VISUALIZING MULTIPLE SCALAR FIELDS ON A SURFACE

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Abstract: We present a new technique for the simultaneous visualization of an arbitrary number of scalar fields defined on a surface. The technique is called *Generalized Atmosphere Upper Bound Level (GAUBL)*, since it is an evolution of our previous AUBL technique, that allowed for the visualization of a single scalar field. The generalized AUBL can highlight the dependencies and interactions between many scalar fields, and can handle a multi-valued scalar field as a special case. We have implemented the GAUBL into a visualization tool that handles triangle-based surface models, and we show here some experimental results.

1 INTRODUCTION

In many applications of computer graphics (e.g., medical imagery, visual data mining) the visualization of a scalar field representing some data relative to a three-dimensional shape is a basic tool to explore and understand the behavior of the field. Several scalar fields may be interesting to be studied simultaneously, to highlight their dependencies and their mutual influence. For example, in medical imagery, oxygen rate and sugar rate can be visualized together to study the brain surface activity. Unfortunately, human perception is limited to three dimensions and the visualization of those scalar fields needs additional independent directions to be achieved. To avoid this obstacle, we need to find a natural way to embed these multi-dimensional data in the Euclidean space R^3 so that the result still has some meaningful interpretation, especially for comparison purposes. Here, we propose a visualization technique that allows this embedding and thus gives us the opportunity to explore and study multi-valued scalar fields defined on the same surface. The basic idea is to convert the scalar fields into a sequence of vector fields on the surface and then display a surface for each vector field according to some constraints. We call the new visualization technique *Generalized AUBL (GAUBL)*, since it generalizes to multi-dimensional scalar fields the *AUBL (Atmosphere Upper Bound Level)* technique introduced in (Mesmoudi et al., 2007). This latter allows the 3*D* visualization of just one scalar field defined over a surface embedded in the three-dimensional Euclidean space. The generalized AUBL technique is easily adapted to handle discrete scalar fields defined over triangulated surfaces. We present here an interactive visualization tool that implements the GAUBL technique for triangle meshes. We use such tool to illustrate the results of the GAUBL visualization technique. The remainder of the paper is organized as follows. In Section 2, we review related work. In Sections 3, we briefly review the *AUBL* visualization technique. In Section4, we introduce the *GAUBL* visualization technique that generalizes *AUBL* to visualizing several scalar fields. In Section 5, we present the visualization tool implementing the GAUBL technique for discrete scalar fields defined on triangulated surfaces, and some results. In the last Section, we draw some concluding remarks.

2 RELATED WORK

To represent multi-dimensional data in the three dimensional space, we need to reduce their dimensionality without loosing important information. Geo-

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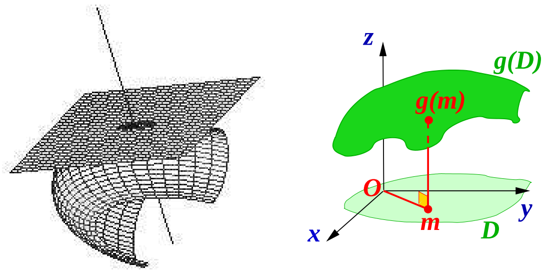
metric projection techniques allow meaningful visualization of multi-dimensional data. Some of them are statistically based techniques (Huber, 1985). Parallel coordinates techniques (Inselberg, 1985) represent attributes as parallel lines in the two-dimensional space. Hierarchical techniques use a partitioning of the space into subspaces. In stacking techniques, the space is partitioned into 2D subspaces that are stacked in a recursive way (Blanc et al., 1990). The *worlds-within-worlds* technique partitions the 3D space into nested subspaces: three attributes are selected and visualized through a 3*D* surface, then, for any point on the surface selected by the user, three other attributes are visualized in the same manner (Feiner and Beshers, 1990). When some attributes are functions of two or three dependent parameters (like in terrain modeling, image processing, medical imagery), the graphical representation of these attributes has more sense if it can be represented in the ambient space by a surface. The *AUBL* technique developed in (Mesmoudi et al., 2007) allows the 3*D* visualization of a scalar field defined over a surface embedded in the 3D Euclidean space. This technique when applied to a constant function is known as offsetting (Rossignac and Requicha, 1985; Frisken et al., 2000; Cohen et al., 1996). In (Taylor, 2002; Kirby et al., 1999; Crawfis and Allison, 1991), techniques to represent multiple scalar fields (at most four fields) on the same surface have been proposed. These techniques combine colors, contour lines, spot noise texture generation, reaction-diffusion texture generation, surface albedo, data-driven spots and oriented slivers.

3 THE *AUBL* VISUALIZATION TECHNIQUE

Two-dimensional manifolds (without boundary) are (smooth) surfaces that are locally diffeomorphic to discs in R^2. At each point p of a surface S, the tangent plane T_pS is defined and a unit normal vector $\overrightarrow{n_p}$ to S at point p can be drawn. This latter correspondence is called the *Gauss map*. Vector $\overrightarrow{n_p}$ with an orthonormal basis of T_pS generates a mobile orthonormal frame of the Euclidean three-dimensional space R^3 whose origin is located at point p (see Figure 1(a)). The key idea of the AUBL visualization technique comes from the graphical representation of 2D scalar fields. The graphical representation of a scalar field g on a two-dimensional domain $D \cong D \times 0$ is a surface embedded in R^3 such that the height of each point on the surface corresponds to the value of g at this point and if the frame is orthonormal then the distance of the point to the Oxy-plane is equal to the absolute value of g at



(a) (b)

Figure 1: (a) A surface with its tangent plane and normal vectorial space at a point. (b) Graphical representation of a function g over a domain $D \subset R^2$.

this point (see Figure 1(b)). By generalizing this idea, we can give a graphical representation of 3D scalar fields.

Definition 1 *Let $\overrightarrow{n_p}$ be the unit normal vector of S at point p. The graphical representation of the scalar field f over S is the surface $\mathcal{S} \subset R^3$ defined by the vector field $\tilde{f}(p) := p + f(p)\overrightarrow{n_p}$, i.e.,*

$$\mathcal{S} = \{p + f(p)\overrightarrow{n_p} : p \in S\} \quad (1)$$

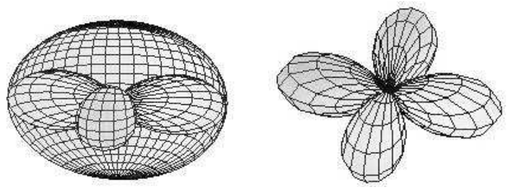
Note that vector $\overrightarrow{p\tilde{f}(p)}$ is normal to S at p and $\| \overrightarrow{p\tilde{f}(p)} \| = | f(p) |$.
The graphical representation of function f defines an *atmosphere layer* over surface S. The thickness of the layer is given by the function values. In (Mesmoudi et al., 2007), we have defined graphical operations which can be used to better analyze the shape of the surface and thus the properties of the field. *Scaling* multiplies the field vector value through a factor; *inflation* and *deflation* translate $\tilde{f}(p)$ in direction of the normal vector $\overrightarrow{N_p}$ by a constant positive and negative value, respectively. Details can be found in (Mesmoudi et al., 2007).

Definition 2 *Under such assumptions, we call the graphical representation $\mathcal{S}$ of f, the atmosphere upper bound layer (AUBL) of the pair (S, f).*

In Figure 2, we illustrate the above situation for the unit sphere $x^2 + y^2 + z^2 = 1$ with an atmosphere corresponding to the function $f(x,y,z) = x^2 - y^2 - 1$.

4 THE GENERALIZED *AUBL* TECHNIQUE

The main idea in generalizing the *AUBL* visualization technique comes from the fact that the AUBL technique gives a *vector field* $\left(\overrightarrow{p\tilde{f}(p)}\right)_{p \in S}$ over surface S.



(a) (b)

Figure 2: (a) A cross section of the unit sphere with an atmosphere defined by function $f(x,y,z) = x^2 - y^2 - 1$. (b) Visualization of $\mathcal{S}$ corresponding to $\tilde{f}$.

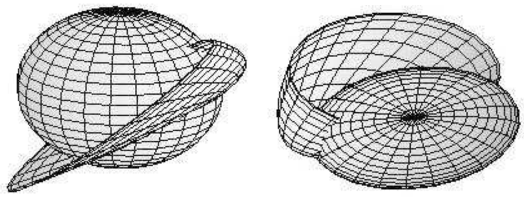
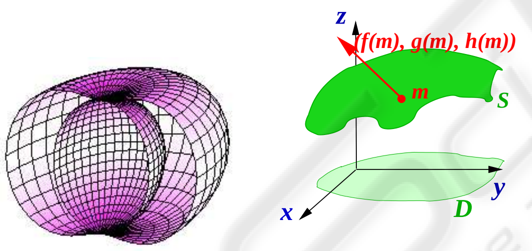


Figure 4: Visualization of three functions $f(x,y,z) = x^2 + y^2$, $g(x,y,z) = z^2$ and $h(x,y,z) = y - z$ defined over the unit sphere, from two different points of view. Functions f, g, h are presented as the surface of a vector field (f,g,h).

We use this idea to generate a vector field over S for any number of functions defined on S. We will show that successive vector fields can be defined depending on the number of functions. Assume that two scalar fields f and g are defined simultaneously on S. *AUBL* visualization technique is used to visualize function f as surface $\mathcal{S}$. To visualize function g with f, a coloring map c is defined on the image $Im(g)$. Then a vector function $\tilde{g}$ can be defined as follows: for each point $p \in S$ we associate the pair $(\tilde{f}(p), c(g(p)))$. Finally, functions (f,g) are visualized as a colored surface, that we denote $\mathcal{CS}$ to distinguish it from $\mathcal{S}$. Figure 3(a) gives an example of two functions f and g defined on the unit sphere S^2. Let now f, g and h be

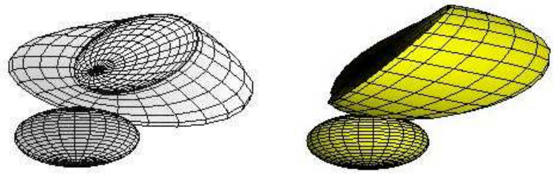


(a) (b)

Figure 3: (a) Visualization of two functions $f(x,y,z) = x^2 + y^2$ and $g(x,y,z) = z^2$ defined over the unit sphere. Function f is represented as a surface containing the unit sphere in its interior, and g is represented by a coloring map where white and magenta correspond to low and high field values, respectively. (b) Graphical representation of three scalar fields over a surface S in R^3.

three scalar fields defined on surface S. For each point $p \in S$, we define a vector $\overrightarrow{V_p} = (f(p), g(p), h(p))$. This gives a vector field $(\overrightarrow{V_p})_{p \in S}$ over S (see Figure 3(b)). We define surface $\mathcal{S} := \{p + \overrightarrow{V_p} : p \in S\}$ as the graphical representation of scalar fields f, g and h over S. The direction and the intensity of each vector is influenced by the values of the three functions f, g and h. Thus, interactions among these functions can be seen (see Figure 4). For four scalar fields f, g, h and k, we embed surface S in R^4 by $S \approx S' := \{(x,y,z,0) : (x,y,z) \in S\}$. For each point $p \in S$ we define a vector $\overrightarrow{V'_p} = (f(p), g(p), h(p), k(p))$. Vectors $(\overrightarrow{V'_p})_{p \in S}$ form a vector field over S'. Visualization of scalar fields f, g, h and k can be achieved in R^4 by constructing a new surface $\mathcal{S}' := \{(p,0) + \overrightarrow{V'_p} : p \in S\} = \{(x_p + f(p), y_p + g(p), z_p + h(p), k(p))\}$. Equivalently, $\mathcal{S}'$ can be seen as a surface $S" := \{(x_p + f(p), y_p + g(p), z_p + h(p)) : p \in S\}$ in R^3 endowed with a scalar field k. Function k can be seen as a coloring function of surface $S"$. Another way to represent function k is to use the *AUBL* visualization technique that permits to define a surface $\mathcal{S}" \subset R^3$ associated with the pair $(S", k)$. We define thus $(S", \mathcal{S}")$ to be the graphical representation of scalar fields f, g, h and k.

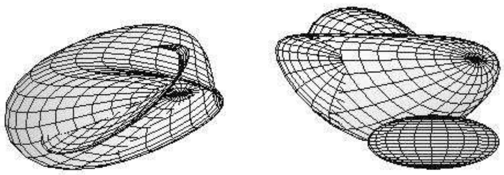
Now, the generalization to five scalar fields can be done as for the case of two scalar fields. We represent the fourth scalar field by the *AUBL* technique as a surface $\mathcal{S}"$ and the fifth scalar field by a coloring function over surface $\mathcal{S}"$. The colored surface $\mathcal{CS}"$ with $S"$ give a graphical representation of the all five scalar fields (see Figure 5). For six



(a) (b)

Figure 5: Visualization of five functions $f(x,y,z) = x^2 + y^2$, $g(x,y,z) = z^2$, $h(x,y,z) = 2y - z + 3$, $k(p) = z$ and $l(p) = x$ defined over the unit sphere S^2. (a) Surfaces S^2, $S"$ and $\mathcal{S}"$. (b) The coloring function l is represented in $\mathcal{CS}"$.

scalar fields f, g, h, k, l and m, we shift surface S to R^6 to get a surface $S \approx S' := \{(x,y,z,0,0,0) : (x,y,z) \in S\}$. Then for each point $p \in S$ we define a vector $\overrightarrow{V'_p} = (f(p), g(p), h(p), k(p), l(p), m(p))$. Vectors $(\overrightarrow{V'_p})_{p \in S}$ form a vector field that traverses over S'. Visualization of scalar fields f, g, h, k, l and m can be achieved in R^6 by constructing surface $\mathcal{S}' := \{(p,0,0,0)+\overrightarrow{V'_p} :\ p \in S\} = \{(x_p+f(p), y_p+g(p), z_p+h(p), k(p), l(p), m(p))\}$. Equivalently, $\mathcal{S}'$ can be seen as a surface $S" := \{(x_p+f(p), y_p+g(p), z_p+h(p)) : p \in S\}$ embedded in R^3 with a vector field defined by vectors $\overrightarrow{V"_p} = (k(p), l(p), m(p))$. Then as for three scalar fields we can construct a surface $\mathcal{S}"$ defined by vectors $\overrightarrow{V"_p}$. The visualization of all scalar fields f, g, h, k, l and m is hence given by two surfaces $(S", \mathcal{S}")$ as in Figure 6. Following the previous reasoning, we can extend (modulo 3) the above visualization techniques to n scalar fields defined over surface S. The *AUBL* technique, the coloring technique and the vector flow technique form a basis of the visualization techniques that can be used together, or separately, following the rank of $n \ (mod\ 3)$. This extension gives a hierarchical representation of scalar fields as described in the *worlds-with-worlds* data mining representation technique (see Section 2 above).



(a) (b)

Figure 6: Visualization of six functions $f(x,y,z) = x^2+y^2$, $g(x,y,z) = z^2$, $h(x,y,z) = 2y-z+3, k(p) = z$, and $m(p) = -y^2$. Functions f, g and h form a first vector field $\mathcal{S}'$ over S^2. Then function k, l and m form a second vector field over $\mathcal{S}'$. (a) Surfaces $\mathcal{S}'$ and $\mathcal{S}"$. (b) Surfaces S^2, $\mathcal{S}'$ and $\mathcal{S}"$.

5 EXPERIMENTAL RESULTS

Our GAUBL tool allows for the visualization of up to four different scalar fields defined on a triangulated surface. It is implemented in C with the OpenGL graphical library and has a very simple user interface developed with Glut. The surface is given as a triangle mesh in indexed format (each vertex as three coordinates, each triangle as three vertex indexes). A scalar field can be provided in two forms: an explicit list of field values at the mesh vertices (e.g., sampled values of temperature, pressure etc.), or a mathematical formula to compute such values. Visualization adapts to the current number of loaded scalar fields, by selecting the appropriate *GAUBL* technique. Additional inputs are represented by coloring functions to map field values to color values specified in the red-green-blue (RGB) format. The user can interactively set parameters, as the translation factor for inflation / deflation, the multiplicative factor for scaling, the coloring function, surface and background colors, transparency effects, and, of course, he can rotate, pan and zoom the entire scene. Figure 7 shows the unite sphere along with the colored mesh representing four scalar fields: $f(p) = x, g(p) = y-z, h(p) = x^2+y^2$ and $k(p) = x$, the last one rendered with a coloring function going from red to blue through green. Figure 8 shows a mesh representing a girl and a field equal to $f(p) = z$. In Figure 9 the same mesh is associated with a scalar field that simulates fattening of the central part of the body, through a gaussian formula. The figure shows the original surface and two fattened versions, with different multiplication factors. Figure 10 shows a terrain with two scalar fields, where the first one (giving the surface) is constant and the second one (rendered as color scale) is $g(p) = z$. Inflation and deflation correspond here to a version of the same terrain after deposit of material (e.g. calcium carbonate on the bottom of a lake in a cavern) or after erosion, respectively.

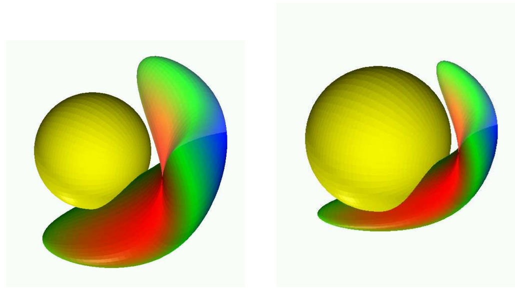


Figure 7: The sphere and four fields, with two different multiplication factors.

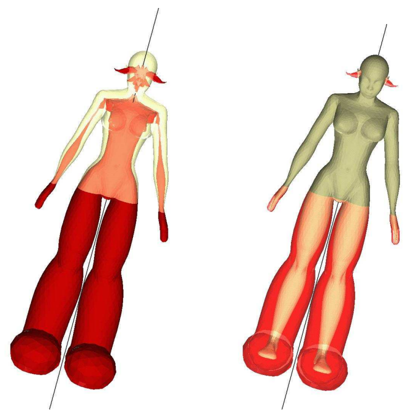


Figure 8: Girl surface with one field $f(p) = z$.

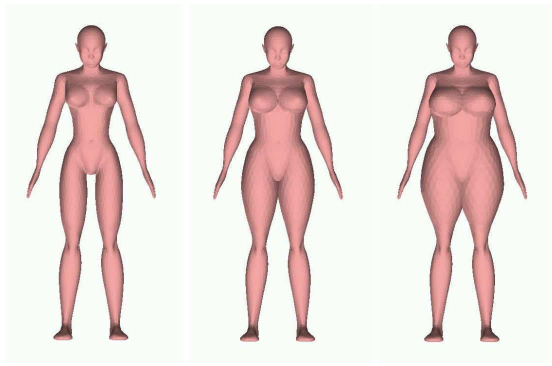


Figure 9: Girl surface with different multiplication factors.

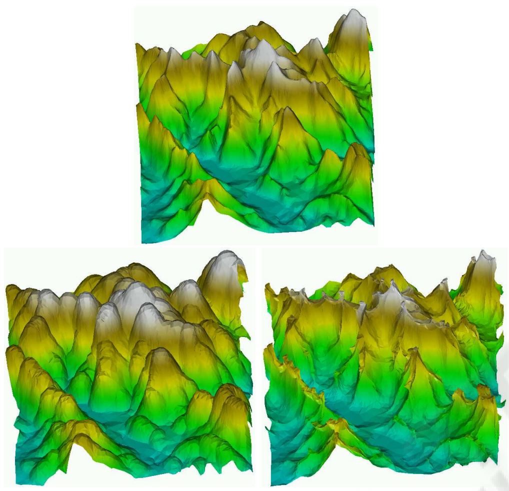


Figure 10: A terrain and its inflated and deflated versions.

6 CONCLUDING REMARKS

We have presented the *GAUBL* visualization technique that allows displaying any number of scalar fields defined on a surface, in the form of another (possibly colored) surface embedded in 3D space. In our ongoing work, we will improve our visualization tool with new functionalities, such as showing algebraic information at a clicked point on the surface (vector length, direction, position with respect to the normal vector of the original surface,...). Moreover, we plan to combine this visualization technique with a mesh-based multi-resolution representation to allow selective and adaptive offsetting of a surface.

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