A NOVEL CHAOTIC CODING SYSTEM FOR LOSSY IMAGE COMPRESSION

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Abstract: In this paper a novel image compression pipeline, by making use of a controlled chaotic system, is proposed. Chaos is a particular dynamic generated by nonlinear systems. Under certain conditions it is possible to properly manage the chaotic dynamics obtaining very feasible and powerful working instruments. In the proposed compression pipeline a linear feedback control strategy has been used to stabilize chaotic dynamic used to track the 1D signal generated by the input image. The pipeline is closed by an entropy encoder. Preliminary experiments and comparison with respect to standard JPEG engine confirm the effectiveness of the proposed chaotic coding system both for natural and graphic images. Also the overall performances in terms of rate-distortion capabilities are promising.

1 INTRODUCTION

Image coding is mainly achieved by making use of well known de-facto standards both for lossless and lossy compression. Coding redundancy is typically attacked by making use of DCT or Wavelet transform.

Main goal of the proposed compression methods is to improve the coding frameworks proposing an alternative compression engine (Amerijckx et al., 1998). A novel algorithm to compress (lossy) colour images by means of controlled chaotic dynamics is discussed. Chaos is a typical dynamic generated by nonlinear systems with at least one positive Lyapunov exponent.

There is no well accepted rigorous mathematical definition of chaos but its properties of high sensitivity to small perturbations as well as the capability to generate many complex dynamics such as limit cycles, attractors and unstable orbits, are well known (Arena et al., 2002; Chen et al., 1997; Perrone, 1997). Due to this extremely sensitivity to tiny perturbations the chaotic trajectories can be controlled to follow a reference dynamic very quickly (i.e. the NASA scientists have used a similar approach in the control system of the spacecraft ISEE-3/IEC). By taking into account some of the mentioned properties, several algorithms have been proposed using chaos for encrypting signals and images (Dedieu et al., 1995; Guan et al., 2005).

Moreover, interesting approaches for coding signal and images have been proposed in (Perrone, 1997, Belkhouche et al., 2003, Nien et al., 2007). An alternative compression approach based on "fractal" theory has been presented in (Li et al., 2000) but without improvement with respect to standard compression engine such as JPEG. In the proposed compression pipeline a linear feedback control strategy has been used to stabilize chaotic dynamic properly tracking the 1D signal generated by a classic raster visit of the input image. Classic differential coding (Gonzalez et al., 2000; Sayood, 2003)) and Huffman coder are used to complete the compression pipeline.

The paper is organized as follows: next section presents some recall about chaos control techniques whereas section III describes the proposed algorithm. In section IV experimental results and future works are briefly sketched.

2 THE CONTROL SYSTEM

There are several reasons for controlling chaos (i.e., nonlinear systems which shows chaotic dynamics such as the Lorenz system, the Logistic map, the Duffing oscillator, the Chua's system and so on).

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Without loss of generality, we concentrate our attention into the Chua's system as dynamical system with chaotic behaviour. The Chua's system was first proposed by L. O. Chua as autonomous nonlinear circuit. There are many implementations of the Chua's system and a lot of complex nonlinear dynamics that can be generated by tuning of the Chua system's parameters. We are referring to canonical representation reported in (Chen et al., 1997) in which the state equations can be re-written in canonical dimensionless form as follows:

$$\begin{cases} \dot{x} = p(-x + y - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -qy \end{cases} \quad (1)$$

$$f(x) = m_0 x + 0.5(m_1 - m_0)(|x+1| - |x-1|)$$

By tuning the parameters (p, q, m_0, m_1) a vast variety of dynamics can be generated (Chen et al., 1997; Manganaro et al., 1999) also including chaos. In the theory of chaos control, several goals can be achieved (Boccaletti et al., 2000). The Fig. 1 shows a typical double-scroll attractor generated by the Chua's system described in (1). Typical approaches to chaos control are: open loop strategies, feedback control systems, adaptive control systems (Boccaletti et al., 2000). A typical open loop control strategy is the so called *entrainment* in which the controlled chaotic dynamic is forced to follow a reference trajectory (Chen et al., 1997).

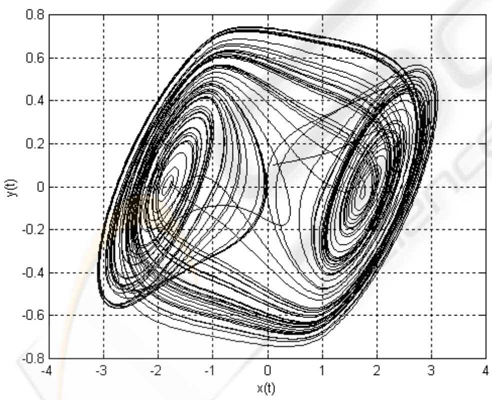

Figure 1: The Double Scroll attractor.

A closed loop (feedback) version of the *entrainment* control strategy was proposed in (Jackson,1998). The general *entrainment* strategy can be defined as follows. Let a dynamical system which shows chaotic behaviour:

$$\dot{x}(t) = f(x(t), t) + u(t) \qquad x(t), u(t) \in \Re^n \quad (2)$$

The goal of the above control scheme is to find a controller "*u(t)*" to force the dynamical system in (2) to follow (asymptotically stability) a target trajectory *r(t)*:

$$\lim_{t\to\infty} \|x(t) - r(t)\| = 0 \qquad \forall r(t) \in \Re^n \quad (3)$$

In realistic applications, the target to be achieved when a chaotic system is controlled is the so called "*near*" control goal (Chen et al., 1997):

$$\|x(t) - r(t)\| < \varepsilon \qquad \forall t \geq t_0 \qquad t_o < \infty \quad (4)$$

In (4) the term ε represents a prefixed allowable tolerance ad t_0 a terminal time. There are several approaches to determine the controller *u(t)* (Mascolo et al., 1999). The controller *u(t)* can be implemented by means of a careful analysis of the dynamics to be controlled or by using adaptive algorithms. Recently, "intelligent" controllers have been widely used in several applications for controlling nonlinear dynamics such as chaos (Arena et al., 2002). In (Chen et al., 1997) a linear feedback control system has been used for controlling Chua's system successfully. With this approach, the state equations of the controlled Chua's system can be written as follows:

$$\begin{cases} \dot{x} = p(-x + y - f(x)) - k_x(x - x_{ref}) \\ \dot{y} = x - y + z - k_y(y - y_{ref}) \\ \dot{z} = -qy - k_z(z - z_{ref}) \end{cases} \quad (5)$$

$$f(x) = m_0 x + 0.5(m_1 - m_0)(|x+1| - |x-1|)$$

where the control gains (k_x ,k_y ,k_z) have to be computed during the design phase whereas (x_{ref}, y_{ref}, z_{ref}) are the reference trajectories.

The design of the above control gains can be made by using several techniques (Arena et al., 2002; Boccaletti et al., 2000).

In (Chen et al., 1997) a Lyapunov's theorem based algorithm has been used for getting the following conditions:

$$k_x \geq -pm_1, k_y \geq 0, k_z \geq 0 \quad (6)$$

The reference trajectories can be generated by a Chua's circuit with different initial conditions or parameters or from another dynamical system. Clearly, the control action can be also applied to a subset of variables of the Chua's system as showed in (Arena et al., 2002). They were able to control the Chua's system acting only to its x and y state variables by means of a linear feedback control scheme with adaptive gains. The experimental results (tested with many kind of complex target trajectories) shows, also in this case, the good performance of the linear feedback control approach in controlling chaotic dynamics. For the proposed compression pipeline, we use the same linear feedback control strategy used in (Arena et al., 2002) applied to the x variable of the Chua's system showed in (1). The so controlled Chua's system, integrated with a classic *Euler* algorithm, can be re-written as follows:

$$
\begin{aligned}
& e_x(k) = x(k) - x_{ref}(k) \\
& \begin{cases} x(k+1) = x(k) + h \cdot \psi_x(k) \\ y(k+1) = y(k) + h \cdot (x(k) - y(k) + z(k)) \\ z(k+1) = z(k) + h \cdot (-qy(k)) \end{cases} \\
& \psi_x(k) = p(-x(k) + y(k) - f(x(k)) - k_x e_x(k) \\
& \forall k = 0,1,..N-1 \\
& p = 10, q = 14.87, m_0 = -0.68, m_1 = -1.27
\end{aligned} \quad (7)
$$

In (7) the term h represents the integration step while the reported parameters (p, q, m_0, m_1) are suitable to generate a *double scroll* chaotic attractor (Chen et al., 1997). The described control theory is the main core of the proposed lossy compression pipeline. The key idea is based on the specific property of chaotic dynamic: high sensitivity to small perturbations. As mentioned in section 1, the previous property leads a controlled chaotic system to follow a desired trajectory very quickly.

3 THE PROPOSED PIPELINE

In the proposed compression pipeline we force the controlled Chua's system showed in (7) to track the 1D representation of bi-dimensional source image (the target trajectory x_{ref}). Due to the above considerations about main chaos properties, we make sure that at least *near* goal feedback control (showed in (4)) can be achieved (Arena et al., 2002). Both for the encoder and decoder sides a Chua's system as showed in (7) is used. The initial conditions are $x(0)=0.1$; $y(0)=0.2$; $z(0)=0.3$ for both encoder/decoder side. The used integration step is $h=0.01$.

3.1 The Encoder

The encoding pipeline starts converting the source colour image $I(x,y)$ of size (m x n) from RGB to YC_bC_r colour space (Gonzales et al., 2000). After that, the chrominance components, C_b and C_r, are down-sampled by a factor 2. In the next step, the encoding scheme is applied for each plane (Y, C_b and C_r) separately. We refer in the next paragraphs to Y plane of the source image but the same consideration may be applied to the chrominance components (down-sampled by factor 2) of the same image. The 2D image plane is than translated into 1D by a classical raster visit. Finally, a normalization in the range [0,1] of the 1D image vector is applied. Let $i(k)$ the obtained vector corresponds to the reference trajectory x_{ref} showed in (7). At this point the tracking error can be defined as follows:

$$e(k) = x_N(k) - i(k), \qquad k = 0,1,..(m \times n) - 1 \quad (8)$$

Each of the Chua's system variables ($x(k)$, $y(k)$, $z(k)$) are normalized in order to define the precision of the non-integer values involved in the proposed algorithm:

$$
\begin{aligned}
& x_N(k) = round(x(k) \cdot RF) / RF \\
& y_N(k) = round(y(k) \cdot RF) / RF \\
& z_N(k) = round(z(k) \cdot RF) / RF \\
& k = 0,1,..(m \times n) - 1
\end{aligned} \quad (9)
$$

where RF is *ad hoc* heuristically defined round-off factor. Finally, in order to re-map the non-integer values of the tracking error to an integer range, before to the Huffman encoding, the following re-mapping equation is used:

$$
\begin{aligned}
& e(k) = round[(round(e(k) \cdot RF) / RF) \cdot RF] \\
& k = 0,1,..(m \times n) - 1
\end{aligned} \quad (10)
$$

By tuning this RF factor we are able to change the compression rate of the proposed algorithm. The encoder defines an RF parameter both for luminance (RF_y) and chrominance (RF_c) quantization. After that, we proceed to compress the error $e(k)$ as showed in (8); really we compress the quantized version of $e(k)$ described in (10) instead of $i(k)$. The linear feedback control system, leads the chaotic dynamic of he x-variable of the Chua's system to follow the target (i.e. the vector $i(k)$) very quickly). The residual entropy of the error signal $e(k)$ is, of course, more achievable than original $i(k)$ signal, allowing to obtain near-optimal rate-distortion performances. A classical differential coding (Gonzales et al. 2000; Sayood, 2003) followed by an Huffman encoder is used to complete the compression pipeline. In the proposed algorithm we have defined *ad hoc* header (just a few bytes) included together with the data stream. This header contains the size of the original image, the round-off factors RF_y and RF_c (both luminance and chrominance) three parameters used by differential coding, codewords length generated by the Huffman coder for each mage plane (Y,C_b,C_r). In Fig. 2 the proposed encoding pipeline is showed. The signal $e_e(k)$ represents the encoded image.

3.2 The Decoder

In the decoding phase all previous steps are simply inverted. Firstly, the included header is parsed to extract the basic information. The compressed encoded signal $e_e(k)$ is processed by an Huffman decoder and then by an inverse differential coding. The reconstructed error signal $e_{rec}(k)$ is used for controlling the x-variable of the Chua's system as showed in (7) (with the same initial conditions, system parameters and control gains used in the encoder).

At this point, the source vector $i(k)$ can be reconstructed as follows:

$$i_{rec}(k) = x_N^{Chua-Decoder}(k) - e_{rec}(k) \qquad (11)$$
$$k = 0,1,..(m \times n) - 1$$

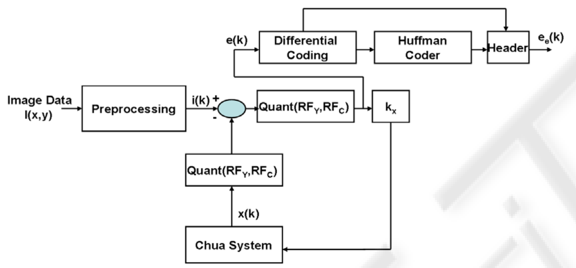

Figure 2: The proposed Encoder.

We reconstruct the 2D image from the 1D $i_{rec}(k)$ vector (using also the information included in the header). For the chrominance components, an up-sampling operation (2x bi-cubic interpolation) is also applied. Finally, the so reconstructed 2D image (in YC_bC_r) is converted into RGB colour image $I'(x,y)$. In Fig. 3 the proposed decoding pipeline is showed.

4 EXPERIMENTAL RESULTS

The proposed lossy image compression pipeline has been tested with different images. The value of the used control gain during the tests execution is $k_x=10$. The tests have been run under MATLAB framework (rel. 7.0.1).

The reported results show good performances in terms of quality (*PSNR*) and compression ratio (*Original image filesize/compressed image filesize*) of the proposed pipeline. We report in Fig. 4 and Fig. 5 some comparisons with JPEG standard applied to the synthetic image named "Benjerry". Moreover, in Fig. 6 we have tracked the PSNR dynamic versus RF_y for the "Benjerry" image. For some synthetic images (466x60) we report the full rate-distortion curve in Fig. 7 while some visual results are showed in Fig. 8. Further results on graphic and natural images are briefly reported in Table 1. Finally in Fig. 9 and Fig. 10 we have tracked respectively the *Bpps* versus *RFy* and *PSNR* versus *RFy* for a single natural image.

Preliminary comparisons with JPEG standard are also reported by measuring the improvement obtained in terms of compression ratio by considering the same amount of visual quality (measured by PSNR values). We are planning to make further experiments to provide a full detailed rate-distortion repository on large images dataset including a JPEG2000 (lossy pipeline) comparisons as well. Future works aims to apply the proposed pipeline for the encryption of digital images.

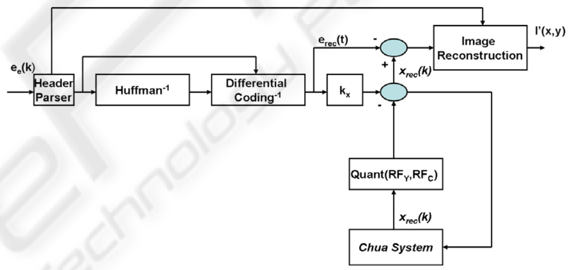

Figure 3: The proposed Decoder.

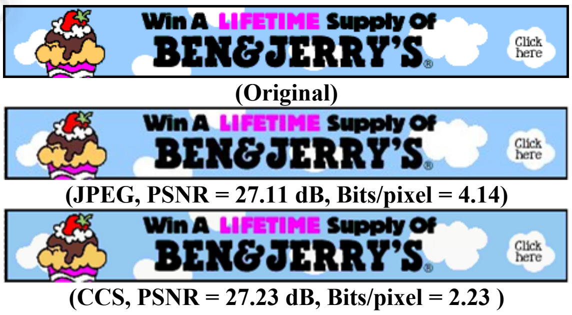

Figure 4: The comparison between the proposed algorithm (CCS – Chaotic Coding System) with respect to JPEG.

Figure 5: A detail of the "Benjerry" image compressed with the proposed pipeline (CCS) and with JPEG codec. The above detail shows the absence of artefacts (ringing) typically showed by JPEG engine.

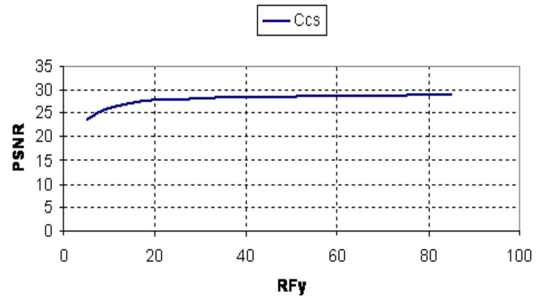

Figure 6: PSNR versus RF_y plot (for Y plane of the synthetic image “Benjerry”).

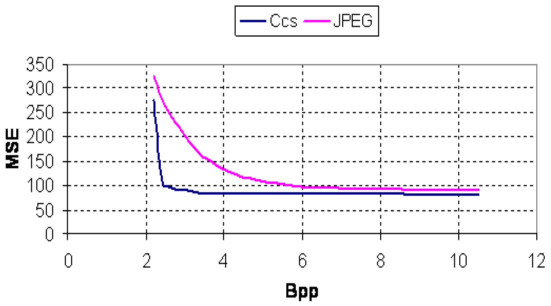

Figure 7: The Rate-Distortion plot of the proposed pipeline with respect to the JPEG standard (for synthetic images).

(a)

(b)

(c)

Figure 8: The above images show a further comparison between the proposed pipeline with respect to JPEG standard. In (a) the original image is reported (gimp_temp.115971). In (b) the JPEG image is reported (Compression ratio=5.54, PSNR=37.4 dB). Finally in (c) the image compressed with the proposed pipeline is reported (Compression ratio=6.24, PSNR=37.71 dB). All the above images contain a superimposed detail to show the absence of typical ringing artefacts (normally present in JPEG coding) in the proposed compression pipeline.

Table 1: Comparison between the proposed pipeline (CCS) with respect to JPEG standard in terms of compression ratio (CR) and PSNR (in dB). The used quality factor for JPEG coding is in the range 90 ÷ 100. The CCS coding has been performed tuning the parameter RF_y in the range 35 ÷ 80.

Images	**JPEG**		**CCS**	
	CR	**PSNR**	**CR**	**PSNR**
Benjerry	3.72	28.31	5.61	28.99
Netscape	3.01	34.47	7.83	34.53
Book	2.87	26.60	3.40	25.34
gimp_temp.115971	5.54	37.41	6.24	37.71
gimp_temp.115977	5.87	37.15	6.98	37.42
gimp_temp.115979	3.11	35.21	4.02	35.31
gimp_temp.1159715	4.53	35.53	4.82	35.24
gimp_temp.1159717	7.98	38.38	4.74	39.25
gimp_temp.1159719	3.94	39.59	4.25	40.93
gimp_temp.26473	5.27	39.93	7.23	39.35
gimp_temp.26475	3.23	37.42	3.87	37.43
gimp_temp.264711	5.14	39.19	5.72	39.23
gimp_temp.264715	3.84	37.10	4.02	37.02
gimp_temp.264723	4.26	39.01	4.15	38.43

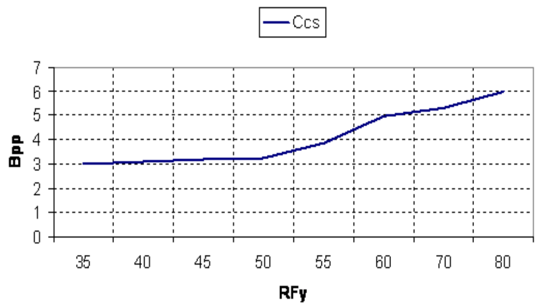

Figure 9: Bit per pixels versus RF_y (computed for the natural image "gimp_test.115979").

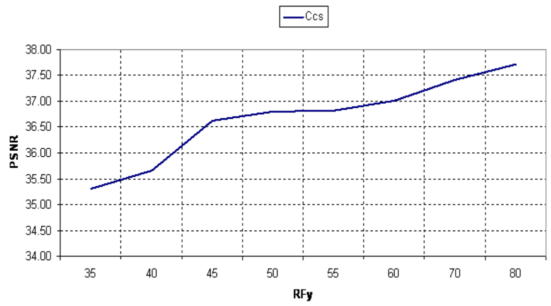

Figure 10: PSNR (in dB) versus RF_y (computed for the natural image "gimp_test.1159715").

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