ON THE SEMI-AUTOMATIC VALIDATION AND DECOMPOSITION OF TERNARY RELATIONSHIPS WITH OPTIONAL ELEMENTS

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Abstract: This paper analyzes the problems that concern the design of databases. CASE tools supply a resources kit for the design and creation of database in a DBMS (Database Management System). Sometimes, these tools only help to draw diagrams. Ideally, they would verify and validate DB design and transform it from Conceptual to Logical Model. In a last step, they would transform the Logical Model to a specific DBMS. Currently, commercial tools do not verify or validate the model in an optimal way. This paper is focused on the validation and checking of database schemas. This work specially analyzes the ternary or higher-order relationships when there are optional components.

1 INTRODUCTION

When a Project Leader develops an application from the beginning, he or she has to think in the data. Once the designer has created the Conceptual Schema, the designer has to transform the Conceptual to Logic schemas, because the Logic Model is nearest to a DBMS. This paper is focused on Entity-Relationship and Relational models. A CASE tool of Database helps to realize the scheme in a particular DBMS. These tools start helping to design the Conceptual Model from UD and after they transform this Conceptual to Logic Model. At the end, they create a schema in a SGBD.

A scheme has to be verified syntactically and semantically. The syntactical verification has to check the rules of building. The semantic

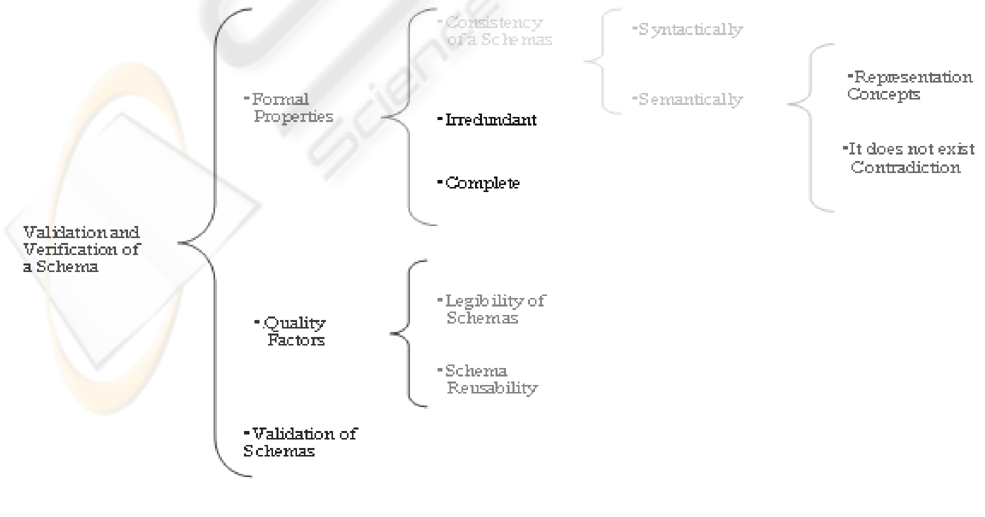

Figure 1: Validation and Verification of a Schema.

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verification tries to find the inconsistency between the semantic constrains and the user (Bouzeghoub, M. et al., 2000). The figure 1 shows the steps of validation and verification of a scheme. A good Conceptual scheme has to have Formal Properties, Quality Factors and Conformance with the user necessities (validation). Formal properties mean that the scheme has to be consistent, complete and irredundant. With respect to formal properties the majority of commercial CASE tools do a good syntactic validation, but no semantic. For example, they check if there is, at least, an entity and that the entities have different names. However, they do not check if there are contradictions among the schema concepts. These tools do not also verify the redundant elements. The completeness of a Conceptual scheme can be defined with respect to the meta-model or the UD represented. The first part the metamodel concerns the mandatory elements that constitute a conceptual schema. The commercial tools do this verification. The represented UD means the validation with respect to the user requirements. Checking whether a Conceptual schema represents all the necessary knowledge for a given information system, which refers to conformance of the Conceptual schema to the real world. With respect to quality factors, we have to look at the things the readability and the reusability. Readability is a desirable property, but it is a subjective valuation. The reusability is far away in commercial tools. For the last, the validation of a schema means that if the schema is adapted to the user requirements. In this topic, some tools have developed the Paraphrasing (NLDB, 2000). This technique generates a textual description from a Conceptual schema and the user can validate the model.

This paper analyzes the validation and correctness of the ternary or higher order relationships with optional elements. The majority of tools do not well implement this type of relationships.

Next section will describe the necessary definitions for this paper. The third section will look over some research works about this topic. The fourth section will explain our contribution and it will analyze and validate the relationships with optionality. The fifth section will observe the semantic anomalies in the ternary relationships with optional elements. The sixth section will show with an example our proposal. The seventh section will show the conclusions.

2 SOME REQUIRED DEFINITIONS

We show in this section the necessary definitions for developing this paper. We begin by defining the Entity and the Relationship element according to Thalheim (2000).

Let be $\mathrm{E}=\left(\mathrm{A}_1,\ldots,\mathrm{A}_n\{id_j(\mathrm{E})/1\le j\le n\}\right)$ an entity with attributes $\mathrm{A}_1,\ldots,\mathrm{A}_n$, each attribute is defined in a domain, where $\{id_j(\mathrm{E})/1\le j\le n\}$ is the set of candidate keys of the entity E, and this property characterizes in a univocal way, every one of the instances of E. We define E^t as a set of instances of E. An element e^t of E^t is a vector of n values, where the component i is denoted by $e_i \in dom(\mathrm{A}_i)$, which verifies that everywhere instance e'^t and $id_j(\mathrm{E})$: $\Pi_{id_j(\mathrm{E})}\left(e^t\right)\neq \Pi_{id_j(\mathrm{E})}\left(e'^t\right)$ where $\Pi_{\mathrm{A}}(\mathrm{E})$ is the projection of A in E.

We define the set of key instances associated to an entity as $\#\mathrm{E}=\pi_{\mathrm{IP}}\left(\mathrm{E}^t\right)$. A relationship of order n with s attributes is defined as R= $\left(r_1\mathrm{E}_1,\ldots,r_n\mathrm{E}_j,\mathrm{A}_1,\ldots,\mathrm{A}_s\right)$, where each r_i is the role and, where the entity $\mathrm{E_K}$ participates in R. We define R^t as the set of instances of R. An element r^t of this set is a vector of n components, where each component depicts a role that contains a key instance of the entity that it takes part with this role. Then, the set of instances R^t of a relationship R is a subset of the product contained of the key instances of the entities that participate in R and domains of attributes that participate in R.

$$R^t \subseteq r_1E_1\times\ldots\times r_nE_j\times dom(\mathrm{A}_1)\times\ldots\times dom(\mathrm{A}_s).$$

We define the *participation cardinality constraint* of an entity $C\left(r_i\mathrm{E}_j,R\right)=\{0 \text{ or } 1\}$ as the optional or mandatory participation, respectively, of the key instances E_j with role r_j in the relationship $R=\left(r_1\mathrm{E}_1,\ldots,r_n\mathrm{E}_j\right)$ where the relationship has order n. Optional constraint is depicted with a white circle, and the mandatory with a black circle, but both by the side of the relationship.

The *Merise's cardinality* for a relationship is defined as CMerise $(r_iE_j,R)=$ (n, m) where r_i is the role in $R=\left(r_1\mathrm{E}_1,\ldots,r_n\mathrm{E}_j\right)$, $n\le m$, and n, m $\in \mathrm{N}$. This means that a key instance of $r_iE_j\in R^t$ is in R^t as minimum and maximum *n, m* times. We depict this cardinality with a label at the end of the line that links the entity and the relationship but by the side of the relationship.

The *Chen's cardinality* (Cuadra, 2003) for a role r_i into $R=(r_1 \mathrm{E}_1,...,r_n \mathrm{E}_j)$ as *CChen* $(r_i \mathrm{E}_j, R)$ = (n, m), where n, m $\in$N and $1 \le n \le m$. This means that for any combinations of key instances $a_1,...,a_{i-1},a_{i+1},...,a_n$ de R^t is in R^t, n and m times as minimum and maximum, respectively. We depict this cardinality in a label at the line that links the entity and the relationship but by the side of the entity.

Let be R= $(r_1 E_1,...,r_n E_j)$ a relationship of grade n. We define a Complementary Relationship of R (Cuadra, 2003) as R_a^c, where *a* shows the roles in which participates this relationship and it has to carry out:

- a<n, that is to say, the number of roles which are applied, it has to be less than the grade of the relationship.
- Let C(aE, R)=0 be the cardinality constraint for every entity, which participates with every one of the roles in E has to be optional.
- $R_a^c \not\subset R$, that is to say, the instances which belong to the relationship, they can not be a subset of the complete relationship.

Let be R= $(r_1 E_1,...,r_n E_j)$ a relationship of order n. We define a Complete Relationship of R as R_a^T, with the same definition of the Complementary but it does not carry out the third point.

We use some definitions from the work of Trevor H. Jones and Il Yeol Song (Trevor H. Jones et al., 1996). If a binary relationship is semantically a subset of the ternary relationship and constraints the instances of the ternary relationship, then the binary relationship is a Semantically Constraining Binary (SBC) relationship. If not the binary relationships is Semantically Unrelated Binary (SUB) relationship. We do not analyze the SUB because it has not an effect on the ternary. They define the Implicit Binary Cardinality (IBC) rule as in any given ternary relationship, regardless of ternary cardinality, the implicit cardinalities between any two entities must be considered M:N, provided that there are no explicit restrictions on the number of instances than can occur. They define the Explicit Binary Permission (EBP) rule for any given ternary relationship, a binary relationship cannot be imposed where the binary cardinality is less than the cardinality specified by the ternary, for any specific entity. In addition, the Implicit Binary Override (IBO) rule is given the imposition of a permitted binary relationship on a ternary relationship, the cardinality of the binary relationship determines the final binary cardinality between the two entities involved.

McAllister, A. (1997 and 1998) defines the MX2 rule, denoted as augmentation rule. He defines r as the total set of roles for a relationship R. The cardinality constraint Cmax(a, b) means that if we fix a role [a] is the number maximum [a, b] permitted in R. The augmentation rule (MX2) defines that Cmax (a, b) $\le$ Cmax (ac, b). The MX2 rule is equivalent to the Explicit Binary Permission (EBP) rule.

On the other hand, we use some definitions of the Relational Model. They are definitions of Millist, W. V. (1994). Let t be a tuple of a relation R. Let t^* be a tuple to insert, update or delete. Let Σ_K be the set of key dependencies. The set of all relations that they satisfy Σ_K is denoted as SAT (Σ_K).

Let R be a relation, $\sum$ a set of dependencies, which apply to R and, r(R) a relation. A tuple t^* is said to be compatible with r if $r \cup \{t^*\}$ is a relation which is in SAT (Σ_K).

3 RESEARCH ABOUT THE VALIDATION OF TERNARY RELATIONSHIPS

In this section, we analyze different proposals about the validation and verification of relationships.

Trevor H J., et al. (1996) focus on cardinality constraints associated to ternary relationships. They analyze the SBC relationships and as they can affect to cardinality constraints in the ternary relationship. They analyze the implicit and explicit binary relationships and demonstrate their theories through the functional dependencies. The proposal does not depict the syntactic or semantic anomalies, they only study the semantic associated to ternary relationship through binary relationships. The paper of James Dullea (Dullea, J. et al., 1998) depicts when an E/R diagram has not redundancy. They do an analysis about the path (cycle path), which can be right or wrong. They analyze the optionality and they study its cardinality. However, they do not look at the E/R model anomalies.

The papers of McAllister, A. (1997 and 1998) describe an analysis about the minimum and maximum cardinality constraints in the relationships. He establishes rules for deducing cardinalities in the schema. If we apply these rules is possible to get a simplification or decomposition of the original schema. However, these works do not explain the problem from the UD. His work shows the minimum cardinality, 0, but he does not resolve the semantic problem of the optionality.

Rafael Camps (Camps, R. 2002) depicts an excellent analysis about the transformation of ternary relationships, with and without imposition binaries from E/R to R Model. In his work he establishes that the "Look across" cardinality constrains with the Chen approximation is richer semantically. We think also it. Furthermore, he shows the problem that the transformation from E/R to R using only functional dependencies has semantic anomalies.

In the R Model Millest W. Vicent (Millist W. Vincent, 1993, 1994, 1999) describes the semantic anomalies that have the relationships.

Santos (Santos, I. et al., 2006) depicts the semi-automatic validation and decomposition of ternary relationships, however this work does not analyze the optionality.

4 VERIFICATION AND VALIDATION OF TERNARY RELATIONSHIPS WITH OPTIONALITY

We use the representation of "Look across" cardinality constraints of Chen and Merise approximations (Cuadra, 2003), because the depicted semantic is very good for the automation in a CASE tool. The Chen approximation can be use for deriving the functional dependencies. We use the MX2 rule of McAllister for validating the Conceptual schema. On the other hand, the "Look across" cardinality constraint with Merise approximation shows us the primary key and, the candidate keys, if they exist. Furthermore, with this approximation we can get complex rules, because the value of an attribute in a relationship for a domain as minimum has to be n and as maximum m times, $\forall n, m \in \mathrm{N}$ (Al-Jumaily, T. H., 2006).

When there are optional elements in a ternary relationship, we have problems in its transformation. A solution is the Complementary binary relationship (Cuadra, D., 2003). In this work, we propose also the Complete binary relationship. Both solutions were defined in the second section. The Complementary relationship is a good solution, because it has not redundancy. However, in the Complete, there is redundancy, but it will be good solution when there is decomposition.

Next, we show two algorithms of validation and simplification of ternary relationships. Theses algorithms are a modification of Santos, I, (Santos, I. et al., 2006). We begin by checking the schema semantic consistency. In a next step, we have to verify if the concepts are according to the definition and, there are not incompatibilities among the concepts and the schema.

In this paper, we analyze only the ternary or higher-grade relationship. For this when we find a ternary relationship in our model, R_i, we have to look for the SBC relationship with R_i. For each entity A_i related to R_i, we have to find other relationships R_j, with $R_i \neq R_j$, and the rest of entities $\{\mathrm{A}_1,...,\mathrm{A}_{i-1},\mathrm{A}_{i+1},...,\mathrm{A}_n\}$ which are related to R_i. These relationships that we find, they are candidates to be semantic related relationship to R_i, and for this, they can restrict the cardinality of the relationship R_i. When we have the relationships, we have to ask to the designer, because he/she has to decide the relationships, which are SBC.

The step next is to check the optional roles of the entities in the ternary relationship. Let be E_i, E_j and $\mathrm{E}_\mathrm{K} \in R_i$. If E_i has an optional role, then we build between E_i and E_j the Complementary binary relationship. However, if between E_i and E_j there is an implicit ternary relationship, then we can build the Complete binary relationship and we delete the implicit binary relationship.

Now we show the algorithm of validation of a relationship with optionality.

The first algorithm depicted in the figure two has the follow steps:

1. We get the ternary relationship and SBC relationships with the ternary to check, with the help of the designer.
2. Are there some optional elements in the ternary relationship?
3. If there are optional roles then we build the Complementary or Complete binary relationships.
4. We verify the Conceptual Design with the MX2 rule of McAllister. Do the relationships carry out MX2? We have to verify the rule $\left(3^n - 2^{n-1} + 1\right)/2$ times.

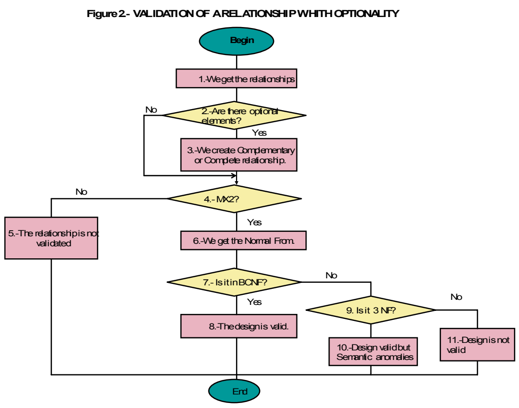

Figure 2 : Validation of Relationship with optionality.

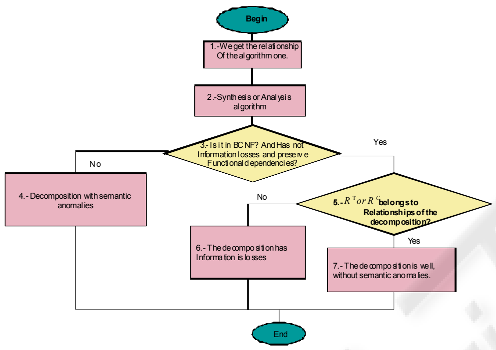

Figure 3 : Simplification Algorithm of a Relationship.

5. Design is incorrect. The E/R Model has to be redesigned.
6. We transform the design from E/R to R Model getting the functional dependencies. We can get the FD from "Look Across" cardinality constrains with the approximation of Chen. We have a FD if maximum cardinality is one. After, with the classic algorithms we verify the normal form of the schema.
7. Is the schema in the Boyce Codd Normal Form?
8. If the schema is in BCNF, the design is good and it has not semantic anomalies.
9. Is it in 3NF?
10. Design good, but with semantic anomalies. We could use the second algorithm.
11. Design with semantic anomalies. We ought to use the second algorithm.

If a ternary relationship with optionality can be decompose and the decomposition or simplification is in BCNF, this decomposition will always have information losses if the Complementary (R_i^C) or the Complete (R_i^T) relationship are not in the decomposition.

Theorem: Let be relation schema R and let be R^{T} a Complete binary relationship and let be R^C the Complementary. The decomposition is in BCNF. Let $R_1, R_2, ..., R_n$ be a set of relationships of the decomposition of R. The $R_1, R_2, ..., R_n \in R$ is information lossless and functional lossless. If R^T or $R^C \neq R_i$, where $i \in N$, then the decomposition is information loss.

Proof: If R^T or $R^C \neq R_i$ will not be the original tuples, because $R = R_1 \bowtie R_2 \bowtie, ..., \bowtie R_n$.

The algorithm implemented in the figure three shows the steps of simplification of a ternary relationship with or without optional elements.

1. We get the relations from algorithm first.
2. We apply the Analysis or Synthesis classic algorithms.
3. Is the result in BCNF? Does not it exists information losses and preserve functional dependencies?
4. The decomposition has semantic anomalies. It is not good solution.
5. Are there Complete relationships? These relationships belong to decomposition?
6. The decomposition has information losses.
7. The decomposition is valid.

We prefer the Complete relationship to the Complementary, only in this case, because if we use Complementary then we have to do an union between the Complementary and its corresponding R_i.

5 ANOMALIES IN THE COMPLEMENTARY RELATIONSHIPS

When there is a ternary relationship, with or without imposition binaries but with at least a Complementary binary relationship, the insertion, deletion, updated and selection operations have anomalies. We will analyze in this section these problems. For this analysis we have use the definitions of Millis W. Vincent (Millist W. Vincent, 1993, 1994, 1999).

Let be three entities E, P and T, a relationship R with attributes e, p, t and a Complementary relationship ought to T is optional. In the insertion, we will have to distinguish when we insert a null or not.

1. Let t^* =<e,p,null> be a tuple to insert and let r(R) the ternary relationship and let $r^c(R^c)$ be the Complementary relationship. If $\left(r \cup \{t^*\}\right) \in SAT(\Sigma_K)$ and $\left(r^c \cup \{t^*\}\right) \in SAT(\Sigma_K)$ then we can insert $t_c^* =< e, p >$ in R^c.
2. Let be t=<e,p,t>, $t \neq null$ and let r(R) be the ternary relationship and $r^c(R^c)$ the Complementary. If $\left(r \cup \{t^*\}\right) \in SAT(\Sigma_K)$ and $\left(r^c \cup \{t_c^*\}\right) \in SAT(\Sigma_K)$, then we can insert t^* in R.

The delete operations have two cases.

1. Let t^*=<e,p,t>, where t=null. If $\left(r^c - \{t_c^*\}\right) \in SAT(\Sigma_K)$, then we can delete t_c^* in $r^c(R^c)$.
2. Let t^*=<e,p,t>, where $t \neq null$. If $\left(r - \{t^*\}\right) \in SAT(\Sigma_K)$, then we can delete t^* of R.

However, the modification operations have four cases.

1. Let be t=<e,p,t> and t^*=<e', p', t'>, where t, t'$\neq null$. If $\left(\{t^*\} \cup (r - t)\right) \in SAT(\Sigma_K)$, then the modification is right and we update only the ternary.
2. Let be t=<e,p,t> and t^*=<e', p', t'>, where t and t'= null. If $(\{t_c^*\} \cup (r^c - t_c^*)) \in SAT(\Sigma_K)$ then the modification is possible, but in the Complementary.
3. Let be t=<e,p,t> and t^*=<e',p',t'>, where t=null and t'$\neq null$. If $\left(r^c - \{t_c^*\}\right) \in SAT(\Sigma_K)$ and $\left(r \cup \{t^*\}\right) \in SAT(\Sigma_K)$, then we can delete t^c of the Complementary and to insert t^* in the ternary.
4. Let be t=<e,p,t> and t^*=<e',p',t'>, where t$\neq null$ and t'=null. If $\left(r - \{t^*\}\right) \in SAT(\Sigma_K)$ and $\left(r^c \cup \{t_c^*\}\right) \in SAT(\Sigma_K)$, then we can delete t of ternary and to insert t_c^* in the Complementary.

The operation of selection is more complex:

Select e, p, t from ternary-relationship
Union
Select e, p, null from Complementary;

When there is decomposition and a binary relationship is overlapped by the Complementary relationship is better to replace the relationships by the Complete relationship.

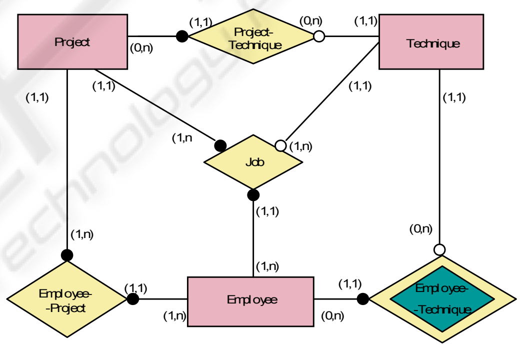

Figure 4 : Case (i).

6 VALIDATION AND REFINING OF TERNARY RELATIONSHIP AND ITS DECOMPOSITION WITH OPTIONALITY

Let be a company that wants to manger the jobs and employees. The management has imposed the next constraints:

- An employee that works in a project, he can only use a technique.
- An employee that works at a technique, he can only work at a project.
- An employee can only work at a unique project.

- In a project is only possible use a technique.
- The last constraint, we distribute in two exclusive cases in our example:
  (i) It can have employees with projects that they do not have allocated technique.
  (ii) It can have employees that use a technique, but they are not allocated to any project.

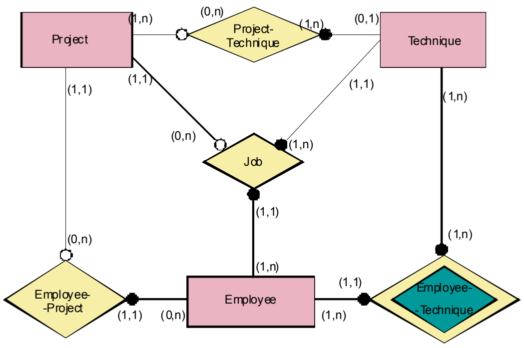

Figure 5 : Case (ii).

We show in the figure fourth and fifth the E/R model of this example. In the figure sixth (case (i)) and seventh (case (ii)) is the solution to optional elements, with the Complementary binary relationship. From the figure forth, we depicts the "Employee-Technique" relationship with double rhombus, because it is a deduced relationship.

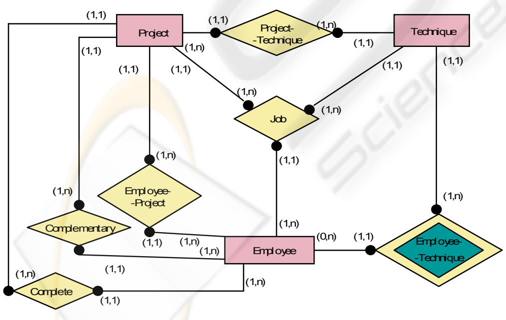

Figure 6 : We transform the optionally case (i).

Through the algorithm one, we verify the relationships and we look at the redundancies.

We can notice that in both cases that carry out the augmentation rule (MX2 rule).

If we get the functional dependencies, in both cases we have: Σ = {(Employee, Project $\rightarrow$ Technique; (Employee, Technique) $\rightarrow$ Project; Employee $\rightarrow$ Project; Project $\rightarrow$ Technique}. The Key is S_K ={Employee} and the minimum cover R_m = {Employee$\rightarrow$Project, Project$\rightarrow$Technique}. The resulting schema is in the 2NF.

If we apply the second algorithm, then we will get two relations; R_1 ={Employee, Project} with Σ_1 = {Employee$\rightarrow$Project} and R_2 ={Project, Technique} with Σ_2 ={Project$\rightarrow$ Technique}.

If we go on the case (i), we replace R_1 ={Employee, Project} and the Complementary relationship by the Complete relationship. In the figure eight depicts this case.

However, in the case (ii), this is not right, because, we lose data. The Complementary is not in the decomposed relationships. Furthermore, we can not select R_1 ={Employee, Project} with Σ_1 = {Employee$\rightarrow$Project} and R_3 = { Employee, Technique} with Σ_3 = { Employee$\rightarrow$ Technique} because we have dependency loss in this decomposition. Then, the decomposition is not valid, as we depict in the figure nine.

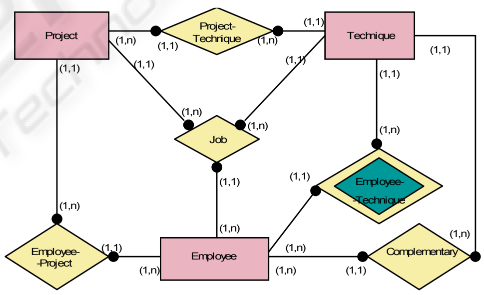

Figure 7 : We transform the optionally Case(ii).

We can resume that the decomposition or simplification of the relationships, although this is in

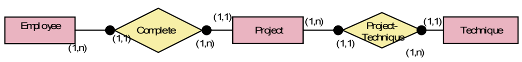

Figure 8 : Simplification of Case (i).

FNBC, if in the decomposition are not complementary relationship, the decomposition is not valid.

7 SOME CONCLUSIONS

The design of a Database is a complex work. The CASE tools help to simplify validation, verification and simplification of a Database design. However, these tools do not implement theses properties or they are very far away.

This paper is based on the ternary relationships, but it ought to be extended to higher grade ones. Two algorithms for verifying, validating and decomposing relationships with or without optional elements have been shown. However, this work is limited to functional dependencies and not to multivalued (MVDs) or join (JDs) dependencies. On

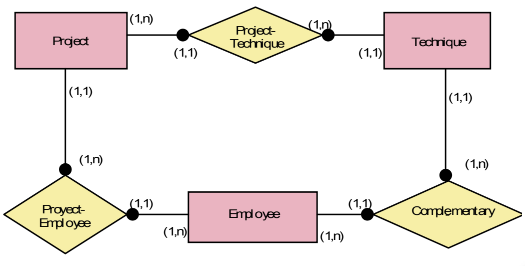

Figure 9 : We transform the optionally case(i).

the other hand, we can conclude that sometimes the simplification is not the better solution, because of the anomalies.

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