APPLICATION OF WEIGHTED LEAST SQUARES TO CALIBRATE A DIGITAL SYSTEM FOR MEASURING THE RESPIRATORY PRESSURES

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Abstract: This article presents the results associated to calibration and evaluation of the measurement uncertainty of a digital respiratory pressures measuring system developed at the Biomedical Engineering Research & Development Laboratory (NEPEB) of the Department of Electrical Engineering – UFMG. The proposed method uses the least squares weighted regression to establish the measurement model and to evaluate the uncertainty. The standard expanded uncertainty estimated by the model was 1.4 kPa.

1 INTRODUCTION

The qualitative and quantitative characterization of the result of a measurement is a demand of metrology organisms, providing knowledge about the reliability, important to quality control of products and services. In Brazil, the National Institute of Metrology, Standardization and Industrial Quality (INMETRO) is the local NMI (national measurement institute) and as such is organism responsible for metrology politics and for establishing criterions and general normalization relative to expression of uncertainty (INMETRO, 2007).

Brazilian Ministry of Health (MS) established the compulsory certification for medical and hospital equipment (MHE), not only attributing conditions for a major credibility of use, but, also causing a huge demand for calibration services in the country. A non-calibrated medical equipment can lead clinic diagnostic errors, wrong therapies and iatrogeny in patients.

Today, regulation of certification process of MHE quality in Brazil is defined by two normative documents: MS and ANVISA (1999) and MIDIC and INMETRO (2006). The former defines the strategies, while the latter describes the technical details for certification.

A digital system was developed at NEPEB (Silva, 2006) in order to measure the maximum pressures exerted by the muscles of the human respiratory system. It was designed as equipment to be used in diagnostic and therapy and, therefore, must be submitted to metrological evaluation according to the standards of the MS. Thus, the measuring system must be certificated, and this process requires *instrument calibration and the evaluation of the measurement uncertainty*.

The measuring system includes a signal acquisition module for pressure and an analogical to digital signal conversion module that allows the digital data to be acquired using a personal computer (PC) through an USB interface.

2 OBJECTIVES

The signal acquisition module of pressure, denominated manovacuometer, uses two sensors of differential pressure, model MPX5050 by Freescale Semicondutor Inc. (Austin, Texas). The sensor possesses two pressure sides (figure 1) and the measured value corresponds to the differential

pressure taken between the two sides. Nevertheless, it is important to emphasize that pressure in one side (P1) must be always higher than that in the other side (P2).
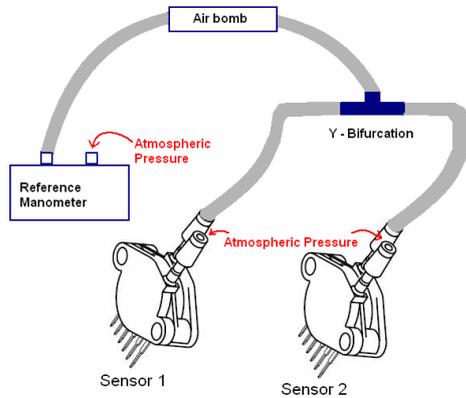
Figure 1: Schematic implementation for prototype calibration.

The purpose of this work is to carry out the calibration of digital manovacuometer developed by NEPEB. The proposed metrological model was built using the weighted least squares method, and, in that way, the measurement uncertainties was evaluated.

3 METHODOLOGY

3.1 Calibration Measurement System Protocol

In order to carry out the calibration of the digital manovacuometer, the protocol describe in (INMETRO, 1997) was used. Two were the motivation for the use of that procedure: (i) the sphygmomanometers with aneroid manometer was one of few clinic equipments that have normalization, regulation and control by INMETRO; (ii) although the normalization is specific for sphygmomanometers with aneroid manometer, this refers to the same type of physical quantity measured by the manovacuometer.

Tests were implemented to get the calibration curves for the two sensors of the developed equipment: i) the maximum indication error test and ii) the hysteresis test.

According to that procedure, the pressure applied to the sensors has to be increased and thereafter decreased. The pressure values (table 1) were applied during approximately five seconds, and an average voltage was evaluated at the output of the manovacuometer. The tests were performed four times, and a curve of the output voltage *versus* applied pressure, for each sensor, was plotted.

Table 1: Reference pressure values applied to prototype.

Pressure (kPa)		
4.0	9.3	12.0
13.3	20.0	26.7
33.3	40.0	46.7
53.3	60.0	–

Curve fitting was implemented using the method of weighted least squares (ABNT and INMETRO, 2003; Lira, 2002; Mathioulakis and Belessiotis, 2000; Press *et al.*, 1992). It was chosen to make the fitting for the average rising and fall curves of each sensor. Hence, two curves (rising and fall) were obtained for each sensor.

The instrument which measure the pressure applied to the prototype was a digital commercial manovacuometer (reference manometer) by Ecil (model BB480003) with reported maximum uncertainty of 0.03% (k=2), traceable to the SI standard. In figure 1, it is illustrated the schematic for calibration of the prototype, built in our laboratory.

3.2 Evaluating Measuring Uncertainty Protocol

The proposed model to calculate the measuring uncertainty, u_{Pc}, which is associated to the pressure corrected value, was built using the regression fitting obtained with the weighted least squares method. The u_{Pc} and the uncertainty of the reference mano-vacuometer lead to the evaluating of the standard combined uncertainty $u_c(P)$ and, consequently, to the expanded uncertainty U_p of the measured pressure with the prototype for a test pressure of 26.7 kPa.

4 RESULTS

Figure 2 shows the average calibration curves (rising and fall) for sensor 1 of the manovacuometer under calibration (similar calibration curves to sensor 2). For fitting implementation, the linear region (up to 53.3kPa) was used. For each calibration curve, the fitting was approximated by the equation:

$$P_{rc} = b + aV_m \qquad (1)$$

where P_{rc} corresponds to pressure given a voltage on the output of the manuovacuometer (V_m) under calibration.
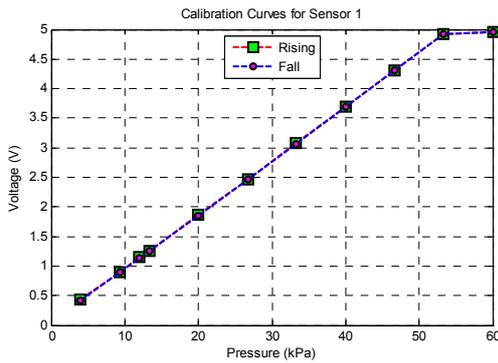
Figure 2: Calibration average curves for sensor 1.

In table 2, are given the reference pressure values associated to measured output voltage values (calibration points) and the respective standard combined uncertainty to rising curve of sensor 2.

Table 2: Reference pressure and output voltage values and associated uncertainties – sensor 2 (rising curve).

P_r (kPa)	u_{Pr} (kPa)	V_m (V)	u_{Vm} (V)
4.0	0.0006	0.4578	0.0024
9.3	0.0014	0.9257	0.0066
12.0	0.0018	1.1753	0.0022
13.3	0.0020	1.2909	0.0036
20.0	0.0030	1.8904	0.0035
26.7	0.0040	2.4955	0.0050
33.3	0.0050	3.0974	0.0047
40.0	0.0060	3.7094	0.0045
46.7	0.0070	4.3333	0.0034
53.3	0.0080	4.9410	0.0050

The uncertainty u_{Pr} associated to reference manovacuometer, was evaluated using the value of the related standard expanded uncertainty, as long as the uncertainty u_{Vm} was estimated based on fluctuation of the repeated readings in each calibration point (Mathioulakis and Belessiotis, 2000), correspondent to type A uncertainty.

The mathematics of linear regression fitting using weighted least squares is described with more details in Lira (2002), Mathioulakis and Belessiotis (2000) and Press *et al.* (1992). The slope *a* and the intercept *b* as well as the associated uncertainties u_a e u_b can be obtained from:

$$\left(K^T \cdot K\right) \cdot C = K^T \cdot L \qquad (2)$$

where, *C* is a vector whose elements are the fitted coefficients *a* and *b*; and $W = (K^T.L)^{-1}$ is a matrix whose diagonal elements are the variances of *a* ($w_{2,2}$) and *b* ($w_{1,1}$). The off-diagonal elements $w_{1,2} = w_{2,1}$ are

the covariances between these parameters. *K* and *L* are a matrix and a vector, respectively, whose elements are weighted by pounds that depend on the uncertainties u_{Pr} and u_{Vm}. Solving (2), one obtains the parameters and their uncertainties for each average curve. The results are shown in table 3.

Table 3: Values for the parameters *a* and *b* and its uncertainties u_a and u_b.

	Sensor 1 - Rising	Sensor 1 - Fall	Sensor 2 - Rising	Sensor 2 - Fall
a (kPa/V)	10.9747	10.9797	11.0159	10.9747
u_a (kPa/V)	0.0050	0.0034	0.0086	0.0026
b (kPa)	-0.5845	-0.5280	-0.9506	-0.7989
u_b (kPa)	0.0169	0.0079	0.0206	0.0077
Cov(b,a) (kPa/V)	-7.44×10^{-5}	-2.13×10^{-5}	-1.43×10^{-4}	-1.71×10^{-5}

Calculation of the uncertainty u_{Pc} associated to calibration points is derived from *the law of propagation of uncertainties* (ABNT and INMETRO, 2003) and equation (1):

$$u_{Pc} = \left(a^2 u_{V_m}^2 + u_b^2 + V_m^2 u_a^2 + 2V_m Cov(b,a)\right)^{1/2} \qquad (3)$$

The standard combined uncertainty, in turn, was obtained by:

$$u_c(P) = u_{Pc}^2 + u_M^2 \qquad (4)$$

where u_M is the uncertainty associated to reference manovacuometer.

The measuring uncertainty was calculated in relation to a calibration point (26.7 kPa) arbitrarily chosen of the average rising curve of the sensor 2. To this point, the average output voltage is $V_m = 2.4955$ V. The uncertainty u_{Vm} is estimated considering the type A uncertainty, u_V, the sensor accuracy, u_E, and the resolution of manovacuometer under calibration, u_R:

$$u_V = \frac{0.0100}{\sqrt{4}} = 0.0050V \qquad u_E = \frac{0.1125}{\sqrt{3}} = 0.0650V$$

$$u_R = \frac{0.0001}{\sqrt{3}} = 0.00006V \quad u_{Vm}^2 = u_V^2 + u_E^2 + u_R^2 \Rightarrow u_{Vm} = 0.0652V$$

Taking account these values, those of third column of table 3 and (3) result $u_{Pc} = 0.7183$ kPa. For the reference pressure of 26.7 kPa, the value of u_M is equal to 0.0040 kPa. Hence, the standard combined uncertainty is estimated as 0.7 kPa. Finally, the value for the standard expanded uncertainty is

evaluated using the expression of Welch-Satterthwaite (ABNT and INMETRO, 2003).

For the confidence interval of 95.45%, the obtained value for the effective number of degrees is $v_{eff} \rightarrow \infty$, which indicates a coverage factor of $k=2$. Therefore, the estimated value for the standard expanded uncertainty is equal to $U_p=1.4$ kPa.

5   DISCUSSIONS

The repeatability of measurement of calibration curves, as discussed in other works of NEPEB, was confirmed here. The maximum value obtained for the type A uncertainty associated to the values of V_m is equal to 0.0066 V.

Calculations of hysteresis were performed considering the fitting curves. The maximum absolute value obtained for hysteresis was equal to 0.1467 kPa for sensor 1 and 0.2957 kPa for sensor 2. These values are much lower than the value established by INMETRO for sphygmomanometers with aneroid manometer. In the same way, when is considered the linear region, it is noticed that the results obtained in maximum indication error test are also inside the tolerance range determined by INMETRO for this equipment (0.4 kPa).

The standard combined and expanded uncertainties estimated for the prototype (for reference pressure of 26.7 kPa) were 0.7 kPa and 1.4 kPa, respectively. The last value is lower than that suggested in INMETRO (2006) for analogical pressures measuring systems, that is, 12 kPa.

6   CONCLUSIONS

The proposed model to calibrate the digital manovacuometer developed at NEPEB uses the weighted least squares. This model indicates that the hysteresis, maximum indication error and uncertainty of the prototype were inside the tolerance range established by INMETRO.

In future works it must be investigated and inserted to the proposed metrological model other possible factors that can be influencing the result prototype uncertainty, as those associated to A/D converter, temperature variation and correlation between the curves parameters and input pressure.

REFERENCES

ABNT; INMETRO, 2003. *Measurement uncertainty guide* (free translation). ABNT, INMETRO, Rio de Janeiro.

Brazilian Ministry of Development, Industry and Foreign Commerce (MIDIC); INMETRO, 2006. Establishes the evaluation of conformity for MHE. Portaria Nº 86 de 3 de abril de 2006.

Brazilian Ministry of Health (MS); Brazilian National Health Vigilance Agency (ANVISA), 1999. Establishes the certification for MHE. Resolução Nº 444 de 31 de agosto de 1999.

INMETRO, 2007. Homepage: http://www.inmetro.gov.br

INMETRO, 1997. *Procedure for verification of the sphygmomanometers with aneroid manometer* (free translation). INMETRO, Rio de Janeiro.

INMETRO, 2006. *Guidelines to carry out the calibration of analogical system measuring pressure* (free translation). INMETRO, Rio de Janeiro.

Lira, I., 2002. *Evaluating the measurement uncertainty:* fundamentals and practical guidance. Bristol and Philadelphia: IoP.

Mathioulakis, E.; Belessiotis, V. 2000. *Uncertainty and traceability in calibration by comparision*. Meas. Sci. Technol., 11: 771-775

Press, W.; Teukolsky, S.; Vetterling, W; Flannery, B. 1992. *Numerical recipes in C*: the art of scientific computing. 2nd ed. Cambridge University Press, Cambridge, 2nd edition.

Silva, J, 2006. *Development of a digital system for measuring maximum respiratory pressures* (free translation). Monografia (Bacharelado em Engenharia de Controle e Automação) – Escola de Engenharia, Universidade Federal de Minas Gerais, Belo Horizonte.