

VIRTUAL RESTORATION OF A MEDIEVAL POLYCHROME SCULPTURE

Experimentation, Modelization, Validation and Visualization in Spectral Ray-tracing

Sylvain Dumazet, Patrick Callet

Applied Mathematics and Systems Lab., Ecole Centrale Paris, grande voie des vignes, Châtenay-Malabry, France

Ariane Genty

INOLAM, 26 bis rue Kléber, Montreuil, France

Keywords: Ray-tracing, spectrophotometry, 4-flux model, sculpture, virtual restoration, gilding, painting.

Abstract: A pluridisciplinary work always in progress involving 3D digitization, simulation, rapid prototyping, virtual restoration of a french medieval sculpture is presented. This work is led in the framework of a general collaboration between three academic labs, industrial partners and Cultural institutions. The main purpose is to virtually represent a polychrome statue of the XIIIth century in high quality spectral rendering, to simulate its visual and original appearance at that period. The complete process used throughout all the phases of the project mainly involves optical devices that ensure no physical contact with the museum object. This article describes the complete chain of engineering resources and the main models we used for accomplishing our objective. From 3D capture without contact to plaster replica, the complete process will be described and illustrated with images and objects during the conference. Some sequences extracted from the didactic and scientific movies produced will also be presented.

1 INTRODUCTION

A collaboration with the "Centre des Monuments Nationaux" (CMN) helps us to elaborate a new project: the study of a polychrome medieval statue, the recumbent statue of Philippe Dagobert de France (circa 1222 - 1232 AC) in the Saint-Denis Basilica (Fig. 1), the royal necropolis in the north of Paris. The CMN being in charge of organizing and managing the public visits in more than one hundred French monuments, is deeply involved in the realisation of the study. The research results are meant to be shown, permanently near the Philippe de France tomb, in order to raise a large public awareness to the scientific research contribution, and for a better understanding of historical heritage. The presented work takes place in a previously defined general framework known "OCRE method" (Optical Constants for Rendering Evaluation) (Callet, 1998), developed by Patrick Callet and describing the optical behaviour of materials on the basis of their fundamental properties.

For metals and alloys we naturally used their complex indices of refraction, either computed or carefully measured, but, in any case always validated by measurements. Plasma physics and spectroscopic ellipsometry, for extracting real and imaginary parts of the complex indices of refraction, were used for that



Figure 1: The recumbent statue of Philippe Dagobert in Saint-Denis Basilica (13th century AC).

pertinent modelling. These previous important results are very useful for the rendering of the golden parts of the statue that consist in a gold alloy containing a small amount of silver. The metallurgical composition of the gold leaves recovering the visible rests will greatly help in the formulation of the visual appearance for virtual restoration or historical reconstitution. The last study about the visual influence of the underlying paint, called "bole", on a gilded surface has shown the importance of the transparency of a thin metallic gold film and of the associated small cracks due to the application process (called "burnishing") (Dumazet et al., 2007).

We here focus on modelling and representing the painting materials using spectrophotometry and extrinsic physical parameters such as the paint film thickness, the concentration of each species of pigments, their mean diameter or the granulometry distribution functions,... Intrinsic parameters, characterizing the nature more than the structure of all the compounds such as complex indices of refraction of all involved materials are used throughout the study. As an extension of the Kubelka and Munk theory, a four-fluxes approach of the multiple scattering of light is used for the simulation. At each step of the study we compare our computed results of simulation with measurements made on handcrafted samples in laboratory (Fig. fig:peinture-atelier) or in situ, i.e. on the statue itself. Helped by the "Centre des Monuments Nationaux" (National Monuments Centre), and with the knowledge about painted stones during the middle-age, we studied the pigments and binders which were probably used by the artist. In a ultimate step we shall replicate the statue to 1/3rd of its original dimensions with its most probable and original colours.

2 HISTORICAL CONTEXT

This recumbent statue has been chosen for its remaining polychrome traces (Fig. 1) and also because it was representative of the fine middle XIIIth century funeral sculpture. At that time, Louis IX, not yet known as king "Saint Louis", was setting up the Saint Denis Royal necropolis and the "Children of France" necropolis in the Royaumont Cistercian abbey at Asnières sur Oise (Erlande-Brandenburg, 1975). It concerns the Philippe-Dagobert's tomb, a young Saint-Louis's brother born in 1222, who was designated to be "clericus", i.e. to have an ecclesiastic career, died in 1232 and buried in Royaumont (North of Paris). His tomb has been raised in the abbey-church chancel between October 1235 (consecration

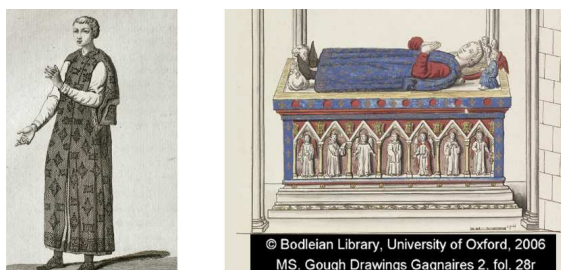


Figure 2: Philippe Dagobert - (left) Stained glass window drawing, (right) Drawing of the tomb with its recess.

date of this church) and middle XIIIth. He is not represented as a child here but as an idealised young man, laying down, opened eyes, joint hands; his head lay on two cushions each of them being held by two angels; his feet are resting upon a lying lion holding in its paws a leg of deer. With the French revolution this tomb encountered several dramatic events. It has been transferred in Saint-Denis in 1791, then damaged in 1793-94 and deserted till 1796 when, it has been preserved in the *Musée des monuments français*. In 1816 it returns to Saint-Denis, where it has been installed in the crypt; then the famous architect Viollet-le-Duc, between 1860 and 1867, inspired by the original pieces, rebuilt entirely the sarcophagus and installed it in the north transept. Concerning the paintings, Baron de Guilhermy published his observations in 1848 (de Guilhermy, 1848): "From the restitution of these tombs in Saint-Denis, the whole sculpture has been restored, and the old paintings disappeared under a new covering which after only thirty years were already eroded". It refers to painted work ordered by the architect François Debré in 1820. Luckily, half a century ago, Millin could study and draw the Philippe-Dagobert tomb in Royaumont as it was in 1790, just before it was transferred to Saint-Denis (Millin, 1791). Millin most likely describes the remnants of medieval paintings namely blue for the cappa magna sprinkled with golden squares and diamonds. This pattern is confirmed by a drawing made in 1694 for Roger de Gaignières, which represents a missing stained glass window in the Royaumont abbey-church with the young prince up and wearing the same clothes than his recumbent statue (left figure 2). As for the tomb, Gaignières made it drawn and painted with great precision. In the right part of figure 2 is exhibited a reproduction of the original coloured drawing conserved in the Bodleian Oxford Library. This research could allow to show once again how colour was present at the medieval epoch. As attested on the picture (Fig. 1) there are still some traces of paint and gilding on the stone at the contrary of the other graves gathered in the basilica. The robe is blue with red sleeves, he has yellow hair, and he

is wearing black breeches. His head rests on a red cushion on top of a green one. The angels' bodies are blue painted and their wings were gilded during the last restorations in XVIII and XIXth centuries. For discriminating all the superimposed layers of paints it was decided to make some takings for chemical analyses.

3 SCIENTIFIC METHODOLOGY AND TECHNICAL PROCESSES

Two kinds of data are necessary for the visual reconstruction of the statue. Shape and spectrophotometric informations are required. The first one is more frequently used in computer graphics while the second is generally replaced by a trichromatic description of the actual colors.

3.1 3D Shape Acquisition and Virtual Reconstruction

As for previous projects the 3D digitization has been performed *in situ* (in Saint-Denis Basilica) after public hours, without contact using an optical system based on structured light projector and a camera (Fig. 3) from Breukmann corp. A regular light and shadow grid is projected on the object and the distortion of the boundaries between light and shadows on the surface is recorded by the camera and analysed, in order to extract a cloud of 3D points with a good accuracy. This gave us 167 clouds of 3D points used for reconstructing the surface with RapidForm, a software from Inus Technology corp.

The final 3D shape of the tomb (recumbent statue on the sarcophagus) in full resolution represents about 7,5 Million triangles (Fig. 3).

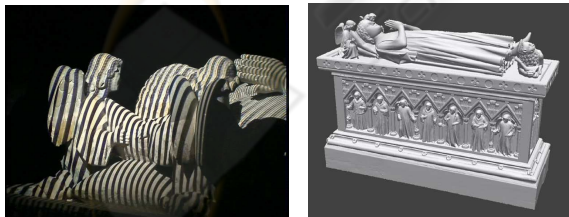


Figure 3: (left) 3D digitization using structured natural white light. (right) 3D colorless model displayed with Catia (Dassault System software).

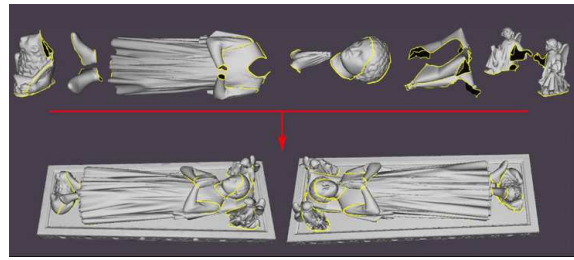


Figure 4: 3D model cut up not completely decomposed in color parts.

3.2 Spectrophotometric Data Acquisition

The portable spectrophotometer (USB 2000 series from Ocean Optics corp.) consists in a set of optical fibers guiding a calibrated light source (a halogen lamp for CIED65 illuminant) surrounding the backscattered light guiding fiber. The latest is connected to a spectrometer where a 1024 photodiodes array transforms the received light in a digital signal. We then obtain the diffuse reflectance factor of the paint according to the visible wavelengths domain (from 380nm to 780nm). Also *in situ*, we captured spectral information on the remaining traces of paint thanks to a spectrophotometer with optical fiber useful for further analyses and material characterization.

The portable spectrophotometer and the recorded reflectance factors are shown in Fig. 5.

As we need to compare and validate permanently our choices and models, we also used pigments and binder samples prepared in the laboratory. We also made our spectral measurements on these samples (Fig. 5).

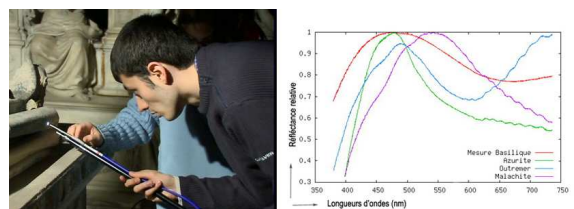


Figure 5: Spectral data acquisition *in situ*, reflectance factors layout and analysis.

3.3 Painting Materials

During the Middle Age and more particularly during the thirteenth century, we already know, (Escalopier, 2004) what pigments are the most commonly used and also how the paints were applied on the sculpture. First, the artists start with two or three layers of ceruse, made out of water and white lead. This



Figure 6: Medieval paint samples preparation.

step enables to waterproof the stone but also enables the painters to rectify the surface defects of the stone by coating. Then they apply several layers of paint, consisting of pigments embedded in a binder like egg (tempera technique), animal protein or gum-resin. The most frequently encountered pigments are:

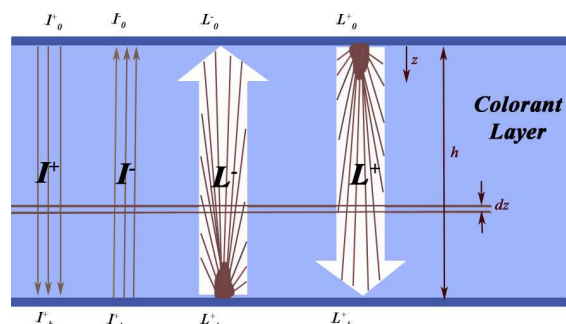
- White : White Lead, White Chalk, Lime;
- Black : Wood Smoke, Black Vine;
- Blue : Azurite, Lapis-Lazuli, Indigo;
- Red : Vermilion, Red Lead, Red Ochre;
- Yellow : Yellow Ochre, Massicot;
- Green : Green Clay, Malachite, Verdigris.

Which of them were used in the specific case of Philippe Dagobert's recombent statue? The recorded diffuse reflectance factors, obtained by the only possible analysis involving a non destructive method allow us to compare them to the results obtained by the chemical analysis for validation and simulation. We also need to specify which part of the global reconstructed shape would be associated with each kind of materials (pigments and binder or/and gils) (Fig. 4).

3.4 The Kubelka-Munk Model Extended to Four-fluxes Theory

Other works are related to spectral representation of colours (Sun et al., 2000), (Rougeron and Perocche, 1997), (Devlin et al., 2002), (Gondek et al., 1994), and pigmented materials in computer graphics (Haase and Meyer, 1992), (Baxter et al., 2004), using the Kubelka-Munk theory. Our first concern here was, to use an extended Kubelka-Munk model to four fluxes approach to better fit a modeling of paintings and to realize our calculations based on 81

wavelengths bands of 5nm over the visible spectrum [380;780]nm. Our second concern is in our entire process to constantly go back and forth between our theoretical models and our measurements on real materials. And above all, helped with our collaborations to apply these concepts in an archeological, and historical approach. The Kubelka-Munk model is almost well suited for the description of pigmented materials. These are described as a scattering medium, laying on a scattering background. The system is illuminated by a diffuse orthotropic incident light. It can be demonstrated (Volz and Teague, 2001; Callet and Zymala, 2004), that an incident orthotropic light flux on such a film of thickness h can be considered equivalent to a collimated directionnal and normal incident flux on a paint film of thickness $2h$. The model gives the reflected and the transmitted fluxes from an incident light, normal to the layers across a paint film of thickness h laid on a substrate. The plain Kubelka-Munk theory (Kubelka and Munk, 1931) is then reduced to an equivalent 2-flux theory involving two directional fluxes of light and, according to the previous remark above, one going downward L^+ and the other upward L^- . The layer of paint is a macroscopic scattering and an absorbing medium so we consider two coefficients: S and K the scattering and absorption ones. These last coefficients depend on the pigments diameters and consequently on the granulometry distribution function. A fine grinding involves an important multiple scattering inside the paint film and gives a desaturated colour obtained without any addition of white pigments. This is a physical whitening only. Further works will use the micro-stratigraphy images for extracting a best estimation of the granulometry distribution function. All the involved terms are wavelength dependent. The substrate has a reflectance factor R_g . Thus, we account for the normal and directional lighting on the outer layer. So the 2-fluxes model is improved with 2 additional fluxes

Figure 7: Four-Fluxes theory: the directional (I^+, I^-) and diffuse (L^+, L^-) fluxes, together with the boundary conditions.

of light l^- and l^+ normal to the interfaces. Including the two previous diffuse fluxes, one downward L^+ and the other upward L^- (as described in (Volz and Teague, 2001)). Figure 7 shows the principle of this model. The incident light is then decomposed in a remaining directional reflected light and of an additional scattered light due to volume and surface scattering from the substrate. Two specular components are then added to the classical Kubelka-Munk model. We operate a local radiative balance and write four equations, where k' , is the absorption coefficient for directional light, and respectively s^+ , the forward scattering coefficient, and s^- , the backward scattering coefficient.

$$dl^+ = -(k + s^+ + s^-)l^+ dz \quad (1)$$

$$-dl^- = -(k + s^+ + s^-)l^- dz \quad (2)$$

$$dL^+ = s^+l^+ dz + s^-l^- dz - (K + S)L^+ dz + SL^- dz \quad (3)$$

$$-dL^- = s^-l^+ dz + s^+l^- dz - (K + S)L^- dz + SL^+ dz \quad (4)$$

Setting l^+ and l^- equal to zero and solving the above system of equations, leads to the plain 2-fluxes Kubelka and Munk expressions of absorption coefficient K and scattering coefficient S . The measurements leads to :

- h : the layer thickness ;
- R_g : the background reflectance ;
- R_∞ : the reflectance for an infinite thickness (totally opaque layer) ;
- R : the layer reflectance (what we need).

We successively calculate: R_0 , the surface reflectance:

$$R_0 = \frac{R_\infty(R_g - R)}{R_g - R_\infty(1 - R_g R_\infty + R_g R)} \quad (5)$$

S , K the macroscopic scattering and absorption coefficients:

$$S = \frac{2.303}{h} \frac{R_\infty}{1 - R_\infty^2} \log \frac{R_\infty(1 - R_0 R_\infty)}{R_\infty - R_0} \quad (6)$$

$$K = \frac{2.303}{2h} \frac{1 - R_\infty}{1 + R_\infty} \log \frac{R_\infty(1 - R_0 R_\infty)}{R_\infty - R_0} \quad (7)$$

The determination of the paint characteristic film thickness h is made using the micro-stratigraphic images (Fig. 8). For the four-fluxes model we add the specification of all the terms k, s_i, s_j from the optical coefficients. Let :

$$a = 1 + \frac{K}{S}, \quad b = \sqrt{\frac{K}{S} \left(\frac{K}{S} + 2 \right)}$$

$$x = bhS, \quad A = asinh(x) + bcosh(x)$$

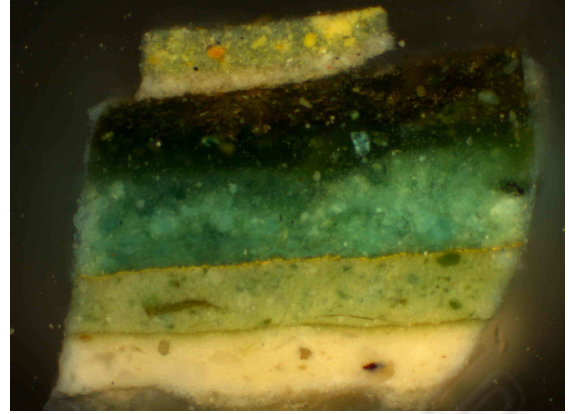


Figure 8: Microstratigraphy of a paint scale taken on the green bottom cushion. Each successive layer gives information about its mean thickness and the granulometric distribution of the embedded pigments. Magnification x20.

from which we deduce the following parameters

$$\tau = \frac{b}{A}, \quad \rho_1 = a - b, \quad \rho_2 = \frac{sinh(x)}{A}$$

We obtain from the previous relationships:

$$T_r = e^{-\mu h} = \left(\frac{R_r}{R_{r,0}} \right)^{\frac{1}{2}} = \left(\frac{R_r^*}{R_{r,0}} \right)^{\frac{h}{2hr}}$$

Then:

$$p = \frac{R_\infty(1 - \tau T_r) - R_b}{\rho_1(1 - \tau T_r) - \rho_2}$$

$$q = \rho_1 p - R_\infty$$

Next, inverting p and q equations leads to the scattering coefficients:

$$\left\{ \begin{array}{l} s_i = p(\mu - aS) + qS \\ s_j = pS - q(\mu + aS) \end{array} \right\}$$

where $\mu = \frac{1}{h} \ln \frac{1}{T_r}$

Finally we can deduce the absorption coefficient for a very thin layer:

$$k = \mu - (s_i + s_j)$$

These results were specially accurate for gilds: gold leaf polished above a colored paint film (the bole). The very small holes observed inside the gold leaf enrich the reflected spectrum with the diffuse component of the underlying paint. An other component of diffuse reflectance is due to the small cracks produced by the polishing process of the gold leaf. That influence has been recently shown(Dumazet et al., 2007).

3.5 Diffuse Reflectance Factors and Paints

We analyzed some paint samples made in laboratory (Fig. 6) and worked on the appearance of the paints

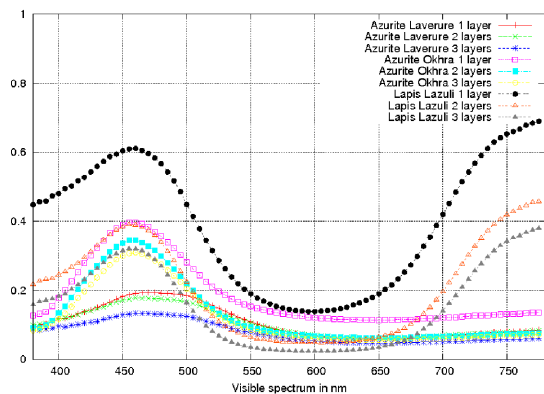


Figure 9: Reflectance factors of the blue pigments, azurite and lapis-lazuli depending on the paint film thickness.

depending on the thickness of each deposited film. We also measured the thickness of each sample. First, we notice the change in reflectance as the thickness increases. The thicker the layer the lower the reflectance. This means that the shade is darker as the thickness is increased, thus masking more and more the white substrate. For several pigments (green clay, malachite, black vine, vermilion, and lemon ochre), the reflectance factor for two and three layers tend to be quite similar, so we will consider that the diffuse reflectance factor for the three layers sample gives the term R_{∞} to determine the coefficients K and S of these pigments. Spectral measurements help in differentiating a lapis-lazuli from an azurite film (Fig. 9). The first one has two peaks, one in the blue wavelengths near 455nm and the other in the red and the near IR wavelengths region (780nm). Only one peak appears for the azurite pigments. Lapis-lazuli and its subtle reddish component is known as natural ultra-marine blue, very efficient for reflecting infra-red radiations (ideal paint used for the shutters in the mediterranean countries). We can also notice that the two pigments have different maxima in the blue region of the visible spectrum. One exhibits a relatively narrow peak while an other has a more flat and extended peak tending to cover the green shades region. Sometimes, a small amount of malachite in natural azurite can explain the reflectance curve aspect and also the greenish color observed on the sample. Now that we have the reflectance factors of the pure pigments, we want to compare them to the ones we measured at the basilica on the recumbent statue. We have identified some of the pigments used on the statue. It seems that the pigment used by the artist was identified as green clay, according to the reflectance curves. We have made this kind of comparison for each *in situ* reflectance factor. The blue areas seem to have been painted with azurite. The reflectance factor of the red cushion approaches that of the red lead. We can notice that the

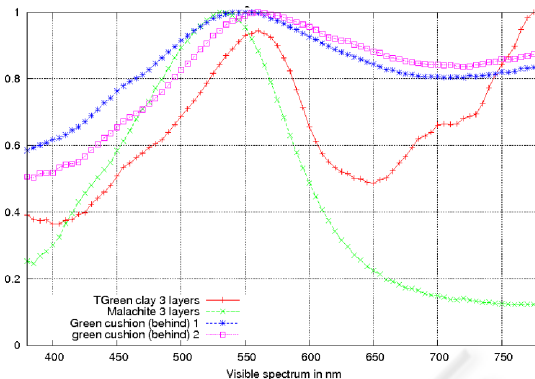


Figure 10: Comparison of the *in situ* measured diffuse reflectance factors with those of their corresponding hand-crafted samples.

recorded spectra in the basilica have a weaker amplitude between their maxima and minima than those of the samples. This phenomenon expresses the fact that the colors of the sculpture are now very de-saturated. There are several reasons for that. First, the paint layers are old and dirty. Then, the deposited dust on the sculpture surface increases the whitening by surface scattering of all incident light.

More studies were necessary to exactly determine the composition of each paint used to achieve this sculpture. More samples of different pigments have been elaborated. Therefore we virtually created mixtures of pigments and compared them to the *in situ* recorded reflectance factors, according to the robust color matching methods.

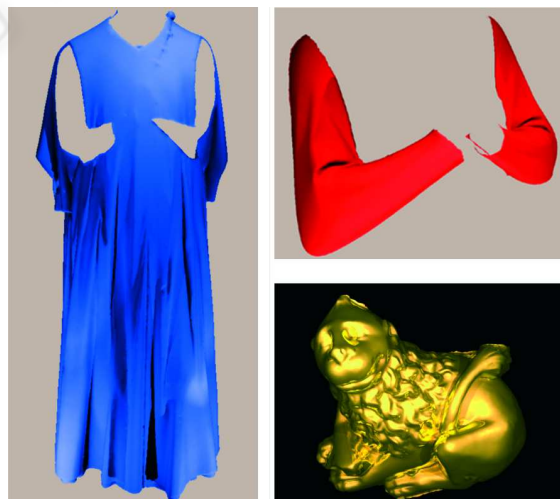


Figure 11: First results obtained with only spectrophotometric data and physical samples preparation. (left) Philippe-Dagobert's *cappa magna* rendered in lapis-lazuli blue pigment by the radiosity software Candelux. (right) Philippe-Dagobert's sleeves - vermilion pigment rendered with Candelux, opaque gold leaf covered lion rendered in ray-tracing with Virtuelium. CIE D65 illuminants.

3.6 Spectral Simulations

With a correct characterisation of all materials and their state of surface (waviness, roughness, paint-film thickness), the lighting conditions and a standard colorimetric observer (CIE 1964 10°), we can compute the restored visual aspect of the museum artifacts (Pitzalis et al., 2007). For paints simulations, including thin or thick metallic reflection the images are computed with *Virtuelium* our multithreaded ray-tracing software running generally over 81 wavelengths bands of 5 nm width, polarization of light, CIE standard illuminants and colorimetric observer. The 3D coordinates of the points are distributed over an octree structure, made to speed up the ray-surface intersections computations. We use the complex indices of refraction which characterise the intrinsic properties of all homogeneous materials. Optical constants are then, the real part and the imaginary part of the complex indices of refraction for the metals or alloys. These can be measured on real polished plates by *spectroscopic ellipsometry*, an optical method based on Fresnel formulas and the analysis of the amount of the reflected and polarized light at large incidence on a smooth surface. The whole theory of reflection of light by a metallic surface is available in optics books such as Born and Wolf's (Born and Wolf, 1975) and for computer graphics and lighting in (Callet, 2006; Callet, 2007) including quantitative data. According to the physical parameters extracted from microstratigraphy and spectrophotometric measurements, we have computed with the 4-fluxes model and *Virtuelium*, the image presented in Fig. 12. Thus, the successive layers with their thickness and concentration as plausible for the XIIIth century can appear. We have made many simulations with different standard illuminants and with characterized real light sources. Some parts of the sculpture as the prince's face or the angel head were probably painted with a cinnabar and vermilion mixture. No visible traces permitted to confirm that point and we decided to render the corresponding parts with the only white lead in a totally opaque layer. We computed the virtual restored aspect of the medieval sculpture with the following materials

- angel tunic: lapis-lazuli layer on azurite ;
- angel wings: 50 % red lead and 50 % cinnabar ;
- Philippe Dagobert hair: 200 nm thick gold leaf on yellow ochre substrate ;
- upper cushion: 50 % red lead and 50 % cinnabar ;
- lower cushion: not completely opaque layer of malachite pigments.



Figure 12: The actual most plausible colors of the medieval polychrome recumbent statue of Philippe Dagobert. Rendering in spectral ray-tracing by *Virtuelium* on an INTEL Pentium4 3GHz dual processors computer in 8 hours for a 1200x1200 resolution, 1Gb RAM and running under the operating system GNU/Linux - Fedora 5.

4 CONCLUSIONS

We went one step ahead in our knowledge and know-how of cultural heritage engineering and physical models for materials rendering in optical simulation. Till now we worked with standard illuminants such as CIE A, D65 or E illuminants. We shall need for these studies a better knowledge on natural lighting by stained glass windows and all other anthropogenic lightings used in the medieval era. For also improving the rendering of the complete sculpture, the canopy and all the gilded ornaments will be added to the 3D model along with photon mapping and texturing. The second part of that project will gather all the acquired new results and will also produce a new video movie translated in many languages.

ACKNOWLEDGEMENTS

In this pluridisciplinary project the authors were greatly helped by the following colleagues, in Ecole Centrale Paris by François-Xavier de Contencin for 3D digitization, Anna Zymla for material analysis, Philippe Denizet and Marie-France Monanges for video assisted by many students. Colleagues in Laboratoire de Recherche des Monuments Historiques, Vincent Detalle, Olivier Rolland and Annick

Texier worked on microstratigraphy, paint analysis and gilding. The Centre des Monuments Nationaux and the Saint-Denis basilica scientific and history team with Jacqueline Maillé, Georges Puchal, Robert Lequément, Alain Erlande Brandenburg, Françoise Perrot and Serge Santos. In Louvre museum, Pierre-Yves Le Pogam gave access to an essential and original colored piece while Nicolas Hueber at Ecole Nationale Supérieure d'Arts et Métiers realized the physical replica assisted by the sculptor Brigitte Bonnet. Gilles Raffier from AXIATEC facilitated the physical realization and sponsorship.

REFERENCES

- Baxter, W., Wendt, J., and Lin, M. C. (2004). Impasto: A realistic, interactive model for paint. In *In Proc. NPAR2004, the 3rd Intern. Symposium on Non-photorealistic Animation and Rendering*, pages 45–56.
- Born, M. and Wolf, E. (1975). *Principles of Optics—Electromagnetic Theory of Propagation, Interference and Diffraction of Light*. Pergamon Press, Oxford.
- Callet, P. (2006). Metals, alloys and lighting: optical properties and spectral simulation in computer graphics. In CIE, editor, *Proceedings of the CIE Expert Symposium on Visual Appearance*, CIE x032:2007. CIE. Held in Muséum National d'Histoire Naturelle, october 19-20th, Paris.
- Callet, P. (2007). Couleur et apparence visuelle ii : l'aspect métallique. In *Sciences fondamentales*, number AF3253 in AF, page 23. Techniques de l'Ingénieur.
- Callet, P. (mars 1998). *Couleur-lumière, couleur-matière – Interaction lumière-matière et synthèse d'images*. Collection Sciences en actes, ISBN : 2-84-352-087-8. Diderot Editeur, Arts et Sciences, Paris, New-York. CDROM d'images inclus, 320p.
- Callet, P. and Zymła, A. (2004). Rendering of binary alloys - example and validation with bronze. In team, I., editor, *Proceedings of ICCVG 04*. ICCVG.
- de Guilhermy, B. F. (1848). *Monographie de l'église royale de Saint-Denis. Tombeaux et figures historiques, Paris, p.240-244.* ” ”.
- Devlin, K., Chalmers, A., Wilkie, A., and Purgathofer, W. (2002). Tone reproduction and physically based spectral rendering. In *EUROGRAPHICS 2002, STAR - State of The Art Report*.
- Dumazet, S., Ariane Genty, a. A. Zymła., de Contencin, F.-X., Texier, A., Ruscassier, N., Bonnet, B., and Callet, P. (2007). Influence of the substrate colour on the visual appearance of gilded sculptures. In Ogleby, C., editor, *Proceedings of the XX1th CIPA conference, Athens*. ISPRS.
- Erlande-Brandenburg, A. (1975). *Le Roi est mort, Etude sur les funérailles, les sépultures et les tombeaux des rois de France jusqu'à la fin du XIIIème siècle, Paris*. Bibliothèque de la Société française d'archéologie,7.
- Escalopier, C. (2004). *Essai sur divers arts. Théophilus moine artisan du XIIe siècle. Texte intégral traduit du latin par Charles Escalopier*. Paleo.
- Gondek, J. S., Meyer, G. W., and Newman, J. G. (1994). Wavelength dependent reflectance functions. In *Proceedings of SIGGRAPH'94*, pages 213–220.
- Haase, C. S. and Meyer, G. W. (1992). Modeling pigmented materials for realistic image synthesis. In *ACM Transactions on Graphics*, pages 305–335.
- Kubelka, P. and Munk, F. (1931). Ein beitrag zur optik der farbanstriche. *Zeitschrift fr Technische Physik*, 12:593–601.
- Millin, A. L. (1791). *Antiquités nationales, ou Recueil de monuments pour servir à l'histoire, tome 2, Paris, p.10-12,pl.4,fig.2 et pl.5, fig.1*. Antiquités nationales.
- Pitzalis, D., Lahanier, C., Aitken, G., Pillay, R., rodriguez Echavarría, K., and Arnold, D. B. (2007). 3d techniques to create interactive virtual museums: the state of the art in the epoch noe. In José Braz, P.-P. V. and ao Madeiras Pereira, J., editors, *GRAPP 2007 – Second Conference on Computer Graphics Theory and Applications*, pages 197–203. INSTICC Press.
- Rougeron, G. and Peroche, B. (1997). An adaptive representation of spectral data for reflectance computations. In *Proceedings of the 8th Eurographics*.
- Sun, Y., Fracchia, F. D., and Drew, M. S. (2000). A composite spectral model and its applications, scottsdale, arizona. In *Eighth Color Imaging Conference*, pages 102–107.
- Volz, H. G. and Teague, B. (2001). *Industrial Color Testing: Fundamentals and Techniques (2nd edition) ISBN: 3527304363*. Vch Verlagsgesellschaft Mbh.