots, m$$

(for deterministic constraints), (24)

$$x_i \geq 0 \qquad i = 1, 2, \ldots, n \quad \text{and} \quad \alpha \in [0,1] \tag{25}$$

where $\mu_D(x), \mu_{Z_j}(x)$ and $\mu_{g_r}(x)$ represent the membership function of solution, objective functions and constraints.

In this solution the relationship between constraints and objective functions in a fuzzy environment is fully symmetric (Zimmermann, 1978). In other words, in this definition of fuzzy decision, there is no difference between the fuzzy goals and fuzzy constraints. Therefore, depending on the mixed model assembly line problem, situations in which fuzzy goals and fuzzy constraints have unequal importance to DM and other patterns, as the confluence of objectives and constraints, should be considered.

The convex fuzzy model proposed by Bellman and Zadeh (1970) and the weighted additive model, is given in equations (26) and (27)

$$\mu_D(x) = \sum_{j=1}^{p} w_j \mu_{Z_j}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x) \tag{26}$$

$$\sum_{j=1}^{p} w_j + \sum_{r=1}^{h} \beta_r = 1 \qquad w_j, \beta_r \geq 0 \tag{27}$$

where w_j and β_j are the weighting coefficients that present the relative importance among the fuzzy goals and fuzzy constraints. The following crisp single objective programming is equivalent to the above fuzzy model:

$$\max \quad \sum_{j=1}^{p} w_j \alpha_j + \sum_{r=1}^{h} \beta_r \gamma_r \tag{28}$$

s.t.

$$\alpha_j \leq \mu_{Z_j}(x) \qquad j = 1, 2, \ldots, p, \tag{29}$$

$$\gamma_r \leq \mu_{g_r}(x) \qquad r = 1, 2, \ldots, h, \tag{30}$$

$$g_p(x) \leq b_p \qquad p = h+1, \ldots, m, \tag{31}$$

$$\alpha_j, \gamma_r \in [0,1] \qquad j = 1, 2, \ldots, p \quad \text{and } r = 1, 2, \ldots, h, \tag{32}$$

$$\sum_{j=1}^{p} w_j + \sum_{r=1}^{h} \beta_r = 1 \qquad w_j, \beta_r \geq 0, \tag{33}$$

$$x_i \geq 0, \qquad i = 1, 2, \ldots, n \tag{34}$$

3 CONCLUSIONS

Mixed model assembly line is a multiple criteria decision-making problem in which the objectives are not equally important. In real cases, many input data are not known precisely for decision-making. In this paper, a fuzzy multi-objective model is developed for mixed model assembly line in order to assign different weights to various criteria. This formulation can effectively handle the vagueness and imprecision of input data and the varying importance of criteria in mixed model assembly line problem.

Also in this model, the α-cut approach can be utilized to ensure that the achievement level of objective functions should not be less than a minimum level α.

In a real situation, the proposed model can be implemented as a vector optimization problem; the basic concept is to use a single utility function to express the preference of DM, in which the value of criteria and constraints are expressed in vague terms and are not equally important.

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