ECG SIGNAL DENOISING
Using Wavelet in Besov Spaces

Shi Zhao, Yiding Wang
Graduate University of Chinese Academy of Sciences, Zhongguancun East Road, Beijing, China

Hong Yang
Graduate University of Chinese Academy of Sciences, Zhongguancun East Road, Beijing, China

Keywords: ECG, noise reduction, wavelet, Besov, nonlinear shrinkage function.

Abstract: This paper proposes a novel technique to eliminate the noise in practical electrocardiogram (ECG) signals. Two state-of-the-art denoising techniques, which both based on wavelet bases, are combined together. The first one is discussing wavelet bases in Besov spaces. Compared to traditional algorithms, which discuss wavelets in $L^2(R)$ spaces, the proposed technique projects ECG signals onto Besov spaces for the first time. Besov space is a more sophisticated smoothness space. Determining the threshold of shrinkage function in Besov space could eliminate Gibbs phenomenon. In addition, instead of using linear shrinkage function, the proposed algorithm uses nonlinear hyper shrinkage function, which is proposed by Poornachandra. The function tends to keep a few larger coefficients representing the function while the noise coefficients tend to be reduced to zero. Combining the two techniques, we obtain a significant improvement over conventional ECG denoising algorithm.

1 INTRODUCTION

Removing noise is an pertinent problem in ECG signals processing. Usually, there are two kinds of noises in ECG, power line frequency noise and white noise. Power line frequency noise can be regarded as the result of an electromagnetic compatibility issues: background electromagnetic field interference from surrounding equipments and from buldings and power conductors. White noise is usually considered from the measure equipment.

Previously, different filters based on Fourier bases are used to eliminate the noises, such as notch filter. The problem of these methods is that they could not reduce the two kinds of noises at the same time. In addition, because the notch has a relatively large bandwidth, which means that the other frequency components around the desired null are severely attenuated, this method brings in signal distortions. In 1995, Donoho (David L Donoho, 1995) proposed a novel denoising algorithm based on wavelet shrinkage. It provides excellent performance and since then, wavelets became a state-of-the-art denoising method. Before long, P. M Agante (P M Agante, 1995) applied soft-threshold method in ECG and achieve good results. However, traditional wavelet method has its drawbacks. They are not shift invariant; therefore, for the signals not smooth enough, it will appear Gibbs Oscillation phenomenon at the location where the signal is sharp changed. In ECG signals, there are R waves, which change sharply. As a result, Traditional wavelet denoising algorithm brings in Gibbs oscillation after R waves.

In this paper, we apply two techniques to eliminate the noise and restrain the Gibbs phenomenon at the same time. First, we determine the threshold of wavelet shrinkage function in Besov spaces. Besov space $B_q^\alpha(L^P)$ is a smoothness space with $\sigma > 0$, $(p,q) \in [1,+\infty)^2$, it is defined by

$$B_q^\alpha(L^P) = \{f \in L^p(R) \mid \|f\|_{B_q^\alpha(L^P)} < \infty\} \tag{1}$$

Where the Besov seminorm $\|\cdot\|_{B_q^\alpha(L^P)}$ is linked to the smoothness modulus of the considered function. Besides that, in stead of linear shrinkage function, we use nonlinear shrinkage model (S. Poornachandra, 2007). Combining the two novel

Zhao S., Wang Y. and Yang H. (2008).
ECG SIGNAL DENOISING - Using Wavelet in Besov Spaces.
In *Proceedings of the First International Conference on Bio-inspired Systems and Signal Processing*, pages 250-254
DOI: 10.5220/0001058302500254

techniques, we obtain a significant improvement over conventional wavelet denoising algorithm. In order to certify our idea, the noises in ECG signals in our experiment are not added by hand. They are from actual interfering. We collect the ECG signals with noises by our own devices.

2 INTRODUCTION TO WAVELET SHRINKAGE FUNCTION IN BESOV SPACE

Wavelet is defined as orthonormal basis functions for the expansion of functions belonging to various function spaces. Usually, it is the space of squared integrable real functions $L^2(R)$ (functions with finite energy). Recently, it has been shown that more sophisticated smoothness spaces, such as Besov spaces, provide a suitable and more refined characterization of real-life signals (Kathrin Berkner, 2000). The wavelet series representation of a function $f(t) \in L^2(R)$ could be express as

$$f(t) = \sum_{k \in Z}^{2^{j_0}-1} c_{j_0 k}\varphi_k(t) + \sum_{j=j_0}^{\infty}\sum_{k \in Z}^{2^{j_0}-1} d_{jk}\psi_{jk}(t) \tag{2}$$

φ is called farther wavelet and ψ is called mother wavelet. $\varphi_{j,k}(t)$ and $\psi_{j,k}(t)$ are the dilation and translation of the wavelet function.

$$\varphi_{j,k}(t) = 2^{j/2}\varphi(2^j - k) \tag{3}$$

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j - k) \tag{4}$$

j , k are the scaling and translation parameters respectively, $j,k \in Z$, $2^{j/2}$ could maintain the unity norm of the basis function at various scales. The coefficients $c_{j_0 k} = \langle g, \varphi_{j_0 k}\rangle$ and $d_{jk} = \langle g, \psi_{jk}\rangle$. Often we set $j_0 = 0$, and in that case there is only one scaling coefficient. The wavelet series are usually discussed in $L^2(R)$ spaces, but in our research, we use a more sophisticated set of functions, Besov spaces $B_q^\alpha(L^P)$ ($0<\alpha<\infty$, $0<p\le\infty$, $0<q\le\infty$). In Besov spaces, for a function $f \in B_q^\alpha(L^P)$, its norm could be defined using its wavelet coefficients as (5) (Kathrin Berkner , 2000)

$$\| f \|_{B_q^\alpha(L^P)} = \left(\sum_k | c_{j_0 k} |^p\right)^{\frac{1}{p}} + \left(\sum_{j>j_0}\left(2^{j(\alpha p + p/2 - 1)}\sum_k | d_{jk} |^p\right)^{q/p}\right)^{\frac{1}{q}} \tag{5}$$

The three Besov parameters have natural interpretations: a p -norm of the wavelet coefficients is taken within each scale j , a weighted q -norm is taken across scale, and the smoothness parameter α controls the rate of decay of the d_{jk} , increasing α corresponds to increasing smoothness.

Based on reference (Antonin Chambolle, 1998), the denoising problem could be described as follow. Given a positive parameter λ and a signal f , find a function $\tilde{f}$ that minimize over all possible function the functional

$$\| f \|_{B_q^\alpha(L^P)}^q + \frac{\lambda}{2}\| f - f_0 \|_{L^2}^2 \tag{6}$$

Choose a proper λ , the $\tilde{f}$ could be the denoising signal of f . For simpleness, we set Besov parameters $p = q = 1$. Then the problem could be expressed as follow:

$$\min \sum_{j,k} 2^{j(\alpha - 1/2)} | d_{j,k} | + \frac{\lambda}{2}\sum_{j,k}\left(d_{j,k}^0 - d_{j,k}\right)^2 \tag{7}$$

That means for each j , k , we estimate the $\hat{d}$ use follow expression:

$$\begin{aligned} \hat{d} &= \arg\min_d \frac{\lambda}{2}(d_0 - \lambda)^2 + 2^{j(\alpha - 1/2)} | d | \\ &= sign(d_0) \cdot \max(| d_0 | - 2^{j(\alpha - 1/2)} / \lambda) \end{aligned} \tag{8}$$

That means the ECG signal has small Besov norm if the wavelet coefficient in each scale have small l_1 norms and those l_1 norms decay rapidly across scale.

Note that any wavelet basis having $r > \alpha$ vanishing moments can be used to measure a Besov norm (Hyeokho Choi, 2004).

3 INTRODUCTION TO NONLINEAR SHRINKAGE MODEL

Donoho and Johnstone were first to formalize the wavelet coefficient thresholding for removal of additive noise from deterministic signals (David L Donoho, 1995). Wavelet thresholding is based on the property that typical real-world signals have sparse representations in the wavelet domain. The small coefficients are usually correlated to noise. Therefore, by choosing an orthogonal basis, which could efficiently approximates the signal with few nonzero coefficients; we could choose a particular threshold and set the coefficient bellow the threshold to zero. Using these coefficients in an IDWT to reconstruct the data, we could kill the noise.

The shrinkage function proposed by Donoho and Johnstone are the hard and the soft shrinkage function. Hard thresholding simply sets the coefficients below a threshold T to zero, as (9). Soft thresholding first shrinks each coefficient by T and then hard thresholds, as (10).

$$\delta_T^H(x) = \begin{cases} 0, & |x| \le T \\ x, & |x| > T \end{cases} \tag{9}$$

$$\delta_T^S(x) = \mathrm{sgn}(x)\left(|x| - T\right)_+ \tag{10}$$

Both hard and soft shrinkages have their disadvantages. Due to the discontinuities of the shrinkage function, hard shrinkage estimate tends to have bigger variance and can be unstable, that is, sensitive to small changes in the data. The soft shrinkage estimate tends to have bigger bias, due to the shrinkage of large coefficients (S. Poornachandra, 2007).

To overcome the drawbacks of hard and soft shrinkage, we decide to use nonlinear shrinkage function. There are two kinds nonlinear shrinkage estimate in our experiment. The first is called nonnegative garrote shrinkage function (M. Vetterli, 1995), which was first introduced by Breiman (1995) as follow:

$$\delta_\lambda^G(x) = x\left[1 - (\lambda / x)^2\right]_+ \tag{11}$$

The shrinkage function $\delta_\lambda^G(x)$ is continuous and it provides a good compromise between the hard and the soft shrinkage functions. It is less sensitive than hard shrinkage to small fluctuations and less biased than soft shrinkage. The second shrinkage function is called hyper shrinkage, which is proposed by S. Poornachandra as follow:

$$\delta_\lambda^{hyp}(x) = \tanh(\rho * x)\left(|x| - t\right)_+ \tag{12}$$

The major advantage of hyper shrinkage is its nonlinearity, that is, the function in wavelet domain tends to keep a few larger coefficients representing the function while the noise coefficient tend to be reduced to zero.

4 NOISE REDUCTION BY OUR METHOD

The objective of this paper is to eliminate the noise buried in practical ECG signals. In our research, we combine the two techniques we mention above. First, we determined the threshold of shrinkage function for each level in Besov spaces. It is obviously that for each subband, the parameter α should be different. We set α_j for each level experimentally. Then we use the two kinds of nonlinear shrinkage functions to obtain the estimated coefficients. Finally, using these coefficients the original ECG signal is thus recovered. The general process is showed bellow. The decomposition level is 6.

Step 1. Choose db3 wavelets, and do DWT.

Step 2. Choose α at each level. For the fist level $\alpha_0 = 0.9$, and $\alpha_j = \alpha_0 + 0.25 * sqrt(\log(j+2))$ for each level.

Step 3. Determine the threshold based on the α_j .

Step 4. Apply hyper shrinkage function and the estimated coefficients obtained.

Step 5. IDWT use the estimated coefficients.

5 SIMULATIONS AND RESULTS

In our research, the ECG signals are obtained by our own devices. Each piece of signal is about 1 min long. The sampling rate is 1200Hz.

In our research, we use five different denoising methods. We show original signal and the processed 4 signals and their spectrums in Fig.1 to Fig.6. In order to see clearly, we show their details of the sample points around R waves. The method in Fig.2 determines the threshold in $L^2(R)$ spaces and use hard thresholding shrinkage function, while in Fig.3 the thresholds is determined in $L^2(R)$ spaces and use soft thresholding method. The other three discuss the thresholds in Besov spaces. Whereas Fig.4 uses soft shrinkage function, Fig.5 use nonnegative garrote shrinkage function and the last one uses hyper shrinkage function.

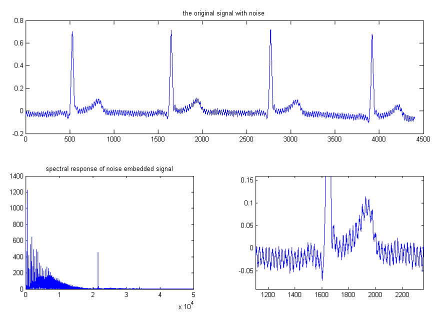

Figure 1: The original signal and its spectrum.

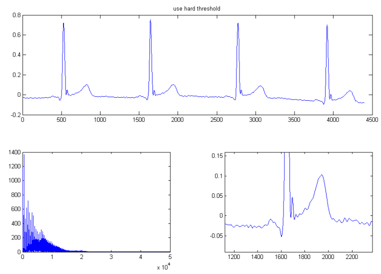

Figure 2: Determine the threshold in $L^2(R)$ and use hard thresholding.

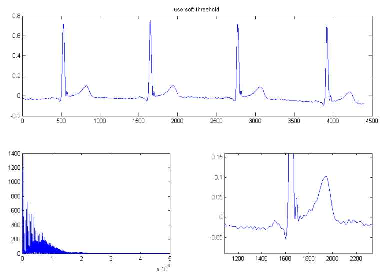

Figure 3: Determine the threshold in $L^2(R)$ and use soft thresholding.

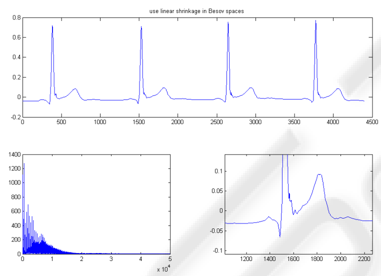

Figure 4: Determine the threshold in Besov spaces and use soft thresholding.

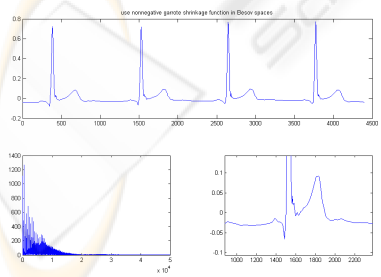

Figure 5: Determine the threshold in Besov spaces and use nonnegative garrote shrinkage function.

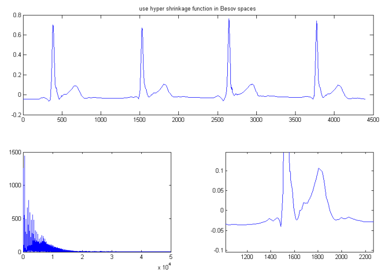

Figure 6: Determine the threshold in Besov spaces and use hyper shrinkage function.

As we seen from the pictures above, combined with threshold determined in Besov spaces and hyper shrinkage function, the recovered signal is the most visually pleasant. The proposed technique almost eliminate Gibbs phenomenon. To describe the oscillation of the recovered signal quantificational, we calculate the total variation of the six signals. Total variation for a uniform sampling discrete signal f is defined as (S. Mallat, 1998).

$$\| f_N \|_V = \sum_n | f_N[n] - f_N[n-1] | \quad (13)$$

Where $\| f_N \|_V$ is the Total Variation. In order to certify the effectiveness of the proposed method, we give 4 pieces of signals' Total Variation. They are show in Table 1.

Table 1: Total Variation of the signals.

	1	2	3	4	average
T1	0.3914	0.3747	0.3801	0.3875	0.3834
T2	0.1789	0.1654	0.1388	0.1252	0.1521
T3	0.1789	0.1654	0.1388	0.1252	0.1521
T4	0.1721	0.1517	0.1006	0.1177	0.1355
T5	0.1758	0.1504	0.1030	0.1206	0.1374
T6	0.1431	0.1371	0.0803	0.0919	0.1131

In the above table, T1 means the original signals' Total Variation. T2 to T6 correspond Fig.2 to Fig.5. In the table, we could notice easily that discussing threshold in Besov space and using nonlinear shrinkage function could obtain good results. And among those, hyper shrinkage is the most effective, it has the least oscillation.

6 CONCLUSIONS

This paper proposes a novel approach to eliminate the noises in practical ECG Signals. First, we use the characterization of Besov space, which is a smoothness spaces, through wavelet decompositions. Then we apply nonlinear shrinkage function instead of linear shrinkage function. The experiment results show that the proposed algorithm is visually pleasant compared to traditional methods. It could eliminate the noise successfully, and at the same time, it suppresses Gibbs oscillation. The proposed technique has potential application in data acquisition systems, which are generally encountered by noise.

ACKNOWLEDGEMENTS

This research is supported by High Technology Research and Development Program of China (863 Program): 2006AA01Z133. The ECG signals collection device is designed by Shen Yadong, who is a graduate student in Tsinghua University, China.

REFERENCES

S. Poornachandra, N. Kumaravel, 2007. A novel method for the elimination of power line frequency in ECG signal using hyper shrinkage function. *Digital Signal Process*, doi:10.1016/j.dsp.2007.03.011.

S. Mallat, 1998. *A Wavelet Tour of Signal Processing*. Academic Press. San Diego, 2nd edition.

David L Donoho, 1995. De-noising by soft thresholding. *IEEE Transactions on Information Theory*, 41(3): 613-627.

P M Agante, J P Marques de Sa, 1995. ECG noise filtering using wavelets with soft-threshold method. *IEEE Computers in Cardiology*, 26:535-538.

M. Vetterli, J. Kovacevic, 1995. *Wavelet and Subband Coding*. Prentice Hall International, Englewood Cliffs, NJ.

Kathrin Berkner, Michael J. Gormish, Edward L. Schwartz, and Martin Boliek, 2000. A new wavelet-based approach to sharpening and smoothing of images in Besov spaces with applications to deblurring. *Proceedings. 2000 International Conference on Image Processing*, Vol 3: 10-13

Hyeokho Choi, Richard G. Baraniuk, 2004. Multiple wavelet basis image denoising using Besov ball projections. *IEEE signal processing letters*, Vol. 11. NO.9.

D. Leporini, J. C. Pesquet, 2000. Bayesian wavelet denoising: Besov priors and non-Gaussian noises. *Elsevier Science Signal Processing*,81: 55-67.

Alexandre Almeida, 2004. Wavelet bases in generalized Besov spaces. *Elsevier mathematical analysis and applications*, Appl.304: 198-211.

Antonin Chambolle, Ronald A. DeVore, Nam-yong Lee, and Bradley J. Lucier, 1998. Nonlinear wavelet image processing: variational problems, compression, and noise removal through wavelet shrinkage. *IEEE Transactions on image processing*, Vol. 7, NO.3.