# APPLICATION OF MODAL ANALYSIS FOR EXTRACTION OF GEOMETRICAL FEATURES OF BIOLOGICAL OBJECTS SET

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Keywords: 3D geometry reconstruction, anthropometric measurements, PCA (Principal Component Analysis), registration, reverse engineering.

Abstract: This article presents application of modal analysis for the computation of data base of biological objects set and extraction of three dimensional geometrical features. Authors apply two types of modal analysis: physical (vibration modes) and empirical (PCA – Principal Component Analysis) for human bones. In this work as the biological objects the fifteen human femur bones were used. The geometry of each bone was obtained by using of 3D structural light scanner. In this paper the results of vibration modal analysis (modes and frequencies) and PCA (mean shape and features – modes) were presented and discussed. Further the possibilities of application of empirical modes for creation three dimensional anthropometric data base were presented.

## 1 INTRODUCTION

Nowadays, many engineering CAD technologies have an application not only in mechanics but also in different disciplines like biomechanics, bioengineering, etc. This interdisciplinary research takes advantage of reverse engineering, 3D modelling and simulation, PCA analysis and other techniques. The 3D virtual models have a numerous applications such as visualisation, medical diagnostics (e.g. virtual endoscopies), pre-surgical planning, FEM analysis, CNC machining, Rapid Prototyping, etc. Several engineering technologies can be used for analysis of biological objects.

Usually the populations of the biological objects like bones, are used to be described only in two dimensional space, by the set of the dimensions (e.g. distance). Thereby traditional anthropometric data base contains information only about some characteristic points, while other parameters are not collected. Generally data acquisition process is made with usage of the conventional measurements equipment (e.g. calliper). For any new research work (when not existing parameter is needed) completely new study and measurements process must be done.

The new methods of statystical analysis and storage of complete parametric data for each of all elements from population are researched. The methods which can be used to describe a geometrical parameters of 3D objects are modal analysis.

## 2 MODAL ANALYSIS METHODS

In this chapter authors present modal analysis methods which are used for geometry description of three dimensional objects and data base creation. These methods are used to simplify and minimize the number of parameters which describe 3D objects.

One of the methods that is based on modal decomposition is PCA (Principal Component Analysis, known also as POD – Proper Orthogonal Decomposition). While empirical modes (PCA) are optimal in the sense of information included inside each of the modes (Holmes, Lumley and Berkooz, 1998), often other decompositions, based on mathematical (e.g. spherical harmonics) or physical modes (vibration modes) are used. The kind of



Rychlik M., Stankiewicz W. and Morzyński M. (2008).
APPLICATION OF MODAL ANALYSIS FOR EXTRACTION OF GEOMETRICAL FEATURES OF BIOLOGICAL OBJECTS SET.
In *Proceedings of the First International Conference on Biomedical Electronics and Devices*, pages 227-232
DOI: 10.5220/0001048902270232

modal method (mathematical, physical or empirical) which is applied to analysis has a fundamental importance for results.

The goal of using mathematical modes is conversion of physical features onto mathematical features (synthetic form). In the case of the mathematical modes the features which describe geometry of 3D object are usually saved as the vectors. Each vector is obtained through splitting of the 3D model onto several classes (different diameter spheres) and calculation of common areas between 3D object and surfaces of individual spheres. All areas are described by a set of vectors (spherical functions). For spherical functions Fourier transformation is used, resulting in easier multidimensional description of feature vectors. For representation of feature vectors spherical harmonics are used (figure 1.).

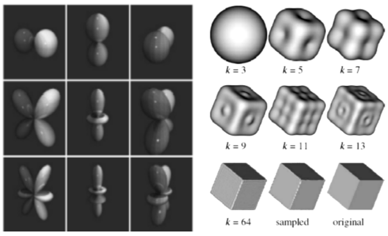


Figure 1: Example of spherical harmonics of 3D model of aeroplane and application of spherical harmonic in reconstructtion of geometry of the cube (Vranic and Saupe, 2002).

Application of spherical modes is not optimal solution and sometimes causes increased computation costs, because all objects are approximated by deformed sphere. Reconstruction of geometry of the cube requires very large number of spherical harmonics. This problem is analogous to Fourier decomposition of rectangular signal.

The second group of modal decompositions of 3D objects is represented by physical (mechanical) modes. These modes – also known as the vibration modes – are obtained by solution of eigenproblem for elastic model of analyzing object. Vibration modal decomposition provides alternative parameterisation of degrees of freedom of the structure (translations of the nodes in x, y, z directions only) based on eigenmodes of the objects and correlated frequencies (eigenvalues). Usually eigenmodes related with low frequencies, describing deformation vectors for individual nodes of FEM grid, are used. This way the deformation of geometry of base object and its fitting into searched object is possible. Vibration modes computed for rigid body represent translations and rotations of 3D model and vibration modes of elastic body describe different variations of the base model's shape (figure 2.).

Figure 2: Graphical representation of seven low frequency vibration modes for surface model of ellipsoid (Syn and Prager, 1994).

PCA transformation gives orthogonal directions of principal variation of input data. Principal component which is related with the largest eigenvalue, represent direction in data space of the largest variation. This variation is described by eigenvalue of largest magnitude. The second principal component describes the next in order, orthogonal direction in the space with the next largest variation of data. Usually only a few first principal components are responsible for a majority of the data variations. The data projected onto other principal components often have small amplitude and can be treated as measurement noise. Therefore, without the loss of accuracy, components related to smallest eigenvalues can be ignored.

## 3 PHYSICAL MODES – VIBRATION MODES

Decomposition basis on vibration modes uses similar procedure like in analysis of dynamical problems. For describing of complicated moving they used set of simple functions (1):

$$u(x, y, z, t) = \sum_{n=1}^{N} q_n(t)\phi_n(x, y, z) \qquad (1)$$

where $N$ is the number of used functions, $\phi_n$ is the vector (mode) of the object's vibration, and $q_n$ is coefficient for $n$-th mode in time $t$.

Linear elastic structures can be described by surface or volume finite elements. After discretization in FEM software the eigenanalysis is done, using the mathematical oscillation model (2):

$$M\ddot{u} + C\dot{u} + Ku = f(t) \qquad (2)$$

where $M, C$ and $K$ are adequately: mass, damping and stiffness matrix, and $u$ is vector of grid node displacements.

### 3.1 FEM Model

Computations have been done using NASTRAN software. Model geometry is based on surfaces resulting from 3D-scanning of real human femoral bone. Finite element mesh consists of approximately 4800 nodes and 5500 elements of two types. External layer of 1940 triangular plate elements represents compact (cortical) bone and 2560 tetrahedral elements represent internal, trabecular bone. Model was fixed in condyles part.

In both cases, orthotropic material, based on measurement data (Ogurkowska et al, 2002), was used.

### 3.2 Eigenmodes

The result of eigenproblem solution is a set of eigenvalues (representing the vibration frequencies) and eigenmodes. The eigenmodes related with lowest frequencies, added to mean shape of femur, are presented at figure 3.

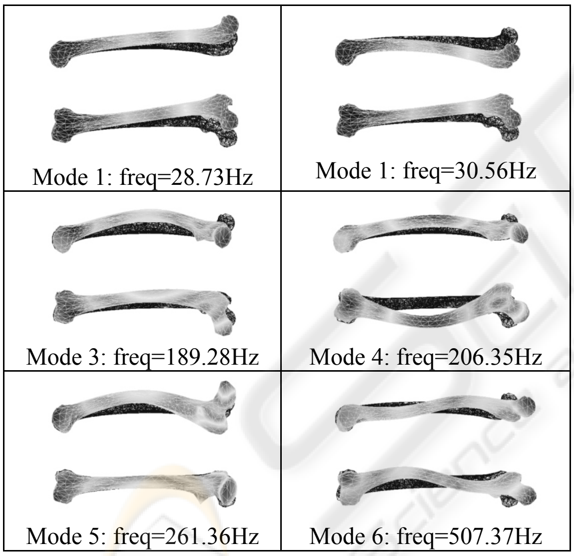


Figure 3: Eigenmodes related with lowest frequencies.

Gray scale levels represent total translation and dark bone is a mean, undeformed shape. Each bone is presented in two views: posteriori and anterior view.

Practical application of vibrational modes is strongly limited due to high number of modes required to reconstruct different geometries. Additionally eigenmodes don't represent any biophysical features of human femoral bones.

## 4 EMPIRICAL MODES – PCA

Despite the fact that the method is called differently in various application areas, the used algorithm is generally the same and is based on statistical representation of the random variables.

The shape of the every object is represented in the data base as the 3D FEM grid and described by the vector (3)

$$S_i = \left[s_{i1}, s_{i2}, \ldots, s_{iN}\right]^T, \; i = 1,2,\ldots,M, \tag{3}$$

where $s_{ij} = (x, y, z)$ describes coordinates of each of the nodes of FEM grid in Cartesian coordinates system. $M$ is the number of the objects which are in database, $N$ is the number of the FEM nodes of every single object. The decomposition is based on computation of the mean shape $\overline{S}$ and covariance matrix $C$ (4):

$$\overline{S} = \frac{1}{M}\sum_{i=1}^{M} S_i \,,\; C = \frac{1}{M}\sum_{i=1}^{M} \widetilde{S}_i \widetilde{S}_i^{\,T} \tag{4}$$

The difference between mean shape and current object from data base is described by the deformation vector $\widetilde{S}_i = S_i - \overline{S}$. The statistical analysis of the deformation vectors gives us the information about the empirical modes. Modes represent the features: geometrical (shape), physical (density) and others like displacement and rotation of the object. Only few first modes carry most of the information, therefore each original object $S_i$ can be reconstructed by using some $K$ principal components (5):

$$S_i = \overline{S} + \sum_{k=1}^{K} a_{ki}\Psi_k \,,\; i = 1,2,\ldots,M, \tag{5}$$

where $\Psi_k$ is an eigenvector representing the orthogonal mode (the feature computed from data base), $a_{ki}$ is coefficient of that eigenvector and $i$-th data base model. The example of low dimensional reconstruction for three different values of the coefficient of the first mode is presented on the figure 4. For $a_{ki} = 0$ we obtain mean value, for different values we get new variants of object's shape.

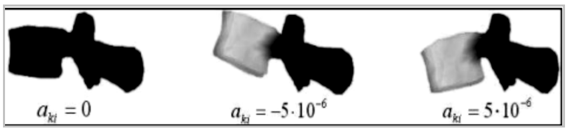


Figure 4: The visualisation of the reconstruction for different coefficient values.

## 4.1 Data Acquisition – 3D Scanning

As the input data (data base) 15 femur bones were measured (6 female, 9 male). For 3D scanning (Rychlik Morzyński and Mostowski, 2001) the structural light 3D scanner – accuracy 0,05mm – was used (figure 5). Each bone was described by individual point cloud (1.5mln points) and triangle surface grid (14000nodes, 30000 elements).

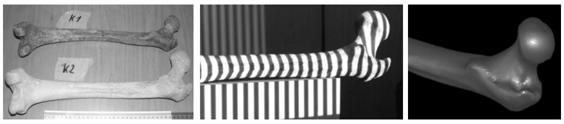

Figure 5: Data acquisition: a) input femur bones, b) measurement process, c) final triangle surface grid.

## 4.2 Data Registration

The Principal Component Analysis requires the same topology of the FEM mesh for all objects (the same number of nodes, connectivity matrix, etc.). To achieve this, every new object added to data base, must be registered. The goal of registration is to apply the base grid onto geometry of the new objects. The registration is made in two steps. First step (preliminary registration) is the rigid registration - a simple geometrical transformation of solid object in three-dimensional space (rotation and translation). The second step is the viscous fluid registration. For this registration the modified Navier-Stokes equation in penalty function formulation (existing numerical code: Morzynski, Afanasiev and Thiele, 1999; source segment: F Bro-Nielsen and Gramkow, 1996) is used (6):

$$\underbrace{\dot{V}_i + V_{i,j}V_j - \frac{1}{\mathrm{Re}}V_{i,jj} + \frac{\varepsilon - \lambda}{\rho}V_{j,ji}}_{\text{existing numerical code}} + \underbrace{(f - g)f_{,i}}_{\text{source segment}} = 0 \qquad (6)$$

where $\rho$ is fluid density, $V_i$ velocity component, Re Reynolds number, $\lambda$ bulk viscosity. In this application parameters $\varepsilon$ and $\lambda$ are used to control the fluid compressibility, $f$ is the base object, $g$ is the target object (input model). The object is described by the FEM grid. The displacements of the nodes are computed from integration of the velocity field. Computed flow field provides information about translations of the nodes (FEM grid) in both sections. After computation we obtain dislocation of nodes of the base grid onto new geometry (figure 6.).

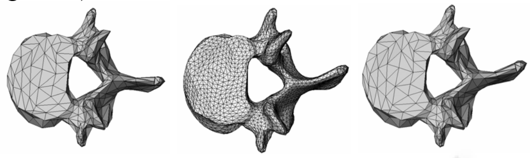

Figure 6: FEM grid deformation (from the left): base object, new object, base FEM grid on geometry of the new objects.

## 4.3 Empirical Modes – PCA

For that prepared database of 15 femur bones the Principal Component Analysis was done. The result of this operation is the mean object, fifteen modes and coefficients (figure 7).

Table 1: Participation of the modes in reconstruction.

| Number of the mode | Participation of the mode [%] | Total participation of the modes [%] |
|---|---|---|
| 1 | 74.9212416 | 74.9212416 |
| 2 | 10.5438352 | 85.4650767 |
| 3 | 4.2699519 | 89.7350286 |
| 4 | 3.3128685 | 93.0478971 |
| 5 | 1.6659793 | 94.7138765 |
| 6 | 1.4234329 | 96.1373093 |
| 7 | 1.0359034 | 97.1732127 |
| 8 | 0.6781645 | 97.8513772 |
| 9 | 0.5866122 | 98.4379894 |
| 10 | 0.4796167 | 98.9176061 |
| 11 | 0.3301463 | 99.2477523 |
| 12 | 0.3080968 | 99.5558492 |
| 13 | 0.2516839 | 99.8075330 |
| 14 | 0.1924670 | 100.0000000 |
| 15 | 0.0000000 | 100.0000000 |

The first fourteen modes include one hundred percent of information about decomposed geometry (table 1.). Fifteenth mode contains only a numerical noise and it is not used for further reconstruction.

Modes describe the features of the femur bones. First mode describes the change of the length of the femur bone, second mode – the change of the position of the head of the bone, third - change of the arc of the shaft (body). Further modes describe more complex deformations. For example fourth mode describes the change of position of the greater trochanter and lesser trochanter and also the thick-

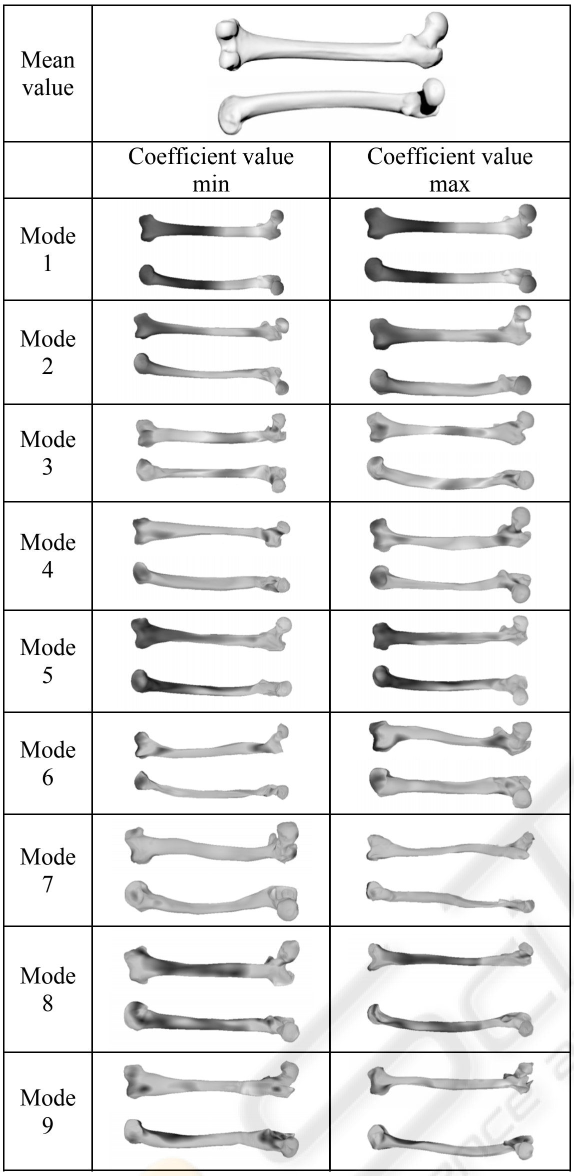


Figure 7: Visualisation of the mean value and first nine empirical modes of femur bones (anterior and posterior view).

ness of the shaft (body). Fifth mode describes deformation of the greater trochanter in other directions. Sixth describes deformation of the greater trochanter and lesser trochanter, and position of the shaft (body) in other directions.

Results of the statistical analysis (empirical modes) can be used for reconstruction of the geometry (in CAD systems) of individual features of the object. Empirical modes give as information about 3D mean shape of population of objects and a set of the geometrical features that describes principal deformations in analyzed population of the objects.

This method can be used for creation of complete 3D anthropometric database and gives us possibility to measure any dimension on the surface of the bone.

Real 3D anthropometric database is also necessary in practical application of the method of reconstruction of 3D biological objects basing on the few RTG images (Rychlik, Morzynski and Stankiewicz, 2005).

## 5 CONCLUSIONS

Although we can use several modal methods to describe the geometry of 3D objects, only empirical modes give us an optimal statistical data base.

Graphical representation of spherical harmonics is very specific and it is impossible to find similarity with input model, with exception of algebraic relations.

There are several differences between methods producing physical (vibration) modes and empirical modes (PCA, POD, Karhunen-Loeve), that are used in modeling of 3D objects.

A large limitation of usage of physical modes (vibration modes) is the impossibility to obtain the modes that describe resizing (scaling) of whole object or it's parts. These features are skipped out and they cannot be used in decomposition. The problem is also the large number of the modes that must be used in description of the shape of the object. Sometimes for reconstruction of a very simply geometry (e.g. cube) we must use a lot of modes (even up to 200 modes).

In case of physical modes, the only available determinant of mode's suitability is the vibration frequency (eigenvalue). One can assume that modes with high frequencies will represent numerical noise only, but the number of modes related with low eigenvalues that have to be used in reconstruction of another 3D object of the population is unknown. While the total number or eigenmodes is equal to the number of degrees of freedom of the model (in our case: 3x4800 DOF), the modal description of the population using physical modes might require larger data storage than input data (separate grids for each of the objects), and might still be incomplete (the scaling mentioned before).

Empirical modes describe features of the object that are dependent on frequent occurrences in population. The largest eigenvalues are related with modes describing the most important features, what makes the reduction of data storage quite simple.

For Karhunen-Loeve analysis of data base which consists of several similar, but not the same objects, differing from each other only in the scale, this feature (size of the object) will be the most dominant empirical mode. Additionally, the number of modes required to reconstruct the whole population of objects without quality losses is assured to be smaller or equal to the number of objects in that population. In practice, a number of empirical modes can be used to describe the population with accuracy higher than in case of any other modes (optimality of PCA mentioned before).

For empirical modes (in data base) it is possible to keep the additional information's, e.g. data from diagnostic systems, density, Young's modulus, and other material properties.

PCA can be used for creation of complete three dimensional anthropometric data base.

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