ELASTIC IMAGE WARPING USING A NEW RADIAL BASIC FUNCTION WITH COMPACT SUPPORT

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Abstract: Thin plate spline (TPS) and compact support radial basis functions (CSRBF) are well-known and successful tools in medical image elastic registration base on landmark. TPS minimizes the bending energy of the whole image. However, in real application, such scheme would deform the image globally when deformation is local. Although CSRBF can limit the effect of the deformation locally, it cost more bending energy which means more information was lost. A new radial basic function named 'Compact Support Thin Plate Spline Radial Basic Function' (CSTPF) has been proposed in this paper. It costs less bending energy than CSRBF in deforming image locally and its global deformation effect is similar to TPS. Numerous experimental results show that CSTPF performs outstanding in both global and local image deformation.

1 INTRODUCTION

Elastic image registration is a significant content in medical image registration. And image deformation plays an important part in elastic image registration. The use of TPS for point-based elastic registration was first proposed by Bookstein (Bookstein, 1989). TPS forces the corresponding landmark to match each other exactly and minimizes the utilization of the bending energy of the whole image, therefore, it is widely used in various fields (Brown and Rusinkiewicz, 2004). However, the deformation of TPS is global, it would be problematic when only local difference exists (Ruprecht and Müller, 1993). N. Arad, D. Reisfeld(Arad and Reisfeld, 1995) has investigated Gaussian radial basis function(RBF) which reduces the global influence. And M. Fornefett, H.S.Stiehl (Fornefett et al.,1999),(Fornefett et al., 2001) used compact support radial basis function(CSRBF) in medical image registration.

Although TPS can minimize the bending energy of the whole image, it can not deform the image locally. CSRBF can deform the image locally; however, it costs much bending energy which means that the warping loses lots of information form original image. This weakness is especially distinct while the deformation is globally.

In this paper, we proposed a new compact support radial basic function to deform the elastic image. This function not only limits the image's deformation in a local domain, but also is a fundamental solution to the biharmonic equation . Its bending costs less energy consequently. Simultaneously, when the support set is wide, its warp effect is similar to TPS. Therefore, this new compact support radial basic function performs well in the local and global registration experiments.

2 THE LIMITATION OF ELASTIC IMAGE REGISTRATION PRESENTLY

TPS models the deformations by interpolating displacements between source and target points. The basic function of TPS is $U(r_i) = r_i^2 \log r_i^2$, r_i is the distance form the cartesian origin. TPS's basic function is a so-called fundamental solution to the biharmonic equation (Bookstein, 1989), which can minimize the bending energy(1).

$$E_{TPS}(f) = \iint \left| \left|\frac{\partial^2 f}{\partial x^2}\right|^2 + \left|\frac{\partial^2 f}{\partial x \partial y}\right|^2 + \left|\frac{\partial^2 f}{\partial y^2}\right|^2 \right| dxdy \qquad (1)$$

Local elastic image registration which bases on RBFs has the same interpolation function as TPS. It can not eliminate the global effect of the deformation. M. Fornefett and K. Rohr(Fornefett et

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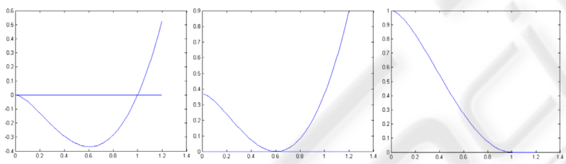
al.,1999), (Fornefett et al., 2001) applied Ψ -function of Wendland as RBFs for elastic registration of medical image. These radial basic functions have compact support.

These compact support radial basic functions can limit the deformation in a local domain. However, they are not the fundamental solution to the biharmonic equation. So they cost more bending energy in deforming the image, especially when their support radius are enormous, the loss of information of the source image is considerable.

3 COMPACT SUPPORT THIN PLATE SPLINE RADIAL BASIC FUNCTION

3.1 Compact Support Thin Plate Spline Radial Basic Function

In this paper, we aim to find out a function that not only is the solution to the biharmonic equation which can limit the bending energy, but have the characteristic of the functions which can deform image locally as well. Therefore, we use TPS's basic function: $U(r_i)=r_i^2\log r_i^2$ to construct a new compact support radial basic function:



(a) (b) (c)

Figure 1: Compact support thin plate spline radial basic function : (a) $U(r_i)=r_i^2\log r_i^2$ (b) fig.(a) displace 1/e upward (c) CSTPF.

As proven in figure.1 (a), TPS's original basic function $U(r_i)=r_i^2\log r_i^2$ decreases at $0-1/\sqrt{e}$, after $1/\sqrt{e}$, the function increases rapidly afterwards. It can be noticed that the decreasing part ($0-1/\sqrt{e}$ part) of this function is similar to compact support radial basic functions which can deform the image locally. So we add a constant (constant value is the min value of this function: $1/e$) to this function (View at fig.1 (b)), and let its increasing part become zero. Then the function becomes:

$$U(r)=\begin{cases} r^2\log r^2+1/e & , \quad r\le r_0 \\ 0 & , \quad r>r_0 \end{cases} \tag{2}$$

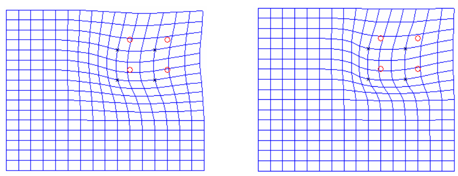
Figure.1 (c) shows that presently this function (2) has the characteristic of CSRBF. It has the max value at $r=0$, it is a decreasing function which is compact supported meanwhile. Therefore, this function can be used for local deform interpolation. Furthermore, this function is the solution to the biharmonic equation and is able to decrease the bending energy. Because this function comes from TPS's original interpolate function, we name it as CSTPF (compact support thin plate spline radial basic function).

3.2 Local Deformation using CSTPF

For the purpose of further investigating elastic image deformation, we hypothesized that the source images have already been rigidly registered with the target image, ignoring the affine part of the interpolation function. In case of affine free deformation, the interpolation function is illustrated as follow: (This interpolation function was utilized in the following 2 chapters.)

$$f_x(x,y)=x+\sum_{i=1}^{n}w_{xi}U(|P_i-(x,y)|) \tag{3}$$

First of all, we compared the performance of local deformation of CSTPF and CSRBF ($\psi_{a,3,2}$) at the same support set. Figure.2 explains the result of the deformation using the elastic registration approach base on CSTPF and CSRBF with four pairs of manual landmarks. It can be noticed that CSTPF is the solution to the biharmonic equation, correspondingly, less bending energy is required than using CSRBF, which means source image's information is better saved and the deformation has been greatly improved.



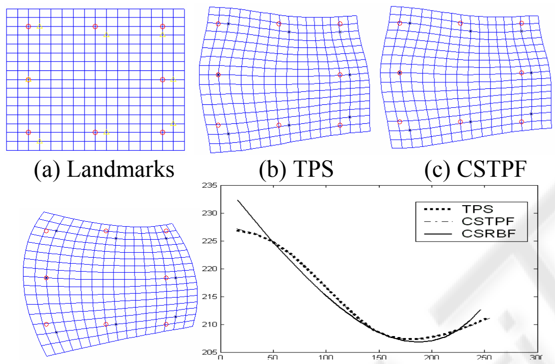
(a) CSTPF (b) CSRBF

Figure 2: Local deformation results (support radius $r=100$) (a)Deformation using CSTPF, the cost of bending energy is3.342; (b) Deformation using CSRBF, the cost of bending energy is 6.323.

3.3 Global Deformation using CSTPF

It has been proved that CSFPF preformed well in local deformation in last paragraph, now let's discuss how it perform in global deformation.

Figure.3 is a contrast of the global deformations using the elastic registration approach base on TPS 、CSTPF、CSRBF with manual landmarks. In this Figure shows we can see that the deformations using the elastic registration approach base on CSTPF (Fig.3 (b)) and TPS (Fig.3 (c)) are almost the same. Figure.3(e) takes one line out of the deformation's results and makes a comparison. It is shown that the deformation's lines of CSTPF and TPS are almost superposed. This result illustrated that image deformation using CSTPF can keep the advantage of TPS in global deformation, which can not be achieved by using CSRBF.



(a) Landmarks (b) TPS (c) CSTPF

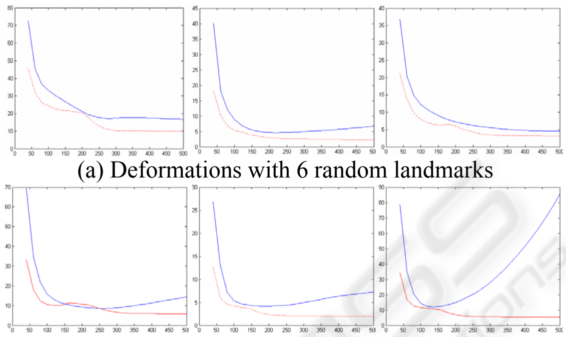
(d) CSRBF (e)

Figure 3: Global deformation contrast (support radius $r = 10000$) : (a) Landmarks' position; (b) global deformations using TPS; (c) global deformations using CSTPF; (d) global deformations using CSRBF; Contrast of the third line of Figure 3 (b)(c)(d), Notice Figure 3 (b) and (c) are almost superposed.

Experimental results have proved that image deformation using the elastic registration approach base on CSTPF is better than CSRBF.

To better illuminate the problem and aiming to compare the bending energy cost at different support radii, we experimented on 6 groups of deformations with random landmarks using the elastic registration approach based on CSTPF and CSRBF. Figure.4 shows the deformations' bending energy cost at different radii. In this graph, it is evident that image deformation using CSTPF costs less bending energy than CSRBF.

Consequently, the analysis and experiments in this chapter indicate that image global deformation using the elastic registration approach bases on CSTPF is similar to those on TPS. Furthermore, it is capable to localize the image deformation domain while TPS can not. In local image deformation, utilization of the elastic registration approach bases on CSTPF costs less bending energy than CSRBF with the same support radius.



(a) Deformations with 6 random landmarks

(b) Deformations with 10 random landmarks

Figure 4: Contrast of deformations' bending energy cost with random landmarks usizng CSTPF and CSRBF (X-axis: support radius, Y-axis is deformation's bending energy cost. real line in figure: CSRBF's energy cost , broken line in figure: CSTPF's energy cost): (a) Deformations with 6 random landmarks; (b) Deformations with 10 random landmarks.

4 EXPERIMENTAL RESULTS OF MEDICAL IMAGES

In this chapter, we have prepared two experiments in practical situations. With manual landmark, we compared the registration results for medical images using the elastic registration approach base on CSTPF and CSRBF.

In Figure 5, we can compare the results of deformation using CSTPF and CSRBF. They look similar but definitely not the same. Observing their edge comparison (Figure 5 (d) and (e)), it is revealed that after deformation, figure 5(d) has more edge information than figure 5(e) (as shown by the arrowhead), which means more information was saved by using CSTPF than CSRBF.

Finally, we employed another experiment to demonstrate that global deformation using CSTPF is better than CSRBF. In this experiment, we used an image of deferent mode, figure.6 (a) is MRI image and figure 6 (b) is CT image. It can be easily noticed that the source image and target image are just the same as they have no deformation. However, because we get landmarks manually, it is liable to have some artificial errors which are, however, considered as allowable errors. Given that these allowable errors are unavoidable, the source image deform globally.

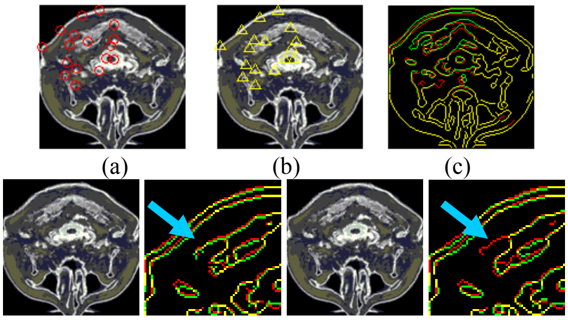


Figure 5: Comparison of local deformation using CSTPF and CSRBF($\psi_{a,3,2}$) ($r = 40$): (a) Source Image(with landmarks); (b) Target Image (with landmarks); (c) Comparison of edge of (a) and (b); (d)Deformation using CSTPF and the edge comparison of deformed image and original image(top left corner); (e)Deformation using CSTPF and the edge comparison of deformed image and original image(top left corner).

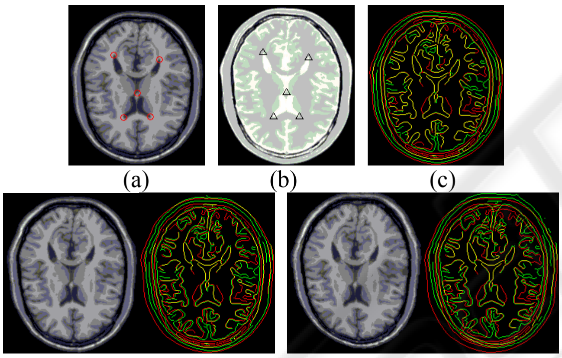


Figure 6: Comparison of global deformation using CSTPF and CSRBF($\psi_{a,3,2}$) ($r = 1000$):(a) Source Image(with landmarks); (b) Target Image (with landmarks); (c) Comparison of edge of (a) and (b); (d)Deformation using CSTPF and the edge comparison of deformed image and original image(global), the deformation cost 0.051327 bending-energy; (e)Deformation using CSTPF and the edge comparison of deformed image and original image(global), the deformation cost 0.19555 bending-energy.

It can be seen in figure.6 (d) that the image after global deformation using CSRBF has changed its shape significantly, while CSTPF kept the shape of source image satisfied. This demonstrated that CSTPF has stronger capability in global deformation than CSRBF.

5 CONCLUSIONS

In conclusion, TPS performs better in image global deformation, but it is not suitable for the local elastic registration. CSRBF can be used in the local registration, but it cost more bending energy which means that it will lose more information during the deformation. Moreover, its global deformation is not as well as TPS. In this paper, we proposed a new radial basic function ‘CSTPF’ which is a solution to the biharmonic equation. In local deformation, using CSTPF cost less bending energy. In global deformation, using CSTPF can keep the topology of source image. Additionally, it can save the information more integrated and it is approaches to TPS. Hence CSTPF is considered superior in image deformation.

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